Effects of Magnetic Island Induced Symmetry Breaking on Plasma Confinement and Island Evolution in Tokamaks

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Abstract Magnetic islands exist in most tokamak discharges. The toroidal symmetry in |***B***| is broken when an island is embedded in the equilibrium magnetic field ***B*** in tokamaks. Plasma confinement properties in the vicinity of an island are different from those in the region away from the island. Physically, this is a result of the modifications on the plasma viscosity in the presence of the island. Interesting plasma confinement properties including, besides the usual particle and energy fluxes, the momentum transport and the bootstrap current, are derived from the island induced plasma viscosity. The consequence of the momentum transport process modifies plasma flow and the radial electric field in the vicinity of an m=1 island, and provides an explanation for plasma confinement improvement in snakes. Here, m is the poloidal mode number. The additional bootstrap current density induced by the presence of an island modifies the island evolution. It is found that island-induced bootstrap current has stabilizing influence on the island dynamics for $m > 1$ islands in high β_p plasmas. Here β_p is the ratio of plasma pressure to the poloidal magnetic field pressure.

1. Introduction

Magnetic islands exist in most tokamak discharges. The toroidal symmetry in |***B***| is broken when an island is embedded in the equilibrium magnetic field ***B*** in tokamaks [1]. Plasma confinement properties in the vicinity of an island are different from those in the region away from the island [2]. Interesting plasma confinement properties include, besides the usual particle and energy fluxes, the momentum transport and the bootstrap current. Here, we present a theory for plasma confinement improvement in snakes, and the effects of the island induced bootstrap current on island evolution.

A snake is a long lasting helical structure usually centered on the $q = 1$ surface with $m = n = 1$ observed in tokamak experiments [3], where q is the safety factor, m is the poloidal mode number, and n is the toroidal mode number. It is believed that this helical structure is a magnetic island. The particle confinement time in snakes can be of the order of the neoclassical value [4,5]. The theory for improved plasma confinement in snakes is developed based on the momentum transport process that results from the symmetry-breaking-induced plasma viscosity, and its consequences on the plasma confinement due to a combination of the turbulence suppression [6,7] and the effects of the orbit squeezing [8]. The mechanism is the same as the plasma confinement improvement in high confinement mode (H-mode) except that the momentum transport mechanism in a naturally occurred H-mode is dominated by the ion orbit loss [9], and in an electrode-induced H-mode by the electrode current [10]. The plasma flow speed and the radial electric field are calculated inside the island using plasma viscosity derived for a model suitable for an $m = 1$ island to illustrate plasma confinement improvement in snakes.

Besides improving plasma confinement in the vicinity of the island, symmetry breaking induced plasma viscosity also generates an additional bootstrap current density [11]. This island induced bootstrap current density has different collision frequency dependence from that of the standard neoclassical tokamak bootstrap current, and it modifies the island evolution equation. A theory is developed to include this island induced bootstrap current density in the island evolution equation for $m > 1$ islands. It is found that the island induced bootstrap current density has stabilizing influence on island stability for high poloidal beta β_p plasmas by imposing a lower limit on the absolute value of the tearing mode stability parameter $|\Delta'|$ for instability [12]. Here, β_p is the ratio of the plasma pressure to the poloidal magnetic field pressure. This may provide an alternative route to stabilize the islands.

We have developed a theory to describe improved plasma confinement in snakes in tokamak discharges, and a theory to include the effects of the island induced bootstrap current on island evolution. These theories can be tested in experiments. They can all be included in the modeling codes to simulate transport phenomena in existing tokamak experiments.

2. Plasma Confinement and Momentum Transport in Snakes

The variation of $|\boldsymbol{B}|$ on an $m = 1$ island can be approximated by

$$B / B_0 = 1 - \Delta \cos\theta, \tag{1}$$

where $\Delta = (r_s/R) - (\xi/R) \{\cos\alpha \pm [\cos^2\alpha + 2\Psi/(\xi^2\Psi_0'')]^{1/2}\}$, $B = |\boldsymbol{B}|$, B_0 is the B on the magnetic axis, θ is the poloidal angle, r_s is the minor radius r evaluated at the rational surface, R is the major radius, ξ is the plasma displacement inside the $q = 1$ surface, $\alpha = \theta - \zeta + \omega t$, ζ is the toroidal angle, ω is the island rotation frequency, Ψ is the helical flux function, $\Psi_0'' = -(B_0 r_s q_s'/R)$, and prime here denotes d/dr.

The electrostatic potential Φ satisfies [13]

$$\Phi = -(\omega q / mc)(\psi - \psi_S) + F(\Psi), \tag{2}$$

where c is the speed of light, ψ_S is the poloidal flux function ψ evaluated at the resonant surface, and $F(\Psi)$ is an integration constant. Here, $F(\Psi)$ plays the same role as the electrostatic potential that is constant on the flux surface in the equilibrium. The function $F(\Psi)$ has the form [13]

$$F(\Psi) = (q/mc)(\omega - \omega_{E0}) H(\Psi), \tag{3}$$

where $\omega_{E0} = -mc\Phi_0'/q$, $H(\Psi) \rightarrow (\psi - \psi_S)$ far away from the island when $x > 0$, and $x = r - r_s$.

The island rotating frequency can be determined from the parallel current density that results from the symmetry breaking induced viscosity driven perpendicular current and has the following form, when the ion viscosity dominates,

$$(\omega - \omega_{E0}) = -\omega_{*pi} - \lambda_i \omega_{*Ti.}, \tag{4}$$

and, when the electron plasma viscosity dominates,

$$(\omega - \omega_{E0}) = -\omega_{*pe} - \lambda_e \omega_{*Te}, \tag{5}$$

where the subscript j denotes plasma species, $\omega_{*pj} = (mcT/e_j q)(p_{0j}'/p_j)$, $\omega_{*Tj} = (mcT/e_j q)(T_{0j}'/T_j)$, the pressure and the temperature gradients relative to the equilibrium poloidal flux prior to the appearance of the island are denoted by p_{0j}' and T_{0j}' respectively, and p_j and T_j are the pressure and the temperature. The parameters λ_j in Eqs.(4) and (5) depend on the plasma collision frequency, and in general they are a function of plasma collisionality as shown in Refs. [2].

The radial electric field in the vicinity of the island in the low collision frequency regimes is determined from the symmetry breaking induced plasma viscosity. In the 1/ν regime, ion particle flux dominates that of the electrons and the radial electric field is approximately

$$e_i \Phi'/T_i \approx - p_i'/p_i - 2.367\, T_i'/T_i\,, \qquad (6)$$

where e_j is the electric charge of the species j. Deep in the ν regime, electron particle flux dominates, and the radial electric field in that limit is approximately

$$e_e \Phi'/T_e \approx - p_e'/p_e + 0.397\, T_e'/T_e\,. \qquad (7)$$

Thus, the sign of the radial electric field can reverse when the collision frequency decreases, a well-known fact in the stellarator transport theory.

Inside the island, the plasma pressure and temperature profiles are usually flat due to the rapid plasma transport along the magnetic field line. However, if the island is fueled by the pellet injection, the density gradient inside the island can be steep. For a 10keV Deuteron plasma in a 2T magnetic field, the banana width is about 10cm if the local aspect ratio is about 10 at $q = 1$ surface. The width of the snakes observed in Joint European Tokamak (JET) has a radial extent about 17cm [4]. Thus, the orbit squeezing factor S, defined as $|S| = |\, 1+ (I/\Omega_i)^2(e_i\Phi''/M_i)|$ [8], with the double prime denoting the second derivative in Ψ, is about 3 or higher. This leads to reduced neoclassical ion heat transport. Because the radial gradients of the $E \times B$ and the diamagnetic angular drift frequency inside the island are similar to those of the plasmas in the edge region of the high-confinement mode (H-mode), they suppress the turbulence fluctuations there. Thus, the plasma confinement inside the island can be improved to the modified neoclassical level, *i.e.*, the level calculated from the neoclassical theory with the orbit squeezing effects included. This mechanism is very similar to that in the H-mode theory [9], and is the same as the mechanism in the theory for the confinement improvement in the vicinity of the low order rational surface proposed in Refs. [14]. The difference is only in the mechanism that determines the radial electric field. In the H-mode, it is the ion orbit loss that determines the radial electric field. Inside the island, it is the symmetry breaking induced toroidal plasma viscosity. The theory proposed here can be tested in experiments quantitatively by measuring the radial electric field profile in the vicinity of an $m = 1$ island.

3. Island Induced Bootstrap Current on Island Evolution

Magnetic islands with finite width degrade plasma performance in high temperature plasma confinement devices such as tokamaks. In the collisionless banana regime, the primary destabilization mechanism is the equilibrium bootstrap current density modified by the presence of the island [15,16]. The simplest version of the theory [15,16] predicts that the magnetic island is unstable for any values of the tearing mode stability parameter $\Delta' < 0$. It is later found that the finite value of the heat conductivity along the magnetic field line has a stabilizing effect on islands with small width where the perpendicular heat conduction (to magnetic field $\boldsymbol{B}$) dominates the parallel heat equilibration to prevent the plasma profiles inside the island from flattening [17,18]. It has also been shown that polarization current density can have the stabilization influence if the island rotates in the direction of the ion diamagnetic flow [13,19]. Besides these effects, there are other effects that can influence the island evolution [20,21].

Here, we present a new physics mechanism that can have effects on the island dynamics by introducing the island induced bootstrap current density in the derivation of the island evolution equation. For simplicity, we neglect all other effects except the perturbed Ohmic current density, the island modified equilibrium bootstrap current density, and the island induced bootstrap current density in the discussions. The key difference between the island modified equilibrium bootstrap current density and the island induced bootstrap current density is that the former remains, while the latter disappears, when the width of the island vanishes. Of course, the perturbed Ohmic current density contribution to the island evolution equation is due to Rutherford, and is independent of the width of the island [22]. The island modified equilibrium bootstrap current density contribution is the key ingredient in the neoclassical tearing mode, and is inversely proportional to the width of the island [15,16]. We find that the island induced bootstrap current density modifies the island evolution equation by introducing a term that is proportional to the width of the island r_w [12],

$$\frac{\partial r_w}{\partial t} = 0.43\frac{\eta c^2}{4\pi}G(\varepsilon,\nu_{*e})[\Delta' - \frac{\beta_p}{\pi}\frac{q_s}{q_s'}\frac{\sqrt{\varepsilon}}{r_w}\frac{1}{p_{to}}\frac{\partial p_{to}}{\partial r}(8.1F + \frac{0.434}{\nu_{*e}}\frac{r_w^2}{r^2})], \qquad (8)$$

and the corresponding time scale is exponential for canonical tokamak safety factor q profiles, and pressure profiles. The modified island evolution equation is structurally similar to that in the quasi-linear theory [23]. Thus, the steady state equation for the island width becomes quadratic instead of linear in the width of the island when the island induced bootstrap current density is included. Examining the solution of the steady state island evolution equation for the width of the island, it is found that there is a lower limit on the absolute value of Δ' for the island to be unstable:

$$|\Delta'| > |\Delta_c'| = \frac{1.2}{\sqrt{\nu_{*e}}}\sqrt{F(\varepsilon,\nu_{*e})}\,\frac{q_s}{rq_s'}\sqrt{\varepsilon}\,\beta_p\left|\frac{dp_{t0}/dr}{p_{t0}}\right|. \qquad (9)$$

The notations in Eqs.[8,9] are: the form factor $G(\varepsilon,\nu_{*e})$ is to take into account the effects of the finite aspect ratio, and finite values of ν_{*e} on plasma resistivity η [24], ν_{*e} is the electron collisionality parameter [24,25], q_s is the safety factor at the mode rational surface where $m = nq_s$, m is the poloidal mode number of the island, n is the toroidal mode number, $q_s' = dq/dr$ evaluated at rational surface, $\varepsilon = r/R$ is the inverse aspect ratio, R is the major radius of the tokamaks, β_p is the ratio of the plasma pressure to the poloidal magnetic field pressure, r is the local radius, and p_{t0} is the total plasma pressure. The form factor $F(\varepsilon,\nu_{*e})$ in Eq.(1) is 1 in the limit of $\varepsilon \to 0$, and is 1/2.4 when ε=1 if $\nu_{*e} \ll 1$ [26]. It is used to take into account the effects of finite values of the aspect ratio, and ν_{*e} on the equilibrium bootstrap current density. For arbitrary value of ε, the value of F can be found in [26]. The dependence of F on ν_{*e} can be found in Ref .[24]. For $|\Delta'| < |\Delta_c'|$, there is no real solution for the width of the island, and thus the island is stable. This lower limit on $|\Delta'|$ depends on the local value of β_p. If β_p is high enough, namely, the inequality sign in Eq.(8) is reversed, the magnetic island can be stabilized. Thus, not all the modes with negative values of Δ' are unstable. This may provide an explanation as to why an m = 2 island is not as commonly observed in tokamak experiments as, *e.g.*, m = 3, 4, or 5 island [27]. The theory also points to alternative routes to stabilize the islands by increasing local β_p, or by tailoring the current density profile to modify Δ'.

Note that we only consider the even (relative to the mode rational surface) component of the island induced bootstrap current density here. The odd (relative to the mode rational surface) component of the island induced bootstrap current density modifies Δ' nonlinearly. We will address this effect together with the effects of the equilibrium current density gradient, and its curvature [28,29] on the island evolution separately.

4. Concluding Remarks

We have shown that the broken toroidal symmetry on $|\boldsymbol{B}|$ when an island is imbedded in a tokamak can have an important impact on the plasma confinement in the vicinity of the island and on the island evolution itself. The plasma confinement improvement in the vicinity of an m = 1 island provides an explanation for the extreme good plasma confinement in snakes observed in the experiments. We also show that an additional bootstrap current density is induced by the presence of the island. This island induced bootstrap current density modifies the island evolution equation and provides a stabilization mechanism in high β_p plasmas. This mechanism may lead to a different way to stabilize the island.

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References

[1] SHAING, K. C., Phys. Rev. Lett. **87**, 245003 (2001).
[2] SHAING, K. C., Phys. Plasmas **10**, 4728 (2003).
[3] WELLER, A., et al., Phys. Rev. Lett. **59**, 2303 (1987).
[4] GILL, R. D., et al., Nucl. Fusion **32**, 723 (1992).
[5] WESSON, J. A., Plasma Phys. Control. Fusion **37**, A337 (1995).
[6]] SHAING, K. C., et al., in *Plasma Physics and Controlled Nuclear Fusion Research*, (International Atomic Energy Agency, Vienna, 1989), Vol. II, p13.
[7] SHAING, K. C., CRUME, E. C. Jr., HOULBERG, W. A., Phys. Fluids B **2**, 1496 (1990).
[8] BERK, H. L., GALEEV, A. A., Phys. Fluids **10**, 441 (1967).[8]
[9] SHAING, K. C., CRUME, E. C., Jr., Phys. Rev. Lett. **63**, 2389 (1989).
[10] TAYLOR, R. J., et al., Phys. Rev. Lett. **63**, 2365 (1989).
[11] SHAING, K. C., Phys. Plasmas **12**, 072523 (2005).
[12] SHAING, K. C., SPONG, D. A., Phys. Plasmas **13**, 022501 (2006).
[13] SMOLYAKOV, A. I., Plasma Phys. Control. Fusion **35**, 657 (1993), and references therein.
[14] SHAING, K. C., Phys. Plasmas **9**, 3470 2002).
[15] CARRERA, R., et al., Phys. Fluids **29**, 899 (1986).
[16] CALLEN, J. D., et al., in *Plasma Physics and Controlled Nuclear Fusion Research* 1986, Kyoto (International Atomic Energy Agency, Vienna, 1987) Vol. 2, p. 157.
[17] FITZPATRICK, R., Phys. Plasmas **2**, 852 (1995).
[18] GORELENKOV, N. N., et al., Phys. Plasmas **3**, 3379 (1996).

[19] CONNOR, J. W., et al., Phys. Plasmas **8**, 2835 (2001).
[20] KOTSCHENREUTHER, M., et al., Phys. Fluids **28**, 294 (1985)
[21] GLASSER, A. H., et al., Phys. Fluids **18**, 875 (1975).
[22] RUTHERFORD, P. H., Phys. Fluids **16**, 1903 (1973).
[23] WHITE, R. B., et al., Phys. Fluids **20**, 800 (1977).
[24] HINTON, F. L., HAZELTINE, R. D., Rev. Mod. Phys. **48**, 239 (1976).
[25] HIRSHMAN, S. P., SIGMAR, D. J., Nucl. Fusion **21**, 1079 (1981).
[26] HIRSHMAN, S. P., Phys. Fluids **31**, 3150 (1988).
[26] SHAING, K. C., Phys. Plasmas **9**, 4633 (2002).
[27] FREDRICKSON, E. D., Phys. Plasmas **9**, 548 (2002).
[28] ESCANDE, D. F., OTTAVIANI, M., Phys. Lett. A, **323**, 278 (2004).
[29] MILITELLO, F., PORCELLI, F., Phys. Plasmas **11**, L13 (2004).