Thermal Diffusion by Stochastic Electromagnetic Fluctuations

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Abstract. A new simple systematic method has been developed to analytically evaluate the thermal diffusion coefficient of guiding center test particles brought by coexisting homogeneously stochastic electrostatic and electromagnetic fluctuations. As a most simple case, thermal diffusion coefficients for electrons and ions are analytically obtained in a large aspect straight tokamak with a small gyro-radius and negligible magnetic shear and negligible equilibrium $\vec{E} \times \vec{B}$ flow shear. Those analytical formulae are applicable to the range beyond so-called quasi-linear limit; thermal diffusion coefficient are squarely (linearly) proportional to fluctuation amplitudes in the quasi-linear (beyond quasi-linear) region. It is shown that the thermal diffusion of electrons (ions) is mainly dominated by magnetic (electrostatic) fluctuations in the experimentally relevant situations, even if both magnetic and electrostatic fluctuations coexist. It is also shown that the resonant electrons do not diffuse when the electric field parallel to the unperturbed magnetic field lines is negligible, even if electrostatic and electromagnetic fluctuations coexist.

1. Introduction

In the standard high-β experimental conditions, electrostatic and electromagnetic micro fluctuations are considered to coexist. The synergetic treatments of both electrons and ions in the gyrokinetic simulations allowing both electrostatic and electromagnetic fluctuations are considered to be quite difficult in the standard experimental situations of LHD and tokamaks. Therefore, it is quite useful to evaluate an analytical formula of the thermal diffusion coefficient of the test guiding particles in coexisting electrostatic and electromagnetic fluctuations, even if such fluctuations are not self-consistent field but given field. There may be cases that the transport of guiding center test particles by the fluctuations is regarded as a diffusion process due to stochastic instability of orbits even in the collisionless limit. The purpose of this paper is to show a new simple systematic method to derive an analytical formula of the thermal diffusion coefficient. As a most simple example, analytical formulae of thermal conductivities for electrons and ions are derived in a large aspect straight tokamak with a small gyro-radius and negligible magnetic shear and negligible equilibrium $\vec{E} \times \vec{B}$ flow shear. Those analytical formulae are applicable to the range beyond so-called quasi-linear limit [1], [2], because a simple renormalization technique and the realization of the stochastic instability [3], [4] are included by regarding the deterministic equations as the stochastic differential equation (SDE).

2. Derivation of mono-energetic conductivity

The deterministic equation of motion of guiding center particles including the electrostatic and electromagnetic fluctuations is derived from the Lagrangian [5];

$$\vec{v} = v_{||} \frac{\vec{B} + \delta\vec{B} + \nabla \times (\rho_{||}\vec{B})}{B + \delta B_{||} + \rho_{||} J_{||}}, \tag{1}$$

where $v_{||}$ is the velocity parallel to the unperturbed magnetic field $\vec{B}$, $\rho_{||} \equiv v_{||}/\Omega$ with the gyro-frequency Ω, and $\delta\vec{B} = \nabla \times \delta\vec{A}$ is a perturbed magnetic field with $\delta\vec{A} = \alpha\vec{B} = \delta A_{||}\hat{n}$ and $\hat{n} \equiv \vec{B}/B$. Note that only perpendicular magnetic perturbations are considered, because a strong longitudinal magnetic field is assumed. An electrostatic perturbation $\delta\phi$ is also included, which appears through $\rho_{||}$. In a large aspect straight tokamak with a small gyro-radius or strong equilibrium magnetic field, the equation of motion of the test guiding center particles is approximately expressed in the cylindrical coordinates (r, θ, ζ) as

$$\frac{dr}{dt} \sim \tilde{g}_r(\vec{r}(t), t), \quad \frac{d\theta}{dt} \sim v_{||}\frac{\iota}{R} - \omega_{E\times B} - \tilde{g}_\theta(\vec{r}(t), t), \quad \frac{d\zeta}{dt} \sim v_{||}\frac{1}{R} \tag{2}$$

where R is the major radius, ι and $\omega_{E\times B}$ are equilibrium rotational transform and $\vec{E}\times\vec{B}$ frequency, respectively. The parts $\tilde{g}_r(\vec{r}(t), t)$ and $\tilde{g}_\theta(\vec{r}(t), t)$ are fluctuating parts due to the electrostatic and electromagnetic perturbations;

$$\tilde{g}_r(\vec{r}(t), t) \equiv v_{||}\frac{1}{rB}\frac{\partial \delta A_{||}}{\partial \theta} - \frac{1}{rB}\frac{\partial \delta\phi}{\partial \theta}, \tag{3}$$

$$\tilde{g}_\theta(\vec{r}(t), t) \equiv v_{||}\frac{1}{rB}\frac{\partial \delta A_{||}}{\partial r} - \frac{1}{rB}\frac{\partial \delta\phi}{\partial r}, \tag{4}$$

where the electromagnetic and electrostatic perturbations are expressed as

$$\delta A_{||} = \sum_{mn} \delta A_{||mn}(r) \cos\left[n\zeta - m\theta + \delta_{mn}^{(\delta A)} - \omega_{mn}^{(\delta A)} t\right], \tag{5}$$

$$\delta\phi = \sum_{mn} \delta\phi_{mn}(r) \cos\left[n\zeta - m\theta + \delta_{mn}^{(\delta\phi)} - \omega_{mn}^{(\delta\phi)} t\right]. \tag{6}$$

with m and n being the poloidal and toroidal mode numbers, respectively. Taking into account the fact that fluctuating quantities cause the stochastic instability of orbits [4], the parts $\tilde{g}_r(\vec{r}(t), t)$ and $\tilde{g}_\theta(\vec{r}(t), t)$ are regarded as Gaussian noises without mean value, so that Eq.(1) becomes a Stochastic Differential Equation (SDE). The formal solution of this SDE is expressed under the locality of the radial diffusion as

$$\begin{aligned} r(t) &= r(t_0) + \int_{t_0}^{t} d\tau \tilde{g}_r(r = r(t_0), \theta(\tau), \zeta(\tau), \tau), \quad \zeta(t) = \zeta(t_0) + v_{||}\frac{1}{R}(t - t_0), \\ \theta(t) &= \theta(t_0) + \left[v_{||}\frac{\iota}{R} - \omega_{E\times B}\right]_{r(t)=r(t_0)} (t - t_0) + \int_{t_0}^{t} d\tau \tilde{g}_\theta(r = r(t_0), \theta(\tau), \zeta(\tau), \tau), \end{aligned} \tag{7}$$

where low magnetic shear $d\iota/dr \sim 0$ and low velocity shear $d\omega_{E\times B}/dr \sim 0$ are assumed. From the definitions of the two time-point Lagrangian auto-correlation function and the running diffusion coefficient for mono-energetic particles, we see

$$\begin{aligned} &\mathcal{R}_r(t, \tau) = \langle \tilde{g}_r(r = r(t_0), \theta(t), \zeta(t), t)\, \tilde{g}_r(r = r(t_0), \theta(\tau), \zeta(\tau), \tau) \rangle, \\ &\mathcal{R}_\theta(t, \tau) = \langle \tilde{g}_\theta(r = r(t_0), \theta(t), \zeta(t), t)\, \tilde{g}_\theta(r = r(t_0), \theta(\tau), \zeta(\tau), \tau) \rangle, \\ &D_r(t, t_0) = \int_{t_0}^{t} d\tau \mathcal{R}_r(t, \tau), \quad D_\theta(t, t_0) = \int_{t_0}^{t} d\tau \mathcal{R}_\theta(t, \tau), \quad t \geq \tau. \end{aligned} \tag{8}$$

where $\langle Q \rangle$ means the ensemble average of Q. By substituting the formal solution Eq. (7), including the perturbed particle orbits through the time integrations of $\tilde{g}_r(r = r(t_0), \theta(t), \zeta(t), t)$

and $\tilde{g}_\theta(r=r(t_0),\theta(t),\zeta(t),t)$, into Eq. (8), the renormalization of the Lagrangian auto-correlation function has been done. Note that in the quasi-linear approximation, unperturbed particle orbits, expressed by Eq. (7) setting $\tilde{g}_r(r=r(t_0),\theta(t),\zeta(t),t)=0$ and $\tilde{g}_\theta(r=r(t_0),\theta(t),\zeta(t),t)=0$, are used. Taking account of the Gaussian statistical properties without the mean value of $\tilde{g}_r(r=r(t_0),\theta(t),\zeta(t),t)$ and $\tilde{g}_\theta(r=r(t_0),\theta(t),\zeta(t),t)$, namely $C_{l=1}=C_{l\geq 3}=0$ (C_l is l-th cumulant and $\left\langle e^{\pm i\xi}\right\rangle = \exp\left[\sum_{l=1}^{\infty}(\pm i)^l C_l/l\right] = e^{-\left\langle \xi^2\right\rangle/2}$), and assuming that the Lagrangian auto-correlation function has a finite correlation time, a long term limit of Eq. (8) will be taken, where the statistical properties are considered to become stationary:

$$\mathcal{R}_r(t,\tau) \sim \mathcal{R}_r(t-\tau) \sim \frac{D_r}{\tau_{ac}^r}\exp\{-\frac{t-\tau}{\tau_{ac}^r}\},\ \ \tau_{ac}^r \sim \frac{1}{\overline{k}_r^2 D_r}, \tag{9}$$

$$\mathcal{R}_\theta(t,\tau) \sim \mathcal{R}_\theta(t-\tau) \sim \frac{D_\theta}{\tau_{ac}^\theta}\exp\{-\frac{t-\tau}{\tau_{ac}^\theta}\},\ \ \tau_{ac}^\theta \sim \frac{1}{(r\overline{k}_\theta)^2 D_\theta}, \tag{10}$$

where τ_{ac}^r and τ_{ac}^θ are the correlation time corresponding to the Lagrangian auto-correlation function $\mathcal{R}_r(t,\tau)$ and $\mathcal{R}_\theta(t,\tau)$, respectively. The resultant mono-energetic diffusion coefficient consisting of non-damping terms is expressed as

$$\begin{aligned}
D_r &= \lim_{t-t_0 \gg \tau_{ac}^r}\int_{t_0}^{t} d\tau \mathcal{R}_r(t,\tau) \sim \lim_{t-t_0 \gg \tau_{ac}^r}\int_{t_0}^{t} d\tau \mathcal{R}_r(t-\tau) \\
&\sim \frac{1}{2}\sum_{mn}\Bigg\{\left[v_{||}\frac{m\delta A_{||mn}}{rB}\right]^2 m^2 D_\theta \\
&\quad - v_{||}\frac{m\delta A_{||mn}}{rB}\frac{m\delta\phi_{mn}}{rB}\left[m^2 D_\theta \cos\Theta(t) + \left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mn}^{(\delta A)}\right]\sin\Theta(t)\right]\Bigg\} \\
&\quad \times \frac{1}{\left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mn}^{(\delta A)}\right]^2 + [m^2 D_\theta]^2} \\
&+ \frac{1}{2}\sum_{mn}\Bigg\{\left[\frac{m\delta\phi_{mn}}{rB}\right]^2 m^2 D_\theta \\
&\quad - v_{||}\frac{m\delta A_{||mn}}{rB}\frac{m\delta\phi_{mn}}{rB}\left[m^2 D_\theta \cos\Theta(t) - \left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mn}^{(\delta\phi)}\right]\sin\Theta(t)\right]\Bigg\} \\
&\quad \times \frac{1}{\left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mn}^{(\delta\phi)}\right]^2 + [m^2 D_\theta]^2}
\end{aligned} \tag{11}$$

where $\Theta(t) \equiv \delta_{mn}^{(\delta A)}-\delta_{mn}^{(\delta\phi)}-(\omega_{mn}^{(\delta A)}-\omega_{mn}^{(\delta\phi)})t$ and $k_{||} \equiv (n-m\iota)/R$. Note that the cross terms between magnetic fluctuations and electrostatic fluctuations exist with oscillatory behaviors. The renomalization is clear in that the diffusion coefficient D_θ appears in the denominator, which removes the singularity by the wave-particle resonances. Physically, it means that diffusive particle orbits remove continuous resonance. When the magnetic fluctuations and electrostatic fluctuations have such a close correlation that $\delta_{mn}^{(\delta A)} \sim \delta_{mn}^{(\delta\phi)}$ and $\omega_{mn}^{(\delta A)} \sim \omega_{mn}^{(\delta\phi)} \sim \omega_{mn}$, the oscillatory parts disappear since $\Theta(t) \sim 0$, so that the mono-energetic diffusion coefficient becomes

$$D_r \sim \frac{1}{2}\sum_{mn}\left[\frac{m}{rB}\left[v_{||}\delta A_{||mn}-\delta\phi_{mn}\right]\right]^2 \frac{m^2 D_\theta}{\left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mn}\right]^2+\left[m^2 D_\theta\right]^2}. \tag{12}$$

Stochastic instability of orbits is brought by simultaneous influences of multiple waves on orbits, so that particles feel infinite number of waves along their perturbed orbits. Because of this stochastic instability, the discrete parallel wave number $k_{||}$ can be treated as a continuous quantity [3], [4]; $\sum_{mn} = L_{||}/(2\pi)\sum_m \int_{\delta k_{||min}}^{\delta k_{||max}} dk_{||}$, where $L_{||}$ and $-\delta k_{||min} \sim \delta k_{||max} \sim L_{||}^{-1}$ are the parallel correlation length in the direction of the unperturbed magnetic field and the parallel wave number contributing to the diffusion around $k_{||} = 0$, respectively. Thus, perturbed quantities are labeled by the m and $k_{||}$. By assuming moderate variations of the amplitude and the frequency, Eq. (11) is modified and the resultant mono-energetic running diffusion coefficient at $r = r_0$ is expressed as

$$
\begin{aligned}
D_r(v_{||}) \;&\sim\; \frac{L_{||}}{4\pi}\sum_m \left\{ \left\langle \left[\frac{m\delta A_{||mk_{||}}}{rB}\right]^2 \right\rangle_{k_{||}} v_{||}^2 - \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\cos\Theta(t)\right\rangle_{k_{||}} v_{||} \right\} \\
&\times\; \int_{\delta k_{||min}}^{\delta k_{||max}} dk_{||}\frac{m^2 D_\theta}{\left[k_{||}v_{||} - \hat{\omega}_m^{(\delta A)}\right]^2 + \left[m^2 D_\theta\right]^2} \\
&-\; \frac{L_{||}}{4\pi}\sum_m \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\sin\Theta(t)\right\rangle_{k_{||}} v_{||} \\
&\times\; \int_{\delta k_{||min}}^{\delta k_{||max}} dk_{||}\frac{k_{||}v_{||} - \hat{\omega}_m^{(\delta A)}}{\left[k_{||}v_{||} - \hat{\omega}_m^{(\delta A)}\right]^2 + \left[m^2 D_\theta\right]^2} \\
&+\; \frac{L_{||}}{4\pi}\sum_m \left\{ \left\langle \left[\frac{m\delta\phi_{mk_{||}}}{rB}\right]^2 \right\rangle_{k_{||}} - \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\cos\Theta(t)\right\rangle_{k_{||}} v_{||} \right\} \\
&\times\; \int_{\delta k_{||min}}^{\delta k_{||max}} dk_{||}\frac{m^2 D_\theta}{\left[k_{||}v_{||} - \hat{\omega}_m^{(\delta\phi)}\right]^2 + \left[m^2 D_\theta\right]^2} \\
&+\; \frac{L_{||}}{4\pi}\sum_m \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\sin\Theta(t)\right\rangle_{k_{||}} v_{||} \\
&\times\; \int_{\delta k_{||min}}^{\delta k_{||max}} dk_{||}\frac{k_{||}v_{||} - \hat{\omega}_m^{(\delta\phi)}}{\left[k_{||}v_{||} - \hat{\omega}_m^{(\delta\phi)}\right]^2 + \left[m^2 D_\theta\right]^2},
\end{aligned}
\tag{13}
$$

where $\langle Q\rangle_{k_{||}}$ is the averaged value of Q with respect to the parallel wave number $k_{||}$ or the replacement of the $k_{||}$ dependence by the typical values at the initial position, and

$$
\begin{aligned}
\hat{\omega}_m^{(\delta A)} \;&\equiv\; \left\langle \omega_{mk_{||}}^{(\delta A)} \right\rangle_{k_{||}} - m\omega_{E\times B}, \\
\hat{\omega}_m^{(\delta\phi)} \;&\equiv\; \left\langle \omega_{mk_{||}}^{(\delta\phi)} \right\rangle_{k_{||}} - m\omega_{E\times B}.
\end{aligned}
\tag{14}
$$

By performing the integration on the parallel wave number $k_{||}$ analytically, the mono-energetic running diffusion coefficient at $r = r_0$ becomes as

$$\begin{aligned}
D_r(v_{||}) \sim\ & \frac{L_{||}}{4\pi}\sum_m \left\{ \left\langle \left[\frac{m\delta A_{||mk_{||}}}{rB}\right]^2 \right\rangle_{k_{||}} v_{||} - \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\cos\Theta(t)\right\rangle_{k_{||}} \right\} \\
\times & \left\{ \mathrm{Tan}^{-1}\left[\frac{\delta k_{||max}v_{||} - \hat{\omega}_m^{(\delta A)}}{\overline{k}_r^2 D_r}\right] - \mathrm{Tan}^{-1}\left[\frac{\delta k_{||min}v_{||} - \hat{\omega}_m^{(\delta A)}}{\overline{k}_r^2 D_r}\right]\right\} \\
- & \frac{L_{||}}{8\pi}\sum_m \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\sin\Theta(t)\right\rangle_{k_{||}} \\
\times & \left\{ \ln\left[\left[k_{||max}v_{||} - \hat{\omega}_m^{(\delta A)}\right]^2 + \left[\overline{k}_r^2 D_r\right]^2\right] - \ln\left[\left[k_{||min}v_{||} - \hat{\omega}_m^{(\delta A)}\right]^2 + \left[\overline{k}_r^2 D_r\right]^2\right]\right\} \\
+ & \frac{L_{||}}{4\pi}\sum_m \left\{ \left\langle \left[\frac{m\delta\phi_{mk_{||}}}{rB}\right]^2 \right\rangle_{k_{||}} \frac{1}{v_{||}} - \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\cos\Theta(t)\right\rangle_{k_{||}} \right\} \\
\times & \left\{ \mathrm{Tan}^{-1}\left[\frac{\delta k_{||max}v_{||} - \hat{\omega}_m^{(\delta\phi)}}{\overline{k}_r^2 D_r}\right] - \mathrm{Tan}^{-1}\left[\frac{\delta k_{||min}v_{||} - \hat{\omega}_m^{(\delta\phi)}}{\overline{k}_r^2 D_r}\right]\right\} \\
+ & \frac{L_{||}}{8\pi}\sum_m \left\langle \frac{m\delta A_{||mk_{||}}}{rB}\frac{m\delta\phi_{mk_{||}}}{rB}\sin\Theta(t)\right\rangle_{k_{||}} \\
\times & \left\{ \ln\left[\left[k_{||max}v_{||} - \hat{\omega}_m^{(\delta\phi)}\right]^2 + \left[\overline{k}_r^2 D_r\right]^2\right] - \ln\left[\left[k_{||min}v_{||} - \hat{\omega}_m^{(\delta\phi)}\right]^2 + \left[\overline{k}_r^2 D_r\right]^2\right]\right\},
\end{aligned} \tag{15}$$

where, D_θ is related to D_r through the relation; $D_\theta \sim \left(\overline{k}_r/(r\overline{k}_\theta)\right)^2 D_r$, where $\overline{k}_r$ and $\overline{k}_\theta$ are the typical values of $k_r \sim (\partial\delta A_{||mn}/\partial r)/\delta A_{||mn} \sim (\partial\delta\phi_{||mn}/\partial r)/\delta\phi_{||mn}$ and $k_\theta \sim m/r$, respectively. The typical radial wave number $\overline{k}_r$ might be related to the perpendicular (radial) correlation length $L_\perp$ as $\overline{k}_r \gtrsim L_\perp^{-1}$.

3. Thermal conductivity in stationary uniform fluctuations without equilibrium flow

In this section, thermal conductivity in stationary fluctuations without equilibrium flow ($\omega_{E\times B}\dim 0$) is considered, where $\hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)} \sim 0$. Since uniform fluctuations are considered, $-\delta k_{||min} \sim \delta k_{||max} \sim \delta k_{||}$ might hold. Under this condition, the cross terms between magnetic and electrostatic fluctuations disappear. The new notations $\delta B_{rmk_{||}} = m\delta A_{||mk_{||}}/r$ and $\delta E_{\theta mk_{||}} = m\delta\phi_{mk_{||}}/r$ are used in this section. By using the approximation; $\mathrm{Tan}^{-1}(x) = x$ for $x \leq \pi/2$, and $\mathrm{Tan}^{-1}(x) = \pi/2$ for $x \geq \pi/2$, the nonlinear equation of $D_r(v_{||})$ given by Eq. (15) is solved under the conditions that $\hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)} \sim 0$ and $-\delta k_{||min} \sim \delta k_{||max} \sim \delta k_{||}$. After the velocity space integration, various limiting cases of the thermal conductivity are obtained.

A) only magnetic fluctuations

$$\chi_M^{(\alpha)} \sim \frac{4v_{T\alpha}\delta k_{||}}{\pi^{3/2}\overline{k}_r^2}\begin{cases} \mathcal{R}_M^2 & \text{for} \quad \mathcal{R}_M \leq 1 \\ \mathcal{R}_M & \text{for} \quad \mathcal{R}_M \geq 1 \end{cases} \tag{16}$$

where $\mathcal{R}_M$ is defined by

$$\mathcal{R}_M \equiv \left[\frac{\pi}{8} \frac{L_{||}\overline{k}_r^2}{\delta k_{||}} \sum_m \left\langle \left(\frac{\delta B_{rmk_{||}}}{B} \right)^2 \right\rangle_{k_{||}} \right]^{1/2} \tag{17}$$

and interpreted as a scale separator independent of particle species; namely the ratio of displacements by the diffusion to the perpendicular correlation length of fluctuations, when $\delta k_{||} \sim L_{||}^{-1}$ and $\overline{k}_r \sim L_\perp^{-1}$. $\mathcal{R}_M \ll 1$ corresponds to the collisionless quasi-linear limit [1], [2], and averaged (unperturbed) orbits are good approximation. For $\mathcal{R}_M \geq 1$, diffusive (perturbed) orbits must be used.

B) only electrostatic fluctuations

$$\chi_E^{(\alpha)} \sim \frac{4 v_{T\alpha} \delta k_{||}}{\pi^{3/2} \overline{k}_r^2} \left\{ \mathcal{R}_E^{(\alpha)} \int_0^{\mathcal{R}_E^{(\alpha)}} dx (1+x^2) e^{-x^2} + \left(\mathcal{R}_E^{(\alpha)} \right)^2 \int_{\mathcal{R}_E^{(\alpha)}}^{\infty} dx \left(\frac{1}{x} + x \right) e^{-x^2} \right\} \tag{18}$$

where $\mathcal{R}_E^{(\alpha)}$ is defined as

$$\mathcal{R}_E^{(\alpha)} \equiv \left[\frac{\pi}{8} \frac{L_{||}\overline{k}_r^2}{\delta k_{||}} \sum_m \left\langle \left(\frac{\delta E_{\theta mk_{||}}}{B v_{T\alpha}} \right)^2 \right\rangle_{k_{||}} \right]^{1/2}, \tag{19}$$

and interpreted as a scale separator of particle species α; namely the ratio of the displacements by the diffusion to the perpendicular correlation length of fluctuations, when $\delta k_{||} \sim L_{||}^{-1}$ and $\overline{k}_r \sim L_\perp^{-1}$. For the electrostatic fluctuations, perturbed (unperturbed) orbits have to be used for low (high) velocity particles independent of the magnitude of the fluctuations.

Note that in both cases A) and B), the diffusion coefficient is proportional to the square (linear) of the fluctuation amplitude for low (high) amplitude cases and that $\mathcal{R}_E^{(\alpha)}$ depends on the particle species α in contrast with $\mathcal{R}_M$.

C) coexisting electrostatic and magnetic fluctuations

Since $(\mathcal{R}_E^{(\alpha)}/\mathcal{R}_M)^2 \sim (c/v_{T\alpha})^2 \mathcal{R}$, with $\mathcal{R} \equiv (\varepsilon_0/2) \sum_m < (\delta E_{\theta mk_{||}}/B)^2 >_{k_{||}} /((1/2\mu_0) \sum_m < (\delta B_{\theta rk_{||}}/B)^2 >_{k_{||}})$, and $\mathcal{R}_E^{(e)} \sim (m_e/m_i)^{1/2} \mathcal{R}_E^{(i)} \ll \mathcal{R}_E^{(i)}$, three interesting cases exist depending on the ratio of the power spectra $\mathcal{R}$

1. $\mathcal{R} \sim 1, \ \mathcal{R}_E^{(i)} \gg \mathcal{R}_E^{(e)} \gg \mathcal{R}_M$
   The dominant thermal transport comes from electrostatic fluctuations given by Eq. (18) for both electrons and ions; $\chi^{(i)} \sim \chi_E^{(i)} > \chi^{(e)} \sim \chi_E^{(e)}$.

2. $\mathcal{R} \sim \dfrac{m_e}{m_i}, \ \mathcal{R}_E^{(i)} \gg \mathcal{R}_M \gg \mathcal{R}_E^{(e)}$
   The thermal transport of electrons is governed by the magnetic fluctuations given by Eq. (16), and the thermal transport of ions is governed by the electrostatic fluctuations given by Eq. (18); $\chi^{(i)} \sim \chi_E^{(i)} > \chi^{(e)} \sim \chi_M^{(e)}$.

3. $\mathcal{R} \ll \dfrac{m_e}{m_i}$
   The dominant thermal transport comes from magnetic fluctuations given by Eq. (16) for both electrons and ions; $\chi^{(i)} \sim \chi_M^{(i)} > \chi^{(e)} \sim \chi_M^{(e)}$.

4. Thermal conductivity in correlated fluctuations without equilibrium flow

In this section, thermal conductivity without equilibrium flow ($\omega_{E\times B} \sim 0$) is considered under the condition that the magnetic fluctuations and electrostatic fluctuations have such a strong correlation that $\delta_{mn}^{\delta A} \sim \delta_{mn}^{\delta\phi}$ and $\omega_{mn}^{\delta A} \sim \omega_{mn}^{\delta\phi}$. In this case, the mono-energetic running diffusion coefficient is given by Eq. (12). In a weak diffusion limit $D \to 0$, taking account of $\vec{E} = -\partial\vec{A}/\partial t - \nabla\phi$, Eq. (12) becomes

$$\begin{aligned} D_r &\sim \frac{\pi}{2}\sum_{mn}\left[\frac{m}{rB}\left[v_{||}\delta A_{||mn} - \delta\phi_{mn}\right]\right]^2 \delta(k_{||}v_{||} - \omega_{mn}) \\ &\sim \frac{\pi}{2}\sum_{mn}\frac{k_\theta^2}{B^2|k_{||}|}\left[\omega_{mn}\delta A_{||mn} - k_{||}\delta\phi_{mn}\right]^2 \delta\left(v_{||} - \frac{\omega_{mn}}{k_{||}}\right) \\ &\sim \frac{\pi}{2}\sum_{mn}\frac{k_\theta^2}{B^2|k_{||}|}E_{||}^2\,\delta\left(v_{||} - \frac{\omega_{mn}}{k_{||}}\right) \end{aligned} \tag{20}$$

Since the phase velocity $\omega_{mn}/k_{||}$ is very fast, only electrons mainly satisfy the resonance condition. Such resonant electrons, however, can not diffuse when the parallel electric field vanishes, even if both magnetic and electrostatic fluctuations coexist.

5. Summary and Discussion

A new simple systematic method is developed in order to analytically evaluate the thermal diffusion coefficient in the coexisting given electrostatic and electromagnetic fluctuations. The analytical formula of the thermal diffusion coefficient is obtained by considering the test guiding center orbits consisting of the free stream along the unperturbed magnetic field lines and motions due to the fluctuating coexisting magnetic and electrostatic fluctuations. It is shown that the thermal diffusion of electrons (ions) is mainly dominated by magnetic (electrostatic) fluctuations in the experimentally relevant situations, even if both magnetic and electrostatic fluctuations coexist. It is also shown that the resonant electrons do not diffuse when the electric field parallel to the unperturbed magnetic field lines is negligible, even if electrostatic and electromagnetic fluctuations coexist.

The present method is applicable to toroidal tokamaks with a circular cross section by using action-angle variables, where the effects of the trapped particles are included. In such a case, it might be expected that passing particles mainly contribute to the thermal diffusion, from the viewpoint of the velocity space integration.

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