Nonlinear Inward Particle Flux in Trapped Electron Turbulence

P.W. Terry 1), R. Gatto 2), D.A. Baver 1), and S. Gupta 1)

1) University of Wisconsin, Madison, WI 53706 USA
2) Universita di Roma "Tor Vergata", 00133 Rome, Italy

e-mail contact of main author: pwterry@wisc.edu

Abstract: Weakly collisional trapped electron mode (TEM) turbulence has a robust inward particle flux component associated with a linearly stable eigenmode that is excited nonlinearly by spectral energy transfer from the unstable TEM mode. The nonlinear mixture of the two eigenmodes achieved in saturation cannot be described by the quasilinear approximation, hence the inward flux component, which combines with the outward quasilinear flux to produce the net flux, is fundamentally nonlinear. The net flux, which remains outward but is significantly reduced by the inward component, depends on the gradients of density and temperature. This dependence, which establishes whether the flux is diffusive, convective, or something else, is sensitive to the details of the saturation. Saturation is calculated asymptotically in an ordered expansion in collisionality and the ratio of density to temperature gradient scale length. Spectral transfer is highly anisotropic and saturation must account for the energy transfer to zonal modes with zero poloidal wavenumber. Even though zonal modes do not contribute directly to the particle flux they change the fluctuation level and gradient scaling of both the unstable and stable eigenmodes. The result is a flux that is neither diffusive nor convective, but is driven by temperature gradient and enhanced by density gradients that are flat or nearly so. Near the instability threshold the inward component is particularly strong.

1. Introduction

Inward particle transport in tokamaks is important for achieving desirable density profiles in discharges without a significant core particle source. In ITER a centrally peaked density profile is favorable for fusion. Yet other considerations make it preferable to heat the plasma with *rf* waves, which provide no central fueling. Under these circumstances, processes that lead to inward particle transport are of interest. An inward particle fluxes can have various density gradient scalings. Moreover, an inward flux generally accompanies some form of outward transport with its own density gradient scaling. The relative magnitude of these components varies, and depending on circumstances, can be greater or lesser than unity.

The first inward flux discovered was a neoclassical effect [1]. Subsequent work showed that turbulence drives inward particle transport under certain conditions. Anomalous inward fluxes operate through additional free energy sources. It was shown that ion temperature gradient instability can produce an inward flux in collisional plasmas [2]. This was subsequently extended to collisionless regimes [3]. The gradient of the safety factor drives a second class of pinch mechanisms through adiabatic invariance [4]. Presently the particle transport in experimental discharges is interpreted using these types of inward transport models [5]-[6].

A striking aspect of the conceptual formulation of inward transport mechanisms is the reliance on quasilinear theory. In quasilinear theory the phase between the electron density and the potential is assumed to be given by a linearized density response. In collisional regimes this assumption is reasonable because the collisional effect is linear, and it dominates the response. In collisionless or weakly collisional regimes the density response can become highly nonlinear. This happens in the hydrodynamic regime of the Hasegawa-Wakatani equation. There is an analogous regime in fluid models of trapped electron mode (TEM) turbulence [7]. When the electron density response becomes nonlinear, particle transport is no longer adequately approximated by quasilinear theory.

This paper describes a new inward transport mechanism arising from nonlinear electron density evolution, and therefore outside quasilinear theory. We calculate its details for a fluid model of TEM turbulence in the weakly collisional limit [8]. The effect arises for other types of turbulence and transport, as shown in the final section [9]. The physics is relatively simple. The nonlinear density mixes the linear eigenmodes of a complete basis set, as it must if there is a deviation from the linear response of the instability. The TEM basis consists of the eigenmode of the instability and a second eigenmode that is damped for all wavenumbers. The excitation of the damped eigenmode drives inward transport. This is a fairly general outcome of damped eigenmode excitation, at least for drift waves: instabilities relax gradients with outward transport, hence stable eigenmodes peak up the profile with inward transport.

2. Anisotropic Saturation of TEM Turbulence

To calculate the transport, inward and outward, the nonlinear density response must be obtained, along with the self-consistent potential. Equivalently, the evolution of unstable and stable eigenmodes can be calculated. This treats the linear instability driving the unstable eigenmode, the mode-mode coupling that both saturates the instability and excites the stable eigenmode, and the damping of the stable eigenmode. The damped eigenmode is readily accessible to the dynamics, and because it reaches finite amplitude, is a potent sink for saturation. The energy transfer that accomplishes saturation occurs in a dual space whose orthogonal manifolds consist of eigenmode space and wavenumber space. In eigenmode space energy is carried from the unstable eigenmode to the damped eigenmode where it is dissipated. In k-space energy is carried in a highly anisotropic fashion from unstable wavenumbers with $k_y \neq 0$ to zonal wavenumbers with $k_y = 0$. Transfer in these spaces is intertwined. Zonal wavenumbers of the stable eigenmode are damped and saturate the instability. These do not contribute directly to particle transport, but they moderate fluctuation levels throughout the spectrum. Hence the nonlinear flux calculation must account for the interplay between zonal modes and the damped eigenmode driving the inward flux component.

Saturation in the dual space of eigenmodes and wavenumbers is described by decomposition of the TEM density and potential fields $n_k(t)$ and $\phi_k(t)$ into nonlinear evolution equations for the amplitudes $\beta_1(k,t)$ and $\beta_2(k,t)$ of the unstable and damped eigenmodes. These evolution equations are given by,

$$\left[\frac{\partial}{\partial t} + i\omega_j\right]\beta_j = -\sum_{k'}\sum_{m=1}^{2}(-1)^j C_m(k,k')\beta'_m\beta''_1 \,, \tag{1}$$

where the notation $\beta'_j \equiv \beta_j(k',t)$, $\beta''_j \equiv \beta_j(k-k',t)$, $\beta_j \equiv \beta_j(k,t)$ is adopted for shorthand and is also applied to the eigenfrequencies $\omega_j(k)$. The factors $C_m(k,k') = -(\mathbf{k}'\times\hat{\mathbf{z}}\cdot\mathbf{k})R_m(k')/[R_1(k) - R_2(k)]$ are the non symmetrized nonlinear coupling coefficients of the eigenmode decomposition. The eigenmode decomposition projects the density and potential onto the linear eigenvectors $[R_1(k),1]$ and $[R_2(k),1]$ of the original evolution equations, diagonalizing the coupling. The projection is

$$\begin{pmatrix} n_k(t) \\ \phi_k(t) \end{pmatrix} = \beta_1(k,t)\begin{pmatrix} R_1 \\ 1 \end{pmatrix} + \beta_2(k,t)\begin{pmatrix} R_2 \\ 1 \end{pmatrix} = \begin{pmatrix} R_1 & R_2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \beta_1(k,t) \\ \beta_2(k,t) \end{pmatrix}$$

$$\equiv \mathbf{M}\left(\begin{array}{c}\beta_1(k,t)\\ \beta_2(k,t)\end{array}\right). \tag{2}$$

The eigenfrequencies and eigenvectors are obtained in the usual way from the evolution equations in the original representation of density and potential given by

$$\frac{\partial n_k}{\partial t} + \nu n_k + [ik_y v_D\hat{\alpha} - \nu]\phi_k = -\sum_{k'}(\mathbf{k}' \times \mathbf{z}\cdot\mathbf{k})\ \phi_{k'} n_{k-k'} \tag{3}$$

$$\frac{\partial \phi_k}{\partial t} - \frac{\varepsilon^{1/2}\nu}{1+k^2-\varepsilon^{1/2}} n_k + \frac{[ik_y v_D(1-\hat{\alpha}\varepsilon^{1/2}) + \nu\varepsilon^{1/2}]}{1+k^2-\varepsilon^{1/2}}\phi_k = -\sum_{k'}\frac{(\mathbf{k}'\times\mathbf{z}\cdot\mathbf{k})(k-k')^2}{1+k^2-\varepsilon^{1/2}}\phi_{k'}\phi_{k-k'}, \tag{4}$$

where $n_k = \varepsilon^{1/2} n_e + \phi_k$ is an effective density, n_e is the density of trapped electrons, ϕ_k is the potential, $\varepsilon^{1/2}$ is the trapping fraction, ν is the detrapping rate, v_D is the diamagnetic drift velocity, $\hat{\alpha} = 1 + 3\eta_e/2$, and η_e is the ratio of gradient scale lengths for the density and temperature. A derivation of this model and the dimensionless normalizations for n, ϕ, t, x, and y are given in Ref. 7. The eigenvector components $R_j(k)$ are the ratio n_k/ϕ_k for each eigenfrequency ω_j. They are obtained by linearizing Eq. (4), replacing $\partial/\partial t$ with $-i\omega_j$, and solving for n_k. The result is

$$R_j(k) = -\frac{1+k^2-\varepsilon^{1/2}}{\nu\varepsilon^{1/2}}\left[i\omega_j - \frac{ik_y v_D(1-\hat{\alpha}\varepsilon^{1/2}) + \nu\varepsilon^{1/2}}{1+k^2-\varepsilon^{1/2}}\right], \tag{5}$$

where the eigenfrequencies ω_j are the roots of the characteristic equation, $\omega^2(1+k^2-\varepsilon^{1/2}) + \omega[-v_D k_y(1-\hat{\alpha}\varepsilon^{1/2}) + i\nu(1+k^2)] - ik_y v_D \nu = 0$. Expressions for these frequencies are given in Ref. 7. There are two approximations that simplify Eq. (1). The first is the neglect of the polarization drift nonlinearity of Eq. (4), appropriate for the long wavelength regime $k < 1$. The eigenmode amplitudes are then governed by the density advection nonlinearity of Eq. (3). The mode coupling in Eq. (1) reflects density advection under the inverse eigenmode decomposition $n_k = R_1\beta_1 + R_2\beta_2$ and $\phi_k = \beta_1 + \beta_2$. The second approximation takes $\phi_k \approx \beta_1$ in the nonlinearity. Numerical solutions show that $\beta_2 \ll \beta_1$ in saturation (but $R_1\beta_1 \sim R_2\beta_2$). The approximated nonlinearity preserves energy conservation.

The saturation of the TEM instability in the dual space of eigenmodes and wavenumber is described by energy-moment equations constructed from Eq. (1). These are equations for $|\beta_1|^2$, $|\beta_2|^2$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle$, and $\mathrm{Im}\langle\beta_1^*\beta_2\rangle$, obtained by forming appropriate moments of Eq. (1).

$$\left[\frac{\partial}{\partial t} + i\omega_j - i\omega_l^*\right]\left\langle\beta_j\beta_l^*\right\rangle = -\sum_{k'}\sum_{m=1}^{2}\left[T_{mjl}(k,k') + T_{mlj}^*(k,k')\right]. \tag{6}$$

Here $T_{mjl}(k,k') = (-1)^j C_m(k,k')\langle\beta_m'\beta_1''\beta_l^*\rangle$ is a triplet correlation of the eigenmode amplitudes. The indices (j,l) take the values (1,1), (2,2), (1,2), and (2,1) to recover equations for the four correlations. As noted in Ref. 10, the coupling coefficients $C_m(k,k')$ become very large when $k_y = 0$. This is due to a near singularity in k-space associated with the vanishing of $R_1(k) - R_2(k)$ for $k_y = 0$, up to collisional terms. Because $\beta_2(k_x, k_y = 0)$ is strongly driven by spectral transfer and damped at a rate slightly larger than the instability growth rate, the instability is saturated by this transfer. The transfer is controlled by correlations with β_2, *i.e.*, $|\beta_2|^2$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle$, and $\mathrm{Im}\langle\beta_1^*\beta_2\rangle$. Consequently it is important to determine the effect of these correlations on particle transport.

The particle flux, $\Gamma = -(c/B_0)\sum_k \mathrm{Im}\langle n_k \phi_{-k}\rangle$, is quadratic in n_k and ϕ_k. Hence it depends on the same correlations $|\beta_1|^2$, $|\beta_2|^2$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle$, and $\mathrm{Im}\langle\beta_1^*\beta_2\rangle$ that mediate saturation. Writing the particle flux in the eigenmode decomposition, $\Gamma = -\sum_k k_y\left[\mathrm{Im}R_1|\beta_1|^2 + \mathrm{Im}R_2|\beta_2|^2 + \mathrm{Im}(R_1 + R_2)\mathrm{Re}\langle\beta_1^*\beta_2\rangle + \mathrm{Re}(R_2 - R_1)\mathrm{Im}\langle\beta_1^*\beta_2\rangle\right]$, where Γ has been normalized to the product of sound speed and mean density, and all quantities are understood to be functions of wave number k. From Eq. (5) for R_j and the roots of the characteristic equation, the flux is

$$\begin{aligned}\Gamma &= \sum_k k_y \Bigg\{ \frac{[\hat{\alpha}(1+k^2)-1][1+k^2-\varepsilon^{1/2}]}{(1-\hat{\alpha}\varepsilon^{1/2})^3}\left(\frac{\nu}{k_y v_D}\right)|\beta_1|^2 - \frac{(1-\hat{\alpha}\varepsilon^{1/2})}{\varepsilon^{1/2}}\left(\frac{k_y v_D}{\nu}\right)|\beta_2|^2 \\ &- \frac{(1-\hat{\alpha}\varepsilon^{1/2})}{\varepsilon^{1/2}}\left(\frac{k_y v_D}{\nu}\right)\mathrm{Re}\langle\beta_1^*\beta_2\rangle - \frac{[2\varepsilon^{1/2}-(1+k^2)(1+\hat{\alpha}\varepsilon^{1/2})]}{(1-\hat{\alpha}\varepsilon^{1/2})\varepsilon^{1/2}}\mathrm{Im}\langle\beta_1^*\beta_2\rangle \Bigg\} . \qquad (7)\end{aligned}$$

The first term of Eq. (7) is the quasilinear flux, while the remaining three terms are the nonlinear components of the flux. They lie wholly outside quasilinear theory. (In quasilinear theory the amplitude of the unstable eigenmode $|\beta_1|^2$ is identical to the fluctuation level $|\phi_k|^2$, and $\beta_2 = 0$.) As expected for an unstable eigenmode of a simple drift wave model, the quasilinear flux is outward. The second term is a contribution coming entirely from the stable eigenmode, and, as expected, it is inward. The signs of the third and fourth terms are not tied in any simple way to transparent physical considerations. They can only be determined from solutions of the saturation equation.

The spectrum evolution equations are solved analytically as follows: 1) The triplet correlations $T_{mjl}(k,k')$ are recast as products of the second order correlations using statistical closure theory. Applying the eddy damped quasinormal Markovian closure,

$$\begin{aligned}T_{mjl}(k,k') = \frac{(-1)^{j+1}C_m(k,k')}{iW_{m1l}}\sum_{p=1}^{2}\Bigg\{(-1)^m\Big[C_p(k',k)\langle\beta_p\beta_l^*\rangle|\beta_1''|^2 + C_p(k',k'-k) \\ \times \ \langle\beta_p''^*\beta_1''\rangle\langle\beta_1\beta_l^*\rangle\Big] - C_p(k-k',-k')\langle\beta_p'^*\beta_m'\rangle\langle\beta_1\beta_l^*\rangle - C_p(k-k',k)\langle\beta_p\beta_l^*\rangle\langle\beta_1'^*\beta_m'\rangle \\ + \ (-1)^l\Big[C_p^*(k,k')\langle\beta_p'^*\beta_m'\rangle|\beta_1''|^2 + C_p^*(k,k-k')\langle\beta_p''^*\beta_1''\rangle\langle\beta_1'^*\beta_m'\rangle\Big]\Bigg\} \qquad (8)\end{aligned}$$

where $iW_{m1l} = i\omega_m' + i\omega_1'' - i\omega_l^* - \Delta\omega_m' - \Delta\omega_1'' - \Delta\omega_l^*$ is the turbulent response function, and $\Delta\omega_m$ is the turbulent (amplitude-dependent) frequency of the eigenmode m. Expressions for $\Delta\omega_1$ and $\Delta\omega_2$ are given in Ref. 7. We assume that W_{m1l} is dominated by the linear frequencies, consistent with a wave-dominated regime valid for $k_y v_D \hat{\alpha} > k^2 n_k$. This is a weak turbulence approximation that is nominally valid for the long wavelengths that dominate the particle flux, and the near threshold conditions explained below. 2) Each spectrum and spectrum evolution equation is projected onto two complementary spectrum subranges, a $k_y = 0$ zonal component denoted with subscript Z, and a spectrum averaged component that excludes $k_y = 0$, denoted with subscript T. 3) The eight resulting spectral components are solved for their dependence on the system parameters residing in ω_j and R_j. These parameters are the electron detrapping rate ν, the diamagnetic frequency ω^*, the ratio η_e of electron density scale length to temperature gradient scale length, and the trapping fraction $\varepsilon^{1/2}$. The dependence of the correlations on the gradient-dependent quantities ω^* and η_e must be determined to derive flux-gradient relationships that characterize the thermodynamics of the transport and set the profiles. Because the closed spectrum-balance equations are very complicated, it is necessary to simplify utilizing an ordered expansion $\nu/\omega^* \sim k \sim \eta_e^2 \ll 1$. The ordering assumes a long wavelength, weakly collisional regime, slightly above the instability threshold $\eta_e = -k^2$. Asymptotic analysis identifies dominant balances. A set of conditions are applied to identify balances that represent a steady state driven by the instability and saturated by anisotropic transfer to the damped zonal models. These conditions are described in detail in Ref. 8.

3. Interplay between Zonal Modes and Damped Eigenmode

The dominant spectrum balances show that the energy flow in the dual space goes first from $|\beta_1|_T^2$, which is driven by the linear instability, to $\mathrm{Im}\langle\beta_1^*\beta_2\rangle_T$. From there it is transferred to $|\beta_2|_T^2$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle_T$, and the zonal spectra $|\beta_1|_Z^2$, $|\beta_2|_Z^2$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle_Z$, and $\mathrm{Im}\langle\beta_1^*\beta_2\rangle_Z$. Linear damping in $|\beta_2|_T^2$, $|\beta_2|_Z^2$, an $\mathrm{Im}\langle\beta_1^*\beta_2\rangle_T$ enters the lowest order balances and allows the system to reach steady state. Net energy transfer to $|\beta_1|_Z^2$ requires residual flow damping beyond electron detrapping to achieve stationarity in $|\beta_1|_Z^2$. The saturation scalings are

$$\begin{aligned}
|\beta_1|_T^2 &= c_1\eta_e\varepsilon^{1/2}\nu^2/\bar{k}^6,\\
\mathrm{Im}\langle\beta_1^*\beta_2\rangle_T &= c_2\eta_e^2\varepsilon\nu^3/\omega_*\bar{k}^6,\\
\mathrm{Re}\langle\beta_1^*\beta_2\rangle_T &= c_3\eta_e\varepsilon^{1/2}\nu^4/\omega_*^2\bar{k}^6,\\
|\beta_2|_T^2 &= c_4\eta_e^2\varepsilon^{1/2}\nu^4/(\omega_*^2\bar{k}^6),\\
|\beta_1|_Z^2 &= c_5\eta_e\nu^2/\bar{k}^6,\\
-\mathrm{Re}\langle\beta_1^*\beta_2\rangle_Z = |\beta_2|_Z^2 &= c_6\eta_e^2\varepsilon^{1/2}\nu^2/\bar{k}^6,\\
\mathrm{Im}\langle\beta_1^*\beta_2\rangle_Z &= c_7\nu\omega_*/\varepsilon^{1/2}\bar{k}^4. \qquad (9)
\end{aligned}$$

The crucial effect on saturation of anisotropic spectral transfer to zonal modes is illustrated by the scalings predicted from the spectrum balance equations when zonal modes are removed from the coupling. The damping of the stable eigenmode still saturates the turbulence, but the scalings are different: $|\beta_1|_T^2 \sim \omega_*^2/\bar{K}^4$, $\mathrm{Im}\langle\beta_1^*\beta_2\rangle_T \sim \nu\omega_*\eta_e\varepsilon^{1/2}/\bar{k}^4$, $\mathrm{Re}\langle\beta_1^*\beta_2\rangle_T \sim \nu^2\varepsilon/\bar{k}^4$, and $|\beta_2|_T^2 \sim \nu^2\varepsilon^{1/2}/\bar{k}^4$. In particular, the energy in the unstable eigenmode is proportional to ν^2 when coupling to zonal modes is included; when excluded, the energy is proportional to ω_*^2. This is consistent with the well-known decrease in fluctuation level associated with zonal modes.

4. Flux-Gradient Relationships of Inward and Outward Flux Components

Because electrons drive the instability in TEM, the net particle flux is constrained by thermodynamics to be outward in steady state. Consequently the inward flux associated with the damped eigenmode cannot exceed the outward flux produced by the unstable eigenmode. Simulations show that the former is a sizable fraction of the latter, reducing the flux to 0.1-0.3 of its quasilinear value. In terms of the eigenmode correlations the flux is given by

$$\Gamma = \sum_k k_y\Big[\eta_e(\nu\omega_*)|\beta_1|^2 - \frac{\omega_*}{\nu}|\beta_2|^2 - \frac{\omega_*}{\nu}\mathrm{Re}\langle\beta_1^*\beta_2\rangle + \frac{1}{\varepsilon^{1/2}}\mathrm{Im}\rangle\beta_1^*\beta_2\rangle\Big]. \qquad (10)$$

As always the presence of the factor k_y prohibits zonal modes from directly contributing to transport. The first term is the quasilinear flux; the remaining terms describe the contribution of the stable eigenmode. Substituting from the saturation scalings given above, the flux becomes

$$\Gamma = \sum_k k_y\Big(\frac{\nu^3}{\omega_*\bar{k}^6}\Big)\eta_e\varepsilon^{1/2}\Big[(c_1+c_2-c_4)\eta_e - c_3\Big]. \qquad (11)$$

This flux is nondiffusive in all terms. The quasilinear term goes as L_n^3/L_T^2, as do the outward directed component from $\mathrm{Im}\langle\beta_1^*\beta_2\rangle$ and the inward component from $|\beta_2|_T^2$. The second inward component, from $\mathrm{Re}\langle\beta_1^*\beta_2\rangle_T$, goes as L_n^2/L_T. Because of the proportionality to L_n^3 and L_n^2 the flux is neither diffusive nor convective. For reference, diffusive transport goes as $1/L_n$, and the thermodiffisive (convective) pinch goes as $1/L_T$. The anisotropic saturation involving zonal modes not only lowers fluctuation levels, but it changes the flux-gradient relationships. If anisotropic transfer to zonal modes is removed all flux terms are diffusive or convective.

5. Damped Eigenmode Excitation

It is important to establish whether the effects described above are peculiar to the TEM model of Eqs. (3)-(4), or whether they are intrinsic to many different kinds of instability-driven plasma turbulence. To begin to answer this question we examine an entirely different type of plasma turbulence. We also formulate general conditions predicting when damped eigenmode excitation will affect transport. We examine a simple model for ion turbulence driven by the ion temperature gradient [9]. The model equations are given by

$$(1+k^2)\frac{\partial\phi}{\partial t} - ik_y v_D\phi(\hat{\eta}k^2-1) + ik_z u_\parallel = -\sum_{k'}(k'\times\hat{z}\cdot k)\phi_{k'}\phi_{k-k'}k'^2 \equiv (1+k^2)N_\phi, \qquad (12)$$

$$\frac{\partial u_\parallel}{\partial t} + ik_z\phi + ik_z p = -\sum_{k'}(k'\times\hat{z}\cdot k)\phi_{k'}u_{\parallel k-k'} \equiv N_{u_\parallel}, \qquad (13)$$

$$\frac{\partial p}{\partial t} + ik_y v_D\hat{\eta}\phi = -\sum_{k'}(k'\times\hat{z}\cdot k)\phi_{k'}p_{k-k'} \equiv N_p, \qquad (14)$$

where $v_D \equiv (cT_e/eB)d(\ln n_0)/dx$ is the drift velocity, $\hat{\eta} = (1+\eta_i)/\tau$, $\eta_i = d(\ln T_i)/d(\ln n_0)$ is the ratio of temperature to density gradient scale lengths, $\tau \equiv T_e/T_i$ is the ratio of electron to ion temperature, and ϕ, $u_\parallel$, and p are the potential, parallel ion flow, and ion pressure at wavenumber k unless otherwise noted. These are normalized according to $\phi \equiv e\Phi/T_e$, $u_\parallel \equiv \tilde{v}_{\parallel i}/c_s$, and $p \equiv [\tilde{p}_i/\langle P_{i0}\rangle](T_i/T_e)$, where $P_i = \langle P_{i0}\rangle + \tilde{p}_i$. Length scales are normalized to $\rho_s = (cT_e/eB)(m_i/T_e)^{1/2}$.

This system differs from the TEM system in several ways. It is a model for ion turbulence with adiabatic electrons. In contrast, the TEM model describes electron turbulence in which nonadiabatic electrons are responsible for the nonlinear density response. The ion model has no particle transport, but does drive ion heat and parallel momentum fluxes. The ion model has three fluctuating fields instead of two. There are therefore three linear eigenmodes. One is unstable for an intermediate wavenumber band, one is stable for all wavenumbers, and one is marginally stable for the intermediate band and stable for higher wavenumbers. This mimics the eigenmode structure of ITG turbulence. Like the TEM model, the ion model is local, *i.e.*, there is no radial eigenmode. It is also a fluid model. Most importantly, the ion pressure and parallel flow fields are governed by the advective nonlinearity $\nabla\phi\times\hat{z}\cdot\nabla$. This is the same nonlinearity of the TEM system, but is also generic to virtually all types of plasma turbulence.

Equations (12)-(14) are already sufficiently complex that doing analytic theory becomes a daunting prospect. Instead we solve the system numerically. This exercise provides a template for numerical analysis of more complicated computational models to determine if damped eigenmodes are excited in them, and what role such eigenmodes play in saturation and transport. An eigenmode solver is applied to the linearized equations to determine the eigenmodes for a given parameter configuration. The full nonlinear equations are then solved numerically as an initial value problem, providing time histories for $u_{\|}(t)$, $p(t)$, and $\phi(t)$. At each point in the temporal evolution $u_{\|}$(t), $p(t)$, and $\phi(t)$ are projected onto the complete basis of the linear eigenmodes. This produces a time history of each eigenmode. The results of this procedure as applied to Eqs. (12)-(14) are shown in Fig. 1. The figure displays the time history of each eigenmode in terms of its energy $|\beta_j(t)|^2$. The total energy is also plotted. It is not equal to the sum of the eigenmode energies $\sum_j |\beta_j(t)|^2$ because cross correlations also contribute to the energy for this system of nonorthogonal eigenvectors. Two features of this evolution are important. First, all three eigenvectors grow exponentially, but only one is linearly driven. The other two eigenvectors are driven by mode-mode coupling. Second, the nonlinearly driven eigenmodes (linearly stable and marginally stable) saturate at a level that is comparable to that of the unstable mode. Indeed, later in the simulation the level of the stable modes exceeds the level of the unstable mode. From the analysis of the TEM system we can anticipate a significant effect on transport fluxes, and because damped modes are involved, we would expect the fluxes to be smaller.

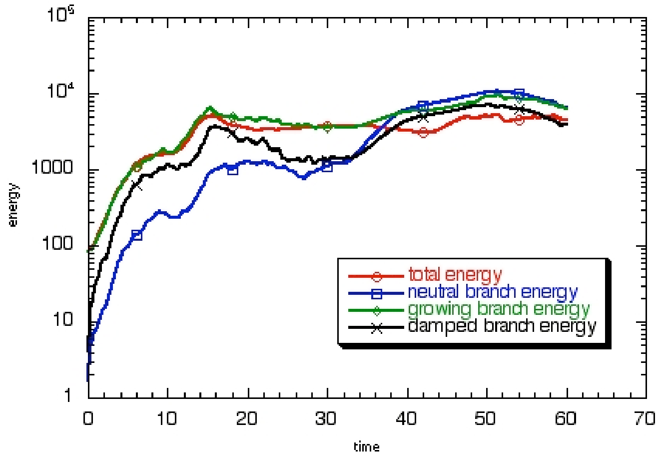

FIG. 1. Nonlinear evolution of energies, showing exponential growth and saturation of the total energy and the energy of the unstable, stable and marginally stable eigenmodes.

The radial heat transport is given by $\chi = \sum_k k_y \langle p_{-k}\phi_k\rangle$. Figure 2 displays this flux along with the quasilinear heat flux. The latter is calculated by taking the linearized response of p_{-k} to the potential ϕ_k and using the simulation results for the value of $|\phi_k|^2$. We see that the true flux is indeed considerably smaller than the quasilinear flux. It is also apparent that the flux and turbulence are highly bursty, and that the flux becomes negative during certain periods. More frequently, the flux is a very small fraction of the quasilinear flux, indicating a large negative contribution from the stable eigenmodes.

A general condition has been formulated to indicate when the inward flux component of a stable eigenmode significantly alters the outward quasilinear flux. To affect the flux it must first grow to finite amplitude. A simple parametric instability analysis applied to the eigenmode decomposition shows that stable eigenmodes generally grow exponentially from infinitesimal levels when the unstable eigenmode initially achieves a higher amplitude. This is not a condition of the nonlinear growth per se, but makes the parametric instability approximation valid. Under that approximation the nonlinearity is dominated by two wavenumbers on the unstable mode directly driving a coupled wavenumber on the stable mode. Because the unstable eigenmode grows promptly from initial conditions and the stable eigenmodes first decay, the parametric instability analysis is generally valid for early times. To determine the conditions required for a nonlinearly driven stable eigenmode to affect saturation we consider a simple model of an unstable eigenmode x_1 generically coupled to a stable eigenmode x_2 according to $\dot{x}_1 = \gamma_1 x_1 + B_1 x_1^2 + D_1 x_1 x_2$ and $\dot{x}_2 = -\gamma_2 x_2 + B_2 x_1^2 + \ldots$. There may be other eigenmodes but we focus on x_1 as the fastest growing linearly unstable eigenmode, and x_2 as the stable eigenmode with the strongest nonlinear drive. This model applies to a decomposition of evolution equations into a representation that diagonalizes the linear coupling. The simple question is the following: given a growth rate γ_1, a stable eigenmode damping rate γ_2, a nonlinear drive of the stable eigenmode $B_2x_1^2$, and a nonlinear feedback $D_1x_1x_2$ of the stable eigenmode on the unstable eigenmode, when does the stable eigenmode saturate at a sufficiently high level to allow it to compete with a saturation $B_1x_1^2$ of the unstable eigenmode by itself? Analysis shows that this occurs when

$$P_t = \frac{D_1B_2}{B_1^2(2+\gamma_2/\gamma_1)} \geq 1. \qquad (15)$$

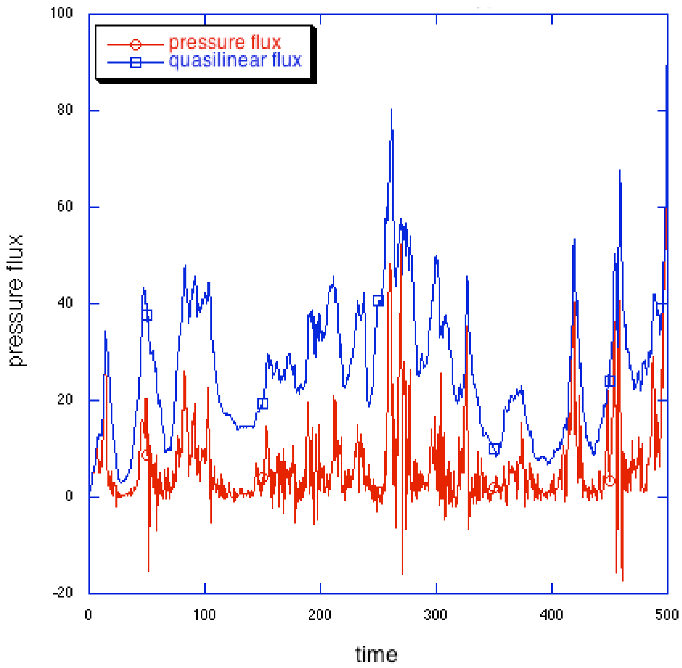

FIG. 2. Heat flux for ion temperature gradient model showing the quasilinear approximation and the smaller true flux.

This condition indicates that damped eigenmodes play a significant role in transport provided their damping is not significantly larger than the instability drive. Note that a damped eigenmode whose damping rate is of the same order as the linear growth rate is at no significant disadvantage relative to a marginally stable mode in terms of playing a role in saturation. This contradicts a common view that only marginally stable or very weakly damped modes (like zonal flows) can be expected to play any role in saturation. For weakly collisional TEM $P_t \gg 1$, consistent with the results reported in this paper. For that system γ_2 is slightly larger than γ_1. For the ion temperature gradient model, $P_t \approx 1$ for both stable eigenmodes. Equation (15) also requires favorable coupling such that D_1B_2/B_1^2 is not much smaller than unity. The coupling coefficients B_1, B_2, and D_1 depend on the projection of the evolution equations in the original representation onto the linear eigenmodes, *i.e.*, on the linear eigenvectors. Because the projection can be very complicated in its dependence on the system parameters and the eigenvector components, general rules are difficult to formulate for the type of eigenmodes and coupling that favor a significant role for damped eigenmodes. This question is presently under study and will be reported elsewhere.

[1] WARE, A.A., Phy. Rev. Lett. **25**, 15 (1970).
[2] COPPI, B., and C.S. Spight, Phys. Rev. Lett. **41**, 551 (1978).
[3] TERRY, P.W., Phys. Fluids B **1**, 1932 (1989).
[4] NYCANDER, J., and V.V. Yankov, Phys. Plasmas **2**, 2874 (1995).
[5] BOURDELLE, C., Plasma Phys. Controlled Fusion **47**, A317 (2005).
[6] ANGIONI, C., et al., Phys. Plasmas **12** , 112310 (2005).
[7] BAVER, D.A., P.W. Terry, R. Gatto, and E. Fernandez, Phys. Plasmas **9**, 3318 (2002).
[8] TERRY, P.W., and R. Gatto, Phys. Plasmas **13**, 062309 (2006).
[9] TERRY, P.W., D.A. Baver, and Sangeeta Gupta, Phys. Plasmas **13**, 022307 (2006).
[10] GATTO, R., P.W. Terry, and D.A. Baver, Phys. Plasmas **13**, 022306 (2006).