Test Particle Statistics and Turbulence in Magnetically Confined Plasmas

M. Vlad, F. Spineanu
Association Euratom-MEdC Romania,
National Institute for Laser, Plasma and Radiation Physics, Bucharest, Romania

E-mail contact of main author: madi@ifin.nipne.ro

Abstract. Test particle statistics in turbulent plasmas is presented as a method to study the transport based on experimental measurements of the characteristics of the turbulence. The trapping or eddying appearing in the ExB stochastic motion in turbulent confined plasmas generates non-standard statistical behavior of the trajectories: memory effects, non-Gaussian probability and coherence. The memory effect produces anomalous diffusion regimes in the presence of weak perturbations (collisions, poloidal rotation). Trajectory quasi-coherent structures appear, which have a strong influence of the evolution of the turbulence in the strong nonlinear regime. These structures provide the physical mechanism for the inverse cascade in drift turbulence.

1. Introduction

The main aim of the studies of turbulence is to determine the diffusion coefficients. The statistics of test particle trajectories provides the transport coefficients in turbulent plasmas without approaching the very complicated problem of self-consistent turbulence that explains the detailed mechanism of generation of the turbulent potential. The statistical characteristics of the potential are considered known from experimental studies or numerical simulations. Alternatively they are described by general models and the dependence of the diffusion coefficients on the parameters of the models is determined.

A component of test particle motion in magnetized plasmas is the stochastic electric drift produced by the electric field of the turbulence and by the confining magnetic field. This drift determines a trapping effect or eddy motion in turbulence with slow time variation. This is a nonlinear process that was analytically studied only in the last decade by developing new statistical methods [1,2]. It was shown that trapping generates non-standard statistical behavior of the trajectories: memory effects, non-Gaussian probability and coherence [2]. The memory effect is represented by long tail of the correlation of the Lagragian velocity, which is shown to strongly influence the transport coefficients and to produce anomalous diffusion regimes in the presence of a weak decorrelation mechanism [3]. The effects of several decorrelation mechanisms (collisions, average flows, parallel motion) were studied in a series of papers [4-6] but considered separately. In realistic plasma all these processes act together and their influence on the transport coefficients is expected to be complex in the presence of trapping when the nonlinear effects are strong. The study of the turbulent transport in the presence of multiple decorrelation mechanisms is presented in the first part of this paper.

The second part of the paper deals with another aspect of the statistics of the trajectories, namely the quasi-coherent behaviour and the trajectory structures that are shown to appear due to trapping [2]. We present here first results on the effects of trajectory trapping on the dynamics of the drift turbulence.

2. Transport coefficients

Test particle studies connected with experimental measurements of the statistical properties of the turbulence provide the transport coefficients with the condition that there is space-time scale separation between the fluctuations and the average quantities. The continuity equation for particle density in electrostatic turbulence leads in these conditions to a diffusion equation for the average density [7]. Recent numerical simulations [8] confirm a close agreement between the diffusion coefficient obtained from the density flux and the test particle diffusion coefficient. Experiment based studies of test particle transport permit to strongly simplify the complicated self-consistent problem of turbulence and to model the transport by means of particle stochastic advection. The diffusion coefficient is obtained as the time derivative of the mean square displacement of test particles or as the integral of the Lagrangian velocity correlation (LVC). The latter has to be determined for given Eulerian correlation (EC) of the fluctuating potential. The Lagrangian velocity correlation determines the time dependent diffusion coefficient and the mean square displacement. It is also a measure of the statistical memory of the stochastic motion.

An important observation is that all the components of particle motion (parallel motion, collisions, average flows, etc.) have to be taken into account. In the non-linear regimes characterized by the presence of trajectory trapping, these components have strong influence on the diffusion coefficients even if they are small. The reason for this behaviour is the shape of the correlation of the Lagrangian velocity for particles moving by the ExB drift in a static potential. In the absence of trapping, the typical LVC for a static field is a function that decay to zero in a time of the order $\tau_{fl}= \lambda/V$, where λ is the correlation length of the stochastic potential and V is the amplitude of the ExB drift. This leads to asymptotic diffusion coefficients $D=cV^2 \tau_{fl} =cV\lambda$, where only the constant c is influenced by the EC of the stochastic field. The presence of trajectory trapping determines a completely different shape of the LVC. A typical example of the LVC is presented in Fig. 1. This function decays to zero in a time of the order τ_{fl} but at later times it becomes negative, it reaches a minimum and then it decays to zero having a long, negative tail. The tail has power law decay with an exponent that depends on the EC of the potential [9]. The positive and negative parts compensate such that the integral of L(t), the running diffusion coefficient D(t), decays to zero. The transport in such 2-dimensional potential is thus subdiffusive. The long time tail of the LVC shows that the stochastic trajectories in static potential have a long time memory.

This stochastic process is unstable in the sense that any weak perturbation produces a strong influence on the transport. A perturbation represents a decorrelation mechanism and its strength is characterized by a decorrelation time τ_d. The weak perturbations correspond to long decorrelation times, $\tau_d >> \tau_{fl}$. In the absence of trapping, such a weak perturbation does not produce a modification of the diffusion coefficient because the LVC is zero at $t> \tau_{fl}$. In the presence of trapping, which is characterized by long time LVC as in Fig. 1, such perturbation influences the tail of the LVC and destroys the equilibrium between the positive and the negative parts. Consequently, the diffusion coefficient depends on the type of decorrelation but it is in general a decreasing function of τ_d. It means that when the decorrelation mechanism becomes stronger (τ_d decreases) the transport increases. This is a consequence of the fact that the long time LVC is negative. This behavior is completely different of that obtained in stochastic fields that do not produce trapping. In this case, the transport is stable to the weak perturbations. An influence of the decorrelation can appear only when the later is strong such that $\tau_d < \tau_{fl}$ and it determines the increase of the diffusion coefficient with the increase of τ_d. This inverse behavior appearing in the presence of trapping is determined by the fact that a stronger perturbation (with smaller τ_d) liberates a larger number of trajectories, which contribute to the diffusion.

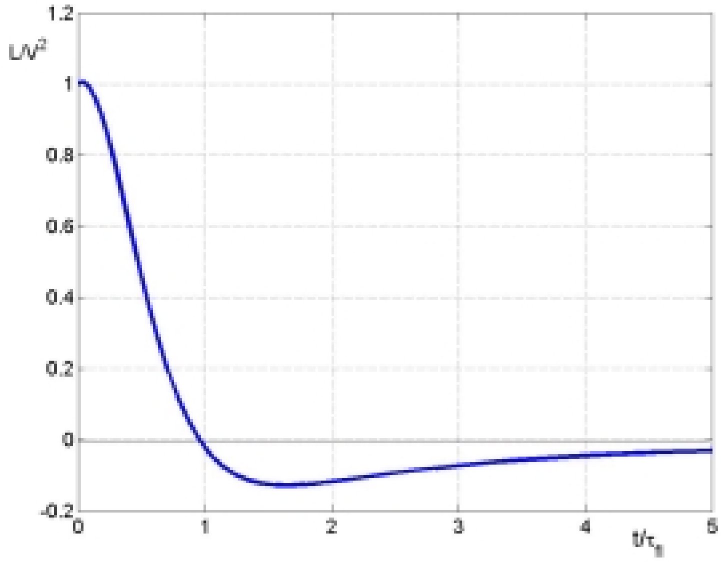

FIG.1. The Lagrangian velocity correlation in static potential.

The decorrelation can be produced for instance by the time variation of the stochastic potential, which produces the decay of both Eulerian and Lagrangian correlations after the correlation time τ_c. The decorrelation time is in this case is τ_c and it is usually represented by a dimensionless parameter, the Kubo number defined by $K= \tau_c / \tau_{fl}$. The transport becomes diffusive with an asymptotic diffusion coefficient that scales as $D_{tr}=c(\lambda_c^2 / \tau_c)K^{\gamma}$, with γ in the interval [-1, 0] (trapping scaling), and thus it is a decreasing function of τ_c.
For other types of perturbations, their interaction with the trapping process produces more complicated nonlinear effects. For instance, particle collisions lead to the generation of a positive bump on the tail of the LVC [4] due to the property of the 2-dimensional Brownian motion of returning in the already visited places. Other decorrelation mechanisms appearing in plasmas are average component of the velocity like poloidal rotation [5] or the parallel motion that determines decorrelation when the potential has a finite correlation length along the confining magnetic field [6].
All these types of decorrelations determine, when trajectory trapping is effective, anomalous diffusion regimes with diffusion coefficients increasing as the decorrelation becomes stronger. These regimes appear for weak decorrelations that correspond to the condition $\tau_{fl} < \tau_d$.

We have developed a model that includes, besides the ExB stochastic drift, particle collisions, an average flow (poloidal rotation) and the parallel motion. The diffusion coefficients are determined using the decorrelation trajectory method. The later is based on a set of smooth trajectories determined by the Eulerian correlation of the turbulence and by all the other components of particle motion. A computer code was developed for determining the diffusion coefficients for given EC of the turbulent plasma.

The diffusion regimes are analyzed and the conditions when they appear are identified. A reach class of anomalous diffusion regimes appears when trajectory trapping is effective, i.e. when the combined action of the decorrelation mechanisms is weak enough. Trapping influences not only the values of the diffusion coefficients but also their scaling laws. A systematic analyze of these regimes and of their specific conditions can be done only when one of the decorrelation mechanisms dominates. This corresponds to the case when one of the characteristic decorrelation times is much smaller than the others. When they have comparable values, the diffusion coefficient is a complicated function of all these characteristic times. The code we have developed provides a tool for the experimental studies of transport.

An example of the strong influence produced by the decorrelation mechanisms is presented in Fig. 2. The diffusion coefficient D_0 due to the ExB drift for a time dependent potential is represented as function of the Kubo number by the blue line. The diffusion coefficient is sensibly increased if particle collisions with a small collisional diffusivity $\chi \ll V\lambda$ are considered (green line). We note that the direct contribution of collisions χ is negligible in Fig. 2. A weak poloidal rotation with a velocity V_p that is smaller than the amplitude of the ExB drift determines a strong decrease of the radial diffusion (black line). When poloidal rotation and collisions act together, the diffusion coefficient (red line) can be smaller or larger than D_0, depending on the values of K, χ and V_p. Fig. 2 also shows that at small Kubo number the diffusion coefficients are not changed by these small perturbations due to collisions and poloidal rotation.

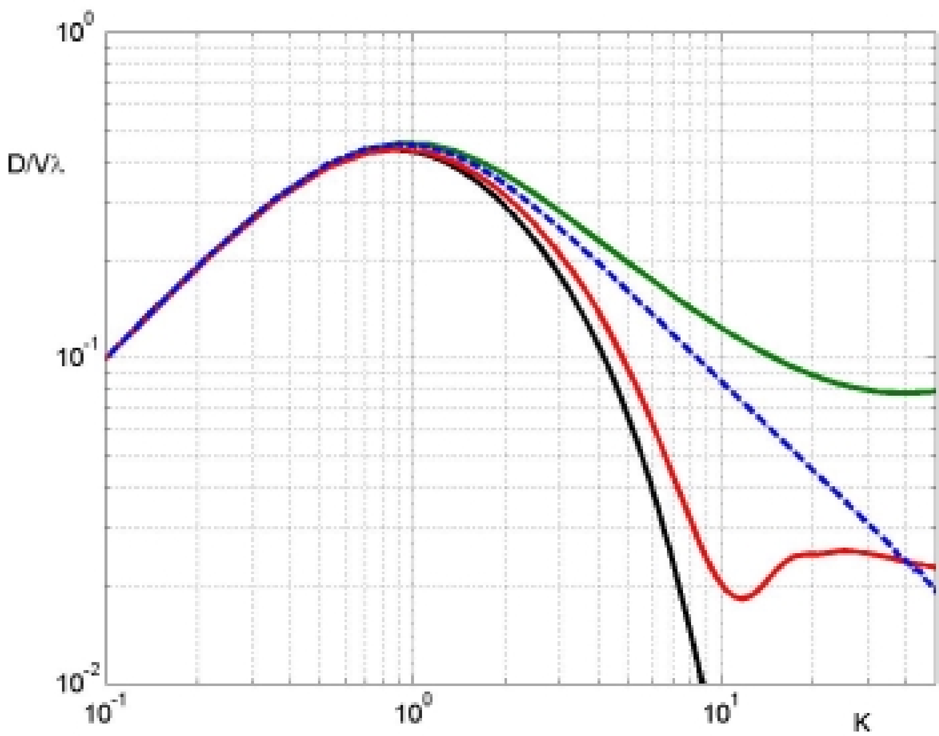

FIG. 2. The diffusion coefficient as function of K for several values of V_p/V, $\chi/V\lambda$: 0, 0 (blue); 0.4, 0 (black); 0, 0.02 (green); 0.4, 0.02 (red).

3. Trajectory structures and drift turbulence evolution

3.1. Coherence and trajectory structures

Detailed statistical information about particle trajectories was obtained using the nested subensemble method [2]. This method determines the statistics of the trajectories that start in points with given values of the potential. This permits to evidence the high degree of coherence of the trapped trajectories.

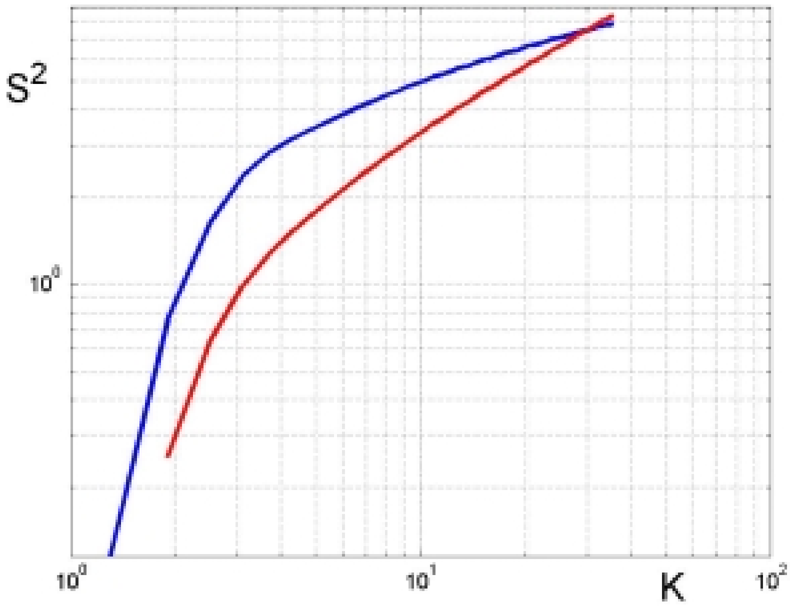

FIG. 3. The average size of the trajectory structures for Gaussian EC (blue) and for an EC that decays as $1/r^2$ (red).

The trapped trajectories correspond to large absolute values of the initial potential while the trajectories starting from points with the potential close to zero perform long displacements. These two types of trajectories have completely different statistical behavior [2]. The trapped trajectories have a quasi-coherent behavior. Their average displacement, dispersion and probability distribution function saturate in a time τ_s. The time evolution of the square distance between two trajectories is very slow showing that neighboring particles have a coherent motion for a long time, much longer than τ_s. They are characterized by a strong clump effect with the increase of the average square distance that is slower than the Richardson law. These trajectories form structures, which are similar with fluid vortices and represent eddying regions. The statistical parameters of these structures (size, build-up time, dispersion) are determined. The dispersion of the trajectories in such a structure is of the order of its size. The size and the built-up time depend on the value of the initial potential. Trajectory structures appear with all sizes, but their characteristic formation time increases with the size. These structures or eddying regions are permanent in static stochastic potentials. The saturation time τ_s represents the average time necessary for the formation of the structure. In time dependent potentials the structures with $\tau_s > \tau_c$ are destroyed and the corresponding trajectories contribute to the diffusion process. These free trajectories have a continuously growing average displacement and dispersion. They have incoherent behavior and the clump effect is absent. The probability distribution functions for both types of trajectories are non-Gaussian.

The average size of the structures S in a time dependent potential is plotted in Figure 3. One can see that for K<1 the structures are absent (S~0) and that they appear for K>1 and continuously grow as K increases. The dependence on K is a power low with the exponent dependent on the EC of the potential. The exponent is 0.19 for the Gaussian EC and 0.35 for a large EC that decays as $1/r^2$.

3.2. Trajectory structures and turbulence dynamics

Test particle trajectories are strongly related to plasma turbulence. The dynamics of the plasma basically results from the Vlasov-Maxwell system of equations, which represents the conservation laws for the distribution functions along particle trajectories. Studies of plasma turbulence based on trajectories [10] were initiated by Dupree [11.12] and developed especially in the years seventies. These methods do not account for trajectory trapping and thus they apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has to be understood is the effect of this non-standard statistical behaviour of the test particle trajectories on the evolution of the instabilities and of turbulence in magnetized plasmas.

We extend the analysis of turbulence based on test particle trajectories to the nonlinear regime characterized by trapping. We study linear test modes on turbulent plasma for the drift instability in slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (non-adiabatic response) and finite Larmor radius of the ions destabilizes the drift waves. We consider a turbulent state of the plasma with known statistical characteristics of the electrostatic potential. The perturbations of the electron and ion distribution functions are obtained from the gyrokinetic equation as integrals along test particle trajectories of the source terms determined by the density gradient.
The background turbulence produces two modifications of the equation for the linear modes. One consists in the stochastic ExB drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity. Both effects are important for ions while the response of the electrons is approximately the same as in quiet plasma.
The average propagator of the modes is evaluated using the above results on trajectory statistics. In the first order it depends on the size S(K) of the structures. The solution of the dispersion relation is obtained as

$$\omega = \omega_{*e} \frac{\Gamma_0\left(\frac{k^2\rho^2}{2}\right)\exp\left(-\frac{k^2 S^2}{2}\right)}{2-\Gamma_0\left(\frac{k^2\rho^2}{2}\right)}$$

$$\gamma = \sqrt{\pi}\,\frac{\omega(\omega_{*e}-\omega)}{2-\Gamma_0}\,\frac{1}{k_z V_{Te}} - k^2 D\cos(\omega\tau_c) + k_i k_j R_{ij} / k_2$$

where ω_{*e} is the diamagnetic frequency, ρ is the ion Larmor radius and $\Gamma_0(b) = \exp(-b)\, I_0(b)$.
The tensor R_{ij} has the dimension of a length and is defined by

$$R_{ij}(\tau,t) = \int_{\tau}^{t} d\theta' \int_{-\infty}^{t-\theta'} d\theta \, M_{ij}(|\theta|)$$

where M_{ij} is the Lagrangian correlation

$$M_{ij}(|\theta'-\theta|) = \langle v_i(\mathbf{x}^i(\theta'), z, \theta') \ \partial_2 v_j(\mathbf{x}^i(\theta), z, \theta) \rangle$$

and **v** is the ExB drift velocity.

The trajectory trapping process has a complex influence on the mode. The ion trajectory structures (the quasi-coherent component of their motion) determine the S-dependent exponential factor in the frequency ω. Its effect is the displacement of the unstable k-range toward small values. The random component in the ion motion determines a diffusive damping term in the growth rate γ that produces the stabilization of the large wave numbers. It is similar with the term obtained in [12], but with the diffusion coefficient influenced by trapping. The fluctuations of the diamagnetic velocity determine the last term in the growth rate. The tensor R_{ij} contributes to the growth of the modes.

The dynamics of the drift turbulence is determined starting from a large spectrum of modes with very small amplitudes (thermal bath). In this quasilinear regime only the diffusion of the ions influences the modes producing the damping of the modes with large wave numbers, $k\rho \gg 1$. The amplitude of the stochastic potential increases continuously while the correlation length remains comparable with ρ and the correlation time is $\tau = \rho / V_*$, where V_* is the diamagnetic velocity. This amplitude increase eventually produces trajectory trapping and coherent motion for a part of the ions. Small trajectory structures are formed and persist during the correlation time of the potential. This ordered motion of the ions acts similarly with the cyclotron gyration: it decreases the frequency of the modes and displaces the maximum of the spectrum toward smaller k. In this stage of the evolution the amplitude of the ExB velocity remain approximately constant, while the correlation length and the correlation time are slowly increasing. The spectrum is continuously displaced toward small wave numbers and narrowed due to the increase of the diffusion coefficient. Thus the energy taken by the instability from the electrons produces a motion of the ions with increasing coherent component. The size of the trajectory structures increases and is reflected in the turbulence that looses the random aspect: large ordered potential cells are produced.
A different perspective on the inverse cascade is thus obtained. It does not appear as wave-wave interaction but as the effect of ion ExB motion on the drift wave stability. Namely, the quasi-coherent motion of the trapped ions produces the destabilization of the modes with wave lengths of the order of the average size of the trajectory structures. These decreases the frequency of the modes and protuces the increase of the size S.

4. Conclusions

We have discussed the problem of stochastic advection of test particles by the ExB drift in turbulent plasmas. We have shown that the nonlinear effects are very strong in the case of static potentials. The trajectories are non-Gaussian, there is statistical memory and coherence, and the scaling laws are dependent on the Eulerian correlation of the stochastic potential.

These properties persist if the system is weakly perturbed by time variation of the potential or by other components of the motion (collisions, poloidal rotation, parallel motion). The memory effect determines anomalous diffusion regimes. A code was developed for the calculation of the diffusion coefficients for given Eulerian correlation of the turbulence, which takes into account multiple decorrelations.
These non-standard statistical properties of the trajectories are shown to be associated with order and structure formation in turbulent magnetized plasmas. Particle trajectories have a high degree of coherence when the perturbations are weak and they form structures. The trajectory structures determine the evolution of the drift turbulence toward large scales (inverse cascade).

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