Interaction of Drift-Tearing (Mesoscopic) Modes with Coherent and Turbulent Microscopic Structures*

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Abstract. Magnetic reconnection in high temperature regimes is an important issue to deal with given that it is observed in existing advanced experiments under conditions not predicted by the original theories. The need to explain these experiments and to attain predictive capabilities requires a more advanced theoretical formulation involving the excitation of mesoscopic modes that produce magnetic reconnection in the presence of background microscopic modes. These can defeat the stabilizing effect of electron Landau damping or longitudinal electron thermal conductivity by increasing the ratio of the transverse to the longitudinal thermal conductivity in high temperature regimes. A new “Neo-Drift-Tearing” mode capable of producing significant magnetic islands in the presence of electron temperature gradients in these regimes is found.

The problem of understanding the interaction of reconnecting drift-tearing and tearing modes in the presence of stochasticity or turbulence is important and has been motivated both by the search for ’fast reconnection’ and by the need for predictive understanding of observed reconnecting modes and NTMs, which intrinsically couple macro-scale reconnection to micro-scale turbulence or structure. Drift-tearing and tearing modes involve both electron and ion dynamics, so their minimal description requires at least a (generalized) Ohms law and a (generalized) vorticity equation. In this paper we investigate the effects of stochasticity-enhanced electron transport and scattering on the excitation of drift-tearing modes (Part 1), and of the coupling to the inverse energy cascade generic to drift wave turbulence on the evolution of the tearing instability (Part 2).

I-1. Introduction and Resolution of the Drift-Tearing Mode Dilemma. (M.I.T. Contribution)

Magnetic reconnection in collisionless and weakly collisional regimes is an important issue both for space and laboratory plasmas, and the advent of systematic experiments on well confined plasmas with low degrees of collisionality has cast new light on the conditions under which reconnection can take place. The drift tearing mode [1] involves the combined effects of magnetic reconnection and electron density and temperature gradients, has an intrinsic frequency of oscillation related to these gradients, is characterized by both a slower growth rate and significantly different eigenfunctions as compared to the purely resistive tearing mode, and is strongly connected with the transport of electron thermal energy. This mode, that can lead to the formation of relatively large magnetic islands, has the same excitation threshold as the conventional resistive tearing mode, as long as the relevant electron equation of state is adiabatic, but it was found to be stable [2] in collisionless regimes as a result of the combined effects of electron Landau damping and temperature gradient [2] or of the parallel longitudinal electron conductivity and temperature gradient [3]. On the other hand, as experiments with lower degrees of

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collisionality have been undertaken [4] modes of this kind, that produce magnetic reconnection, have been observed to persist together with the formation of the resulting magnetic islands.

As a realistic resolution of this paradox we consider that drift-tearing modes develop from a background of "micro-reconnecting" modes with short scale distances of the order of c/ω_{pe} that generate a series of strings of magnetic islands and are driven by the transverse electron temperature gradient. The effects of these modes are to produce a depression of the thermal conductivity along the field lines and a significant transverse thermal conductivity. The combination of both effects is shown to restore the excitation of the drift-tearing mode and lead to the emergence of a different kind of mode involving the combined effects of finite resistivity, electron thermal conductivities, and temperature gradients. This "Neo-Drift-Tearing" mode can be excited for negligible or slightly negative values of $\Delta'/|k_\perp|$ and can be related to the formation of slowly evolving magnetic islands in high temperature regimes. Here $|k_\perp| = m^0/r_0$ is the transverse mode number and Δ' is the well known driving factor of tearing modes.

I-2. "Micro-reconnecting" and "Neo-Drift-Tearing" Modes

In view of extending our results to more complex configurations we refer to a plane geometry where the magnetic field around a surface $x = x_0$ is represented as $\boldsymbol{B} \simeq B_z(x)\boldsymbol{e}_z + (x - x_0)B'_y\boldsymbol{e}_y$. The drift-tearing mode that we consider is characterized by a transition layer localized around $x - x_0$. This mode is assumed to develop from a background of modes that produce magnetic islands on smaller scales and are localized over distances of the order of c/ω_{pe} around successive surfaces $x = x_j$ contained in a finite interval centered on $x = x_0$ and its excitation is shown to depend critically on the ratio $\Delta_{th} \equiv D_\perp^{th}/D_\parallel^{th}$ where $D_\perp^{th}$ and $D_\parallel^{th}$ are the transverse and longitudinal background thermal conductivities, respectively.

The effective transverse thermal diffusion coefficient that we associate with the micro-reconnecting modes, described in Section 3, is of the order of $D_\perp^{th} \sim \alpha_T(d_e/r_{Te})cT_e/(eB)$ where α_T is a finite numerical coefficient, $d_e = c/\omega_{pe}$, and $1/r_{Te} \equiv -\mathrm{d}\log \mathrm{T_e}/\mathrm{d}x$ and we consider the ratio Δ_{th} to be increased further relative to the classical value by the simultaneous reduction of $D_\parallel^{th}$ resulting from the excitation of the same modes. Thus, a possible scaling for Δ_{th} could be $\Delta_{th} \sim (\rho_e/r_{Te})(d_e/r_{Te})^{1/2}$.

The perturbed magnetic field, for the drift tearing mode, is represented by $\hat{\boldsymbol{B}} = \tilde{\boldsymbol{B}}\exp(-i\omega t + ik_y y)$ and the relevant longitudinal momentum conservation equation is

$$0 \simeq -\nu_{ei}^{\parallel} n m_e \hat{u}_{e\parallel} - en\hat{E}_\parallel - \left[\widehat{\boldsymbol{B}\cdot\boldsymbol{\nabla} p}_e + \alpha_T n \widehat{\boldsymbol{B}\cdot\boldsymbol{\nabla} T}_e\right]/B \qquad \text{(I-1)}$$

where $\alpha_T \simeq 0.7$. The adopted electron thermal energy balance equation is

$$\frac{3}{2}n\left(\frac{\partial\hat{T}_e}{\partial t} + \hat{V}_{Ex}\frac{\partial T_e}{\partial x}\right) - T_e\left(\frac{\partial\hat{n}_e}{\partial t} + \hat{V}_{Ex}\frac{\partial n}{\partial x}\right) \simeq \boldsymbol{\nabla}\cdot\left(\widehat{\frac{\boldsymbol{B}}{B}q_{e\parallel}}\right) + \boldsymbol{\nabla}_\perp\cdot\hat{\boldsymbol{q}}_{e\perp} \qquad \text{(I-2)}$$

where $\hat{V}_{Ex} \equiv c\hat{E}_y/B \simeq -ick_y\hat{\Phi}/B$ and we define $\hat{\xi}_x \equiv i\hat{V}_{Ex}/\omega$. Moreover $\boldsymbol{\nabla}_\perp\cdot\hat{\boldsymbol{q}}_{e\perp} \simeq D_\perp\partial^2\hat{T}_e/\partial x^2$, $\boldsymbol{\nabla}\cdot(\boldsymbol{B}\hat{q}_{e\parallel}/B) \simeq ik_\parallel\left\{-D_\parallel\left[ik_\parallel\hat{T}_e + (\hat{B}_x/B)(\mathrm{d}T_e)/\mathrm{d}x)\right]\right\}$ and $k_\parallel \simeq k_yB'_y(x - x_0)/B \equiv k_y(x-x_0)/L_s$.

The drift-tearing mode was found originally in the (adiabatic) limit where the thermal conductivities are not important while for the regimes we consider the relevant terms are prevalent in Eq. (I-2). Thus the profile $\tilde{T}_e(x)$ in the transition region is given by $(\widehat{\boldsymbol{B}\cdot\boldsymbol{\nabla} q}_{e\parallel}) + B\boldsymbol{\nabla}_\perp\cdot\hat{\boldsymbol{q}}_{e\perp} \simeq 0$

that is by the inhomogeneous equation

$$D_\perp \frac{\mathrm{d}^2\tilde{T}_e(x)}{\mathrm{d}x^2} - D_\parallel \frac{k^2}{L_S^2} x^2 \tilde{T}_e(x) \simeq -\frac{\mathrm{d}T_e}{\mathrm{d}x} i k_\parallel \frac{\tilde{B}_x}{B} D_\parallel \tag{I-3}$$

where $\tilde{B}_x \simeq \tilde{B}_{x0} = \mathrm{const}$. Thus the width of this region is of the order of

$$\delta_T = \left(\frac{L_s}{k}\right)^{1/2} (\Delta_{th})^{1/4} < \frac{1}{k} \sim a \tag{I-4}$$

where a represents the plasma radius. The mode frequency is $\omega \simeq \omega_o^o + \delta\omega$ where $\omega_o^o \simeq \omega_{\parallel e}^T \equiv -(k_y c)/(eB)[(dp_e/dx)/n + \alpha_T dT_e/dx]$ and $|\delta\omega| \ll \omega_o^o$. We note that for $|x| > \delta_T$ we have $\tilde{T}_e/(\mathrm{d}T_e/\mathrm{d}x) \simeq i\tilde{B}_x/(\boldsymbol{k}\cdot\boldsymbol{B}) \simeq -\tilde{\xi}_x$ if we consider that in the outer region $\tilde{B}_x \simeq i(\boldsymbol{k}\cdot\boldsymbol{B})\tilde{\xi}_x$.

Clearly Eq. (I-3) is valid for $D_\perp/\delta_T^2 > \omega_0$. The relevant theory [5] is characterized by another small parameter that is

$$\epsilon_I \equiv \frac{D_m \omega_0(\omega_0 - \omega_{di})}{V_A^2 |\omega_{**}^T| \Delta_{th}} \equiv \frac{\epsilon_*}{\Delta_{th}} \tag{I-5}$$

where $\omega_{**}^T \equiv -k_y c(\mathrm{d}T_e/\mathrm{d}x)/(eB)(1+\alpha_T)$, $\omega_{di} \equiv k_y c(dp_i/dx)/(enB)$, and by an inner-most asymptotic region to be considered whose width is of the order of

$$\delta_c \equiv \epsilon_*^{1/4} (L_s/k)^{1/2} \tag{I-6}$$

Whithin this region the function $\tilde{\mathcal{L}} \equiv \tilde{\xi}_x + \tilde{T}_e/T'$ has a peak such that $|\tilde{\mathcal{L}}|/|\tilde{\xi}_x| \sim \epsilon_I^{3/4}$ while outside this region, within δ_T, $|\tilde{\mathcal{L}}|/|\tilde{\xi}_x| \sim \epsilon_I$. The condition $\delta_c < \delta_T$ that characterizes the considered asymptotic regime gives a transition value for Δ_{th}

$$\frac{D_\perp}{D_\parallel} \sim \epsilon_* \tag{I-7}$$

that is relatively low. Then, following the same procedure as that indicated in Ref. [1], we find a growth rate that has two components. One related to the plasma current density gradient in the outer region represented by $\Delta' \geq 0$ and the other, independent of Δ', related to the electron temperature gradient in the transition region, that is $\gamma = \gamma_1(\Delta') + \gamma_2(|\omega_{**}^T|)$ where $\gamma_1 \sim (k\Delta') D_m^{3/4} [V_A^2 \Delta_{th}/\omega_0]^{1/4}$, $\gamma_2 \sim |\omega_{**}^T|^{1/4} D_m^{3/4} [\omega_o/V_A^2]^{3/4} \Delta_{th}^{-3/4}$, and $\omega_{**}^T \sim \omega_0$. Thus $\gamma_1/\gamma_2 \sim \Delta_{th}(k\Delta')V_A^2/\omega_0^2$ and this, clearly, increases with $D_\perp/D_\parallel$.

I-3. Properties of Micro-Reconnecting Modes.

These modes are electromagnetic [6] in the sense that the perturbed longitudinal electric field $\hat{E}_\parallel$ has both $\nabla_\parallel \hat{\phi}$ and $(1/c)\partial \hat{A}_\parallel/\partial t$ as significant components. In particular we take $\hat{A}_\parallel = \tilde{A}_\parallel(\Delta x_j)\exp(-i\omega t + i k_y y)$ where $\Delta x_j \equiv x - x_j$ and $\tilde{A}_\parallel(\Delta x_j)$ is an even function of Δx_j. This parity is associated with breaking the frozen-in law by the effects of finite electron inertia. Only $T_{e\parallel}$ is involved in these modes as we consider $k_\perp^2 \rho_e^2 \ll k_\perp^2 d_e^2 \sim 1$ where $k_\perp^2 = k_y^2 - \partial^2/\partial x^2$. Moreover $\omega^2 \gtrsim k_\parallel^2 v_{the\parallel}^2$ for $v_{the\parallel}^2 = 2T_{e\parallel}/m_e$ and $\omega^2 < k_\perp^2 v_{thi\perp}^2$ where $v_{thi\perp}^2 = 2T_{i\perp}/m_i$ and $k_\perp^2 \rho_i^2 \gg 1$. Here ρ_e and ρ_i indicate the electron and the ion gyroradius, respectively. We note that $\omega = \omega_R + i\omega_I$ and that the mode frequency depends strongly on the values of $\eta_e = (\mathrm{d}\log T_{e\parallel}/\mathrm{d}x)/(\mathrm{d}\log n/\mathrm{d}x)$ in the sense that ω_R/ω_I is finite for $\eta_e \gg 1$ and can vanish if $\eta_e > 1$ but not too large.

Depending on the value of $\bar{k}_0 \equiv (k_y d_e)^{2/3} C^{1/6}$ where $C \equiv 2T_i r_{Te}^2/(\beta_e T_e L_s^2)$ we can indentify three subclasses of modes: i) marginally stable modes for $\bar{k}_0 \lesssim 0.78$, for which a representative

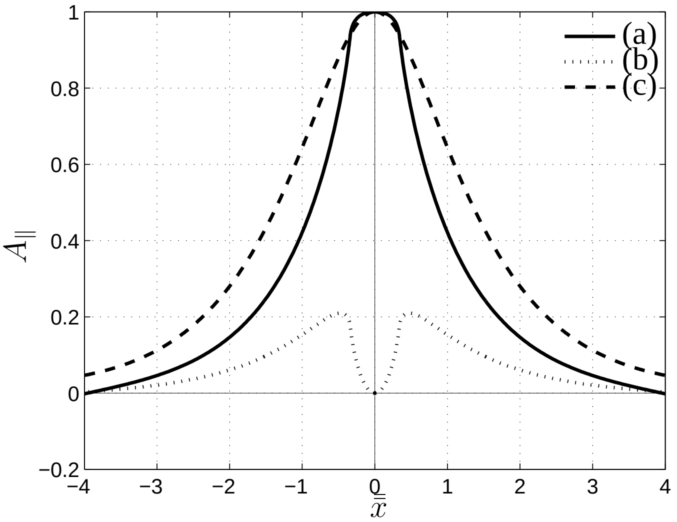

Fig. 1: Curves (a) and (b) represent an unstable mode with $\bar{k}_0 = 0.8745$ *and where* $\bar{\omega} \simeq 0.246 + 0.418i$, $\bar{\omega} \equiv \bar{k}_0^{-1/2} C^{-1/4} (L_s/c_{se})\omega$, $\bar{k}_0 \equiv (k_y d_e)^{2/3} C^{1/6}$, *and* $\bar{\bar{x}} \equiv \bar{k}_0^{3/2} C^{-1/4} \Delta x_j/d_e$. *Curve (a) is the real part and curve (b) is the imaginary part of* $\tilde{A}_{\|}$. *Curve (c) represents a stable mode with* $\bar{k}_0 = 0.75$, *where* $\bar{\omega} \simeq 0.831$ *and* $\tilde{A}_{\|}$ *is real.*

eigenfunction is given in Fig. *1* as a function of $\bar{\bar{x}}$ where $\bar{\bar{x}} \equiv \bar{k}_0^{3/2} C^{-1/4} \Delta x_j/d_e$., ii) unstable regular modes for $0.78 \gtrsim \bar{k}_0 \lesssim 0.88$ whose kind of eigenfunction is represented in Fig. *1*, and iii) weakly singular modes for $\bar{k}_0 \gtrsim 0.88$. The equation for $\tilde{A}_{\|}$ that has been solved is, in the limit where $kd_e > 1$, $\omega < \omega_{*Te}$, and $\omega^2 > k_{\|}^2 v_{the}^2$,

$$\left[\omega^3 + \omega_{*Te} k_y^2 c_{se}^2 \frac{(\Delta x_j)^2}{L_s^2}\right] \left(d_e^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2 d_e^2\right) \tilde{A}_{\|} + \omega^2 \omega_{*Te} \tilde{A}_{\|} \simeq 0 \qquad \text{(I-8)}$$

where $\omega_{*Te} \equiv -k_y c/(eB) \mathrm{d}T_{e\|}/\mathrm{d}x$ is the frequency representing the mode driving factor and $c_{se} = (T_i/m_e)^{1/2}$ is the "electron sound" velocity. Weakly singular modes correspond to $\omega^3 = -\omega_0^3$, where ω_0/ω_{*Te} is real and positive. From this we can derive the following quadratic form

$$\left\langle \bar{\bar{x}}^2 \left| \left(\frac{\mathrm{d}^2}{\mathrm{d}\bar{\bar{x}}^2} - 1\right) \tilde{A}_{\|} \right|^2 \right\rangle + \bar{\omega}^3 \left\langle \left| \left(\frac{\mathrm{d}^2}{\mathrm{d}\bar{\bar{x}}^2} - 1\right) \tilde{A}_{\|} \right|^2 \right\rangle - \frac{\bar{\omega}^2}{\bar{k}_0^2} \left\langle \left| \frac{\mathrm{d}\tilde{A}_{\|}}{\mathrm{d}\bar{\bar{x}}} \right|^2 + \left| \tilde{A}_{\|} \right|^2 \right\rangle = 0 \qquad \text{(I-9)}$$

where $\langle\ \rangle$ indicates integration over $\bar{\bar{x}}$, and $\bar{\omega} \equiv \bar{k}_0^{-1/2} C^{-1/4} (L_s/c_{se})\omega$.

The string of magnetic islands produced by this mode around the surface $x = x_j$ is represented by the magnetic surface function given by

$$\Psi_j \propto (x - x_j)^2 + \delta_I^2 \frac{\tilde{A}_{\|}(x - x_j)}{\tilde{A}_{\|0}} \cos(\omega_R t - k_y y) = c_j \qquad \text{(I-10)}$$

with $c_j = \mathrm{const.}$, $\delta_I \equiv (\tilde{A}_{\|}(0)/B_y')^{1/2} \exp(\omega_I t/2)$ and $\tilde{A}_{\|}(0) = \tilde{B}_{x0}/k_y$. Clearly the linear theory is valid for $\delta_I \sim |\tilde{B}_{x0}/\bar{B}_y|^{1/2} (d_e r_{Te})^{1/2} < d_e$ where $\bar{B}_y \sim |B_y'| r_{Te}$. Thus we consider that a sequence of these strings of islands is produced over a macroscopic scale distance represented by $|B_y'/B_y''|$. We consider these strings of islands to be a hindrance for the propagation of temperature along the field and to modify, considerably, mode-particle resonances that are relevant to the excitation of the drift-tearing mode in the collisionless regime.

Finally, we note that electrostatic modes, so called ETG modes, involving smaller scale lengths than the micro-reconnecting modes can also be excited. Mode packets (quasi-modes) can be constructed [7] by a proper superposition of elementary electrostatic modes that are sharply localized in the x-direction like those found for ITG modes [7]. The quasi-modes can cover a macroscopic interval in the x-direction and can maintain temperature profiles that are localized in the z-direction, along the field. In particular, the expression for the electron temperature is given by

$$\hat{\hat{T}}_e \simeq \tilde{T}_{e0} W(x) \exp\left[-i\omega_R t + ik_y\left(y - x\frac{z}{L_s}\right) + \frac{i}{2}\frac{z^2 k_y^2 \sigma_I}{L_s^2|\sigma|^2}\right] \exp\left[-\frac{1}{2}\frac{z^2 k_y^2}{L_s^2|\sigma|^2}\sigma_R + \omega_I t\right] \tag{I-11}$$

where $W(x)$ is a "weight" function localized over a macroscopic distance, elementary modes represented by $\tilde{T}_e(x) = \tilde{T}_{e0}\exp(-\sigma(\Delta x_j)^2/2)$ are considered, and $\sigma = \sigma_R + i\sigma_I$. Thus, we see that $\hat{\hat{T}}_e$ is both oscillatory and localized along the magnetic field over the scale distance $\delta_z \sim L_s|\sigma|/k_y$ where $|\sigma|k_y \sim 1$.

We consider this to be a further contribution toward having a state of reduced effective longitudinal thermal conductivity and increasing the ratio $D_\perp/D_\parallel$ above the values required for the excitation of drift tearing modes. Finally, we argue that the maximum transverse diffusion of thermal energy can be obtained for $k_\perp \sim 1/d_e$ and $\omega \sim \omega_{*Te}$. In this case all the terms in the electron momentum conservation equation, $m_e n \partial \hat{u}_{e\parallel} = -(\widehat{\boldsymbol{\nabla}\cdot\boldsymbol{P}_e})_\parallel - en\hat{E}_\parallel$, become of the same order and Eq. (I-8) for which $(\widehat{\boldsymbol{\nabla}\cdot\boldsymbol{P}_e})_\parallel$ is prevalent and is relatively simple, is replaced by one that has significantly different features.

II-1. Introduction (U.C.S.D. Contribution)

Here we consider a minimal model of a low-m resistive tearing mode in the presence of electrostatic drift wave turbulence in a cylinder. The tearing mode dynamics are described by reduced MHD (RMHD) and the small scale, large-m mode dynamics are described by an electrostatic fluid model, such as the Hasegawa-Mima, Hasegawa-Wakatani, or fluid ion temperature gradient (ITG) equations. Moreover, even further simplification is made possible by exploiting the disparity in space-time scales between the tearing mode and the background drift waves. In particular, for a tearing mode with wave-vector $\mathbf{q}$ (here $\mathbf{q} = (q_x,\ q_\theta,$ and $q_z)$, where q_x is comparable to the inverse layer width and q_θ, q_z are standard notations), and for drift waves with wave vector $\mathbf{k}$, it is the case that $\gamma_q \lll \omega_k$, $q_\theta \lll k_\theta$, and $q_x < k_x$. It is thus apparent that the tearing mode adiabatically modulates the background drift wave population, and the interaction may be treated using a wave kinetic equation (WKE) for the evolution of an adiabatic invariant of the drift wave population. Thus, the minimal model ultimately reduces to:

1. a WKE for $N(\mathbf{k}, x, t)$, the drift wave population density proportional to the spectral density. Here N is strained and advected by the tearing mode flows.
2. RMHD for the tearing mode, including the effects of stresses and fluxes driven by the drift waves.

Note that albeit simple, the 'minimal model' defines a closed self-consistent feedback loop for the interaction of low-m MHD and high-$\mathbf{k}$ drift waves.

II-2. Wave Kinetics for Small Scale Drift Waves

A WKE for the evolution of the drift wave potential enstrophy density (an adiabatic invariant) in the presence of a slowly varying background can be written (see Ref. [8] for details)

$$\begin{gathered}\frac{\partial}{\partial t}N_k + \frac{\partial}{\partial \mathbf{k}}\left(\omega_k + \mathbf{k}\cdot\mathbf{v}_0\right)\cdot\frac{\partial}{\partial \mathbf{x}}N_k - \frac{\partial}{\partial \mathbf{x}}\left(\omega_k + \mathbf{k}\cdot\mathbf{v}_0\right)\cdot\frac{\partial}{\partial \mathbf{k}}N_k = S\ , \\ \omega_k = \frac{v_e^* k_y}{1+\rho_s^2 k_\perp^2}, \quad \mathbf{v}_0 = \frac{c}{B_0}\left(\hat{\mathbf{z}}\times\nabla\phi^<\right), \quad N_k = \left(1+\rho_s^2 k_\perp^2\right)^2 I_k\ .\end{gathered} \tag{II-1}$$

Here N_k is the enstrophy density, I_k is a Wigner function defined as $I_k = \int d\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{x}}\left\langle \phi^>_{k+q}\phi^>_{-k}\right\rangle$, $\phi^<$ and $\phi^>$ correspond to the tearing mode fields and drift wave fluctuations respectively, and the brackets represent an average over the small, rapidly varying scales. The second term and third terms on the left of Eq. (II-1) correspond to advection and refraction terms modulated by the tearing mode fields. The source term S can be written as $S = \gamma_k N_k - \Delta\omega_k N_k^2$, where the first term corresponds to linear growth, and the second term corresponds to the nonlinear like-scale interaction.

II-3. Mean Field Equations for Large Scale Tearing Mode

As has been observed in drift wave-zonal flow systems, the background turbulence behaves as a *source* of energy for the large scales, and drives mean flows via the stress term. Thus, we seek to understand how the inclusion of this external drive affects the evolution of the tearing mode. For this reason, it is convenient to begin with as simple a description as possible. Separating the tearing mode fields from the small scale fluctuations and averaging, gives the tearing mode equations:

$$0 = \frac{\partial}{\partial t}\psi^< + \mathbf{v}_0\cdot\nabla\psi^< - v_A\frac{\partial}{\partial z}\phi^< - \eta_c\nabla_\perp^2\psi^<\ , \tag{II-2}$$

$$0 = \frac{\partial}{\partial t}\nabla_\perp^2\phi^< + \mathbf{v}_0\cdot\nabla\nabla_\perp^2\phi^< - v_A\frac{\partial}{\partial z}\nabla_\perp^2\psi^< - \mathbf{B}_\perp\cdot\nabla\nabla_\perp^2\psi^< + \left\langle \mathbf{v}^>\cdot\nabla\nabla_\perp^2\phi^>\right\rangle\ . \tag{II-3}$$

In order to close the drift wave-tearing mode system, it is necessary to explicitly write the Reynolds stress term within the RMHD equations in terms of the drift wave enstrophy. After integrating by parts and Fourier transforming, and only keeping the lowest order piece, the Reynolds stress $\langle \mathbf{v}^>\cdot\nabla\nabla_\perp^2\phi^>\rangle$ can be written as:

$$\left\langle \mathbf{v}^>\cdot\nabla\nabla_\perp^2\phi^>\right\rangle(\mathbf{x},t) = -\frac{c}{B_0}\frac{\partial^2}{\partial x^2}\int d\mathbf{k}\frac{k_x k_y}{\left(1+\rho_s^2k_\perp^2\right)^2}N_k(\mathbf{x},t)\ . \tag{II-4}$$

From this expression it is clear that for isotropic turbulence, both integrals vanish. Considering small deviations from the equilibrium drift wave spectrum N_k^0 (i.e. seed asymmetries), Eq. (II-1) can be linearized for small perturbations of the form $(\delta N_k,\ \phi^<) \sim e^{i\mathbf{q}\cdot\mathbf{x}-i\omega_q t}$, yielding an expression for the response of the drift waves to the tearing mode field, which to lowest order is given by $\delta N_k = (c/B_0)\, R(\mathbf{q},\mathbf{k})\,(\mathbf{k}\times\mathbf{q})_z\, \mathbf{q}\cdot\partial N_k^0/\partial\mathbf{k}\phi^<$. Here, $R(\mathbf{q},\mathbf{k}) = -i/\left((\omega_q - \mathbf{q}\cdot\mathbf{v}_{gr}) + i\gamma_k\right)$, ω_q corresponds to the frequency of the MHD mode, and $\mathbf{v}_{gr} = \partial\omega_q/\partial\mathbf{k}$. Substituting δN_k into the polarization drift term of the vorticity equation (Eq. (II-4)) gives to lowest order:

$$\left\langle \mathbf{v}^>\cdot\nabla\nabla_\perp^2\phi^>\right\rangle = -c_s^2\int d\mathbf{k}\frac{\rho_s^2k_y^2}{\left(1+\rho_s^2k_\perp^2\right)^2}\frac{\gamma_k}{\left(\gamma_k^2+(\mathbf{q}\cdot\mathbf{v}_{gr})^2\right)}k_x\frac{\partial N_k^0}{\partial k_x}\frac{\partial^4\phi^<}{\partial x^4}$$

This term has the form of an anomalous viscosity which we shall denote by ν_T. Note that for $k_x\frac{\partial N_k^0}{\partial k_x} < 0$ (universally observed), the value of ν_T will be *negative*, so that small scales transfer energy to large scales, as in an inverse cascade. The presence of a negative viscosity on large scales due to nonlocal interactions with the background microturbulence is a result familiar from considerations of drift wave-zonal flow systems.

We estimate the magnitude of the anomalous viscosity, using a mixing length argument. For drift waves, we can approximate $e\phi^{>}/T_e \sim 1/(k_\perp L_n)$, where L_n is the perpendicular length scale over which the density varies. The magnitude of the turbulent viscosity can then be estimated to be $|\nu_T| \approx \frac{c_s^2}{\gamma_k}\frac{1}{k_\perp^2 L_n^2} \approx \frac{c_s^2}{\gamma_k}\frac{\rho_s^2}{L_n^2}$, where we have used ρ_s to estimate the mixing length. Finally, estimating the linear drift wave growth rate to be of the order of the drift wave linear frequency, $\gamma_k \approx v_e^* k_y \approx c_s/L_n$, yields an estimate of the turbulent viscosity as $|\nu_T| \approx \frac{\rho_s}{L_n}\omega_{ci}\rho_s^2 \sim D_{GB}$. Here D_{GB} denotes the gyro-Bohm diffusivity, which is far in excess of the ion-ion collisional viscosity ρ_i^2/τ_{ii}, or the neoclassical viscosity. To estimate the relative sizes of the turbulence driven flux and linear inertia, we compare $\nu_T \sim D_{GB}$ with $\gamma_T x_T^2$, where γ_T and x_T are the values derived by Furth, Killeen, and Rothenbluth (FKR) [9]. A simple calculation yields the conclusion that turbulent stresses will exceed inertia for

$$D_{GB} > \frac{a^2}{\tau_\eta}\left(\Delta' a\right)^{6/5}\left(1/S\right)^{2/5}\left(L_s/am\right)^{2/5} \ .$$

Here $S = \tau_\eta/\tau_A$, where $\tau_\eta^{-1} = \eta/a^2$ and $\tau_A^{-1} = v_A/a$. This expression can be rewritten as $(\omega_{ci}\tau_\eta)(\rho_s/a)^2 > (L_s/\rho_s)(1/S)^{2/5}(L_s/am)^{2/5}(\Delta' a)^{6/5}$. For realistic parameters this condition will nearly always be satisfied. Thus, in practical terms, the turbulent stresses *always* exceed inertia. Hence, the turbulent Reynolds stress is seen to be the *dominant microscopic* effect on the large scales for the case of electrostatic turbulence.

II-4. Tearing Mode Evolution in Presence of Drift Wave Turbulence

A negative viscosity will have a strong impact on the linear dynamics of the reconnecting mode, i.e. the counterpart of the traditional tearing mode. Integrating Eq. (II-2) across the resistive layer, and writing Eq. (II-3) in dimensionless units gives

$$0 = sgn\left(\nu_T\right)\frac{\partial^4\Phi}{\partial\sigma^4} - \frac{1}{\alpha}\frac{\partial^2\Phi}{\partial\sigma^2} + \sigma\left(1+\sigma\Phi\right) \ , \qquad \text{(II-5)}$$

$$\Delta' = \frac{i\omega_q}{\eta_c}x_\nu\int d\sigma\left(1+\sigma\Phi\right) \ , \qquad \text{(II-6)}$$

where $\Delta' = \left(\psi'\left(0^+\right) - \psi'\left(0^-\right)\right)/\psi_0$, $\alpha = i\left|\nu_T\right|/\omega_q x_\nu^2$, $\sigma = x/x_\nu$, $\Phi = \frac{q_y v_A}{\omega_q}\frac{x_\nu}{L_s}\frac{\phi^<}{\psi_0^<}$, $x_\nu = \left(\eta_c\left|\nu_T\right|\right)^{1/6}\left(\frac{L_s}{q_y v_A}\right)^{1/3}$. In the viscous dominated regime, the inertial term (second term on the right) can be neglected. It is convenient to introduce the Fourier transform defined by $\Phi\left(q_x\right) = \int_{-\infty}^{\infty} d\sigma e^{-iq_x\sigma}\Phi\left(\sigma\right)$. Eqs. (II-5) and (II-6) then become

$$\frac{d^2\Phi\left(q_x\right)}{dq_x^2} - sgn\left(\nu_T\right)q_x^4\Phi\left(q_x\right) = 2\pi i\frac{d}{dq_x}\delta\left(q_x\right) \ , \qquad \text{(II-7)}$$

$$\Delta' = \frac{i\omega_q}{\eta_c}x_\nu\left(2\pi\delta\left(q_x\right) + i\frac{d\Phi\left(q_x\right)}{dq_x}\ |_{q_x=0}\right) \ . \qquad \text{(II-8)}$$

Introducing the transformation [10] $\Phi\left(q_x\right) = i\pi sgn\left(q_x\right)\Phi_{hom}\left(\left|q_x\right|\right)/\Phi_{hom}\left(0\right)$, Eq. (II-8) becomes

$$\Delta' = i\pi\frac{\omega_q}{\eta_c}x_\nu\frac{1}{\Phi_{hom}\left(0\right)}\frac{d\Phi_{hom}}{dq_x}\ |_{q_x=0} \ . \qquad \text{(II-9)}$$

The homogeneous solution of Eq. (II-7) can be easily seen to be:

$$\Phi_{hom}\left(q_x\right) = A\sqrt{q_x}J_{\frac{1}{6}}\left(\frac{q_x^3}{3}\right) + B\sqrt{q_x}Y_{\frac{1}{6}}\left(\frac{q_x^3}{3}\right) \ . \tag{II-10}$$

Two observations concerning this equation are possible. First, upon inverse Fourier transforming Eq. (II-10), the solutions in real space can be seen to undergo oscillations which are ninety degrees out of phase with one another. Thus, fixing the ratio of the amplitudes A/B is equivalent to setting the phase of the oscillations. Second, since both of these solutions converge for $q_x \to \infty$, neither solution can be dropped. These considerations leave us with an undetermined constant A/B in the eigenvalue relation, which can be written as

$$\Delta' = -i\frac{\pi}{2}\frac{1}{6^{1/6}}\frac{\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{7}{6}\right)}\frac{\omega_q}{\eta_c}x_\nu\left(\frac{3^{1/3}}{2^{1/6}} + \frac{1}{6^{1/6}}\frac{A}{B}\right) \ . \tag{II-11}$$

From this expression it is clear that another boundary condition is needed in order to specify A/B, Eq. (II-11). As discussed in Ref. [11] outgoing wave boundary conditions are necessary to determine the phase of the oscillations. After applying outgoing wave boundary conditions, A/B is shown to be pure imaginary, thus the dispersion relation takes the form

$$\gamma_q \sim Re\left(\omega_q\right) \sim \frac{\eta_c}{x_\nu}\Delta' \sim \frac{\eta_c^{5/6}}{\left|\nu_T\right|^{1/6}}\left(\frac{q_y v_A}{L_s}\right)^{1/3}\Delta' \ , \tag{II-12}$$

where Δ' is purely real. Here a real frequency is induced via the strong flow shear introduced by the inverse cascade from the drift wave turbulence. Also note that the background turbulence has a stabilizing effect on the tearing mode. This is a result of the Reynolds stress driven flow distorting the flow pattern near the resonant surface, thus reducing the rate of reconnection. Also, we note that the resistive layer width of the tearing mode is substantially broadened by the inclusion of stress emanating from the drift wave turbulence. This follows from the Reynolds stresses overwhelming the interia term within the vorticity equation, thus establishing a new lowest order balance between Reynolds stresses and $\mathbf{J} \times \mathbf{B}$ forces. A comparison with the FKR tearing layer width, gives the scaling form $x_\nu/x_T \sim S^{7/30}\beta^{1/12}\rho_*^{1/3}$, which for $S \approx 10^7$, $\beta \approx 1/10$, and $\rho_* \approx 10^{-3}$, $x_\nu/x_T \approx 5$, corresponding to a substantial increase in the width of the resistive layer. The onset of the regime of finite island dynamics (i.e. Rutherford phase) is thus concomitantly delayed.

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