Evolution of Anomalous Transport in Shear Flow of Toroidal Devices.

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Suppression of turbulence and anomalous transport by a sheared flows is a crucial issue Ref.[1] in fusion plasma physics. In the edge region of tokamaks, an abrupt increase in the poloidal sheared equilibrium $E \times H$ velocity results in the quenching of the edge turbulence, reducing the anomalous transport in the plasma edge and formation the regime of improved plasma confinement. The time scales associated with such transition (known as the L–H transition) usually occurs in times of the order of submillisecond or microsecond, which are less, or of the order of, the period of the observed drift waves and are significantly less than their inverse growth rate. The velocity shear, dv_0'/dr, of the formed at this transition edge shear flows is of the order of Ref.[2] or even exceeds Ref.[3] the frequency of the observed drift waves. Under such conditions the conventional modal approach to drift and drift Alfven turbulence is principally invalid because the evolution of drift–type disturbances due to flow shear occurs in times much less than the period of the modal drift wave in the shearless plasma.

In this report we present the results of the analysis of the temporal evolution of drift waves and instabilities and of the resulted anomalous transport under conditions of a strong flow shear. This analysis grounds on the non–modal approach, developed in the series of papers Ref.[4]–[8]. The transformation of spatial coordinates to coordinates convected with sheared flow, which is used in this approach, greatly facilitates the solution of the initial value problem for the plasma with shear flow. In these coordinates the system of governing equations does not contain any more the spatial dependency connected with the flow shear and the spatial dependence is transformed to the time domain. That gives the possibility to obtain the initial value problem solution for the waves and instabilities considered for any desired times of interest and for any magnitude of the velocity shear. The developed approach is easily generalized onto time–dependent shear flow. It was shown Ref.[4]–[8] that the conventional modal structure of drift, Alfven waves or eta–i modes holds only for a weak flow shear and a limited time at the initial stage of its evolution. At the longer finite times the waves and instabilities become non–modal with a time dependent amplitude, frequency and wavelength as the results of stretching the waves patterns by shear flow.

Here we present and discuss the temporal evolution of the drift waves on the base of the Hasegawa–Wakatani system for the perturbations of electron density $\tilde{n}$ and electrostatic potential φ, and interchange modes with Rayleigh-Taylor forcing. For the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature) the Hasegawa–Wakatani system of equations is Ref.[4]

$$\rho_s^2 \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} \left(n - \phi \right), \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) n + v_* \frac{\partial \phi}{\partial y} = a \frac{\partial^2}{\partial z^2} \left(n - \phi \right), \tag{2}$$

where $\mathbf{V} = \mathbf{v}_0(x) + (c/B^2)[\mathbf{B} \times \nabla\varphi]$ is the drift velocity, $a = T_e/n_0 e^2 \eta_\parallel$, $\eta_\parallel$ is the resistivity parallel to the homogeneous magnetic field $\mathbf{B}_\parallel \mathbf{z}$, ρ_s is the ion Larmor radius at electron temperature, $v_* = c\kappa T_e/eB$ is the diamagnetic drift velocity, $\kappa = -d \ln n_{0e}(x)/dx$. We consider the case of the flow with constant shear, for which $\mathbf{v}_0(x) = (c/B^2)[\mathbf{E}_0(x) \times \mathbf{B}] = v_0' x\, \mathbf{e}_y$ and v_0' is independent of x. For this case the analysis is greatly facilitated by a transformation to the coordinates convected with the sheared flow. This transformation is defined by the relations Ref.[4]–[**?**]

$$\tau = t, \qquad \xi = x, \qquad \eta = y - v_0(x)\, t = y - v_0' x t, \qquad z = z \tag{3}$$

Introducing Fourier expansion in z–coordinate with z–axis directed along the magnetic field, we find from system (1), (2) the following equation for the potential $\phi(t, \xi, \eta, k_z)$

$$\rho_s^2 \frac{\partial}{\partial \tau}(\Delta_{\perp c}\phi) - \frac{\partial \phi}{\partial \tau} - v_* \frac{\partial \phi}{\partial \eta} + \frac{\rho_s^2}{a k_z^2}\frac{\partial^2}{\partial \tau^2}(\Delta_{\perp c}\phi) = 0, \tag{4}$$

where

$$\Delta_{\perp c} = \frac{\partial^2}{\partial \eta^2} + \left(\frac{\partial}{\partial \xi} - v_0' t \frac{\partial}{\partial \eta}\right)^2. \tag{5}$$

The time dependence in $\Delta_{\perp c}$ is responsible for the shearing of the waves pattern by the basic flow. Note, that equation (4), doesn't contain spatially dependent coefficients. Performing Fourier–transformations of Eq.(4) over variables ξ and η,

$$\phi(\tau, k_\perp, l, k_z) = \iiint d\xi\, d\eta\, dz\, e^{-i(k_\perp \xi + l\eta + k_z z)} \phi(\tau, \xi, \eta, z) \tag{6}$$

equation for $\phi(\tau, k_\perp, l, k_z)$ is found from Eq.(4) to be

$$\frac{1}{C}\frac{\partial^2}{\partial T^2}\Big[\left(1+T^2\right)\phi\Big] + \frac{\partial}{\partial T}\Big\{\left[1 + l^2\rho_s^2\left(1+T^2\right)\right]\phi\Big\} + iSl\rho_s\phi = 0, \tag{7}$$

where a dimensionless time variable T is defined by $T = v_0'\tau - (k_\perp/l)$ and parameters C and S are equal respectively to $C = a k_z^2/\rho_s^2 l^2 v_0' = T_e k_z^2/\rho_s^2 l^2\, v_0' n_0 e^2 \eta_\parallel$, $S = l v_*/v_0' l \rho_s$.

The solution to the equation (7) for large values of the parameter $C \gg 1$ was obtained in Ref.[4] and it is equal to

$$\begin{aligned}
\phi(\tau, k_\perp, l, k_z) &= \phi(\tau = 0, k_\perp, l, k_z)\, \frac{1 + \rho_s^2\left(l^2 + k_\perp^2\right)}{1 + \rho_s^2 l^2 + \rho_s^2\left(l v_0'\tau - k_\perp\right)^2} \\
&\times \exp\left\{-i\frac{S}{\sqrt{1+\rho_s^2 l^2}}\left(\arctan\frac{\rho_s\left(l v_0'\tau - k_\perp\right)}{\sqrt{1+\rho_s^2 l^2}} + \arctan\frac{k_\perp\rho_s}{\sqrt{1+\rho_s^2 l^2}}\right)\right. \\
&+\frac{1}{C}\left[-2\left(v_0'\tau - \frac{k_\perp}{l}\right) - \frac{3}{2}i\frac{S}{l\rho_s} - \frac{1}{4}\frac{S^2\left(v_0'\tau - \frac{k_\perp}{l}\right)}{\left(1 + l^2\rho_s^2\right)}\right]\frac{1}{\left(1 + l^2\rho_s^2 + \rho_s^2\left(l v_0'\tau - k_\perp\right)^2\right)^2} \\
&+\frac{1}{C}\left[-2\frac{k_\perp}{l} + \frac{3}{2}i\frac{S}{l\rho_s} - \frac{1}{4}\frac{k_\perp}{l}\frac{S^2}{1+l^2\rho_s^2}\right]\frac{1}{\left(1 + \left(l^2 + k_\perp^2\right)\rho_s^2\right)^2} \\
&+\frac{1}{C}\left[i\frac{S}{l\rho_s} + \frac{1}{8}\frac{S^2\left(l v_0'\tau - k_\perp\right)\left(1 + 4l^2\rho_s^2\right)}{\left(1 + l^2\rho_s^2\right)^2 l}\right]\frac{1}{1 + l^2\rho_s^2 + \rho_s^2\left(l v_0'\tau - k_\perp\right)^2} \\
&+\frac{1}{C}\left[-i\frac{S}{l\rho_s} + \frac{1}{8}\frac{S^2 k_\perp\left(1 + 4l^2\rho_s^2\right)}{l\left(1 + l^2\rho_s^2\right)^2}\right]\frac{1}{1 + \rho_s^2\left(k_\perp^2 + l^2\right)} \\
&\left.+\frac{1}{8C}\frac{S^2\left(1 + 4l^2\rho_s^2\right)}{l\rho_s\left(1 + l^2\rho_s^2\right)^{5/2}}\left(\arctan\frac{\rho_s\left(v_0' l\tau - k_\perp\right)}{\sqrt{1+\rho_s^2 l^2}} + \arctan\frac{k_\perp\rho_s}{\sqrt{1+\rho_s^2 l^2}}\right)\right\}.
\end{aligned} \tag{8}$$

For Texas Experimental Tokamak (TEXT)[**?**] $T_e = 20$ eV, $n_{0e} = 2 \cdot 10^{12}$ cm^{-3}, $B_0 = 0.7 \cdot 10^4$G, $L_{v_0} = 0.5$ cm, $L_n = 1.5$ cm, $\rho_s = 0.06$ cm, $k_\perp \simeq l = 5$cm^{-1}, drift frequency $\omega_{dr} = 7.5 \cdot 10^5$ s^{-1},

$v_0' = 4.5 \cdot 10^5$ s^{-1} we have $S = 6$ and $C = 16$. For these parameters the temporal evolution of the potential $\phi(\tau, k_\perp, l, k_z)$, determined by the equation (8), is presented on fig.1a. For the Uragan–3M (U–3M) torsatron data [3] ($T_e = 50$ eV, $n_{0e} = 5 \cdot 10^{11}$ cm^{-3}, $B_0 = 0.7 \cdot 10^4$ G, $L_{v0} = 1$ cm, $L_n = 1.5$ cm, $k_\perp \simeq l = 1$cm^{-1}, $\rho_s = 0.1$ cm, $\omega_{dr} = 10^5 \div 1.4 \cdot 10^6$ s^{-1}, $v_0' = 7 \cdot 10^5$s^{-1}, $S = 6.6$ and $C = 43$) the function $\phi(\tau)$ is presented on fig.1b. It follows, that the suppression of the drift resistive instability for the above presented data for TEXT and U-3M is a non–modal process.

Numerical analysis of the solution (8) shows that the effect of the nonmodal suppression of the drift waves potential $\phi(\tau, k_\perp, l, k_z)$ depends predominantly on the parameter $S = v_*/v_0'\rho_s$, which is independent from the wave numbers of the unstable drift waves. We find that for $S \leq 7$, drift waves rapidly suppressed without temporal amplitude growth by the time of the order of one drift wave period. That admits to use for the electrostatic potential of the drift waves considered the power like time dependence (in the convective set of reference) in the form

$$\phi(\tau, k_\perp, l, k_z) \simeq \phi(\tau = 0, k_\perp, l, k_z) \frac{1 + \rho_s^2 (l^2 + k_\perp^2)}{1 + \rho_s^2 l^2 + \rho_s^2 (l v_0' \tau - k_\perp)^2} e^{i\alpha} \tag{9}$$

with $\alpha = const$ as of a convective cell with zero frequency. That asymptotic follows from the condition $|\rho_s l v_0' \tau| > 1$, which is the condition of the dominance of the non-modal evolution of the potential, resulted from the shear enhanced dispersion of drift waves. For the above parameters this asymptotic form is valid on times of the order of $\triangle t \simeq 10 v_0' \simeq 10^{-4}$s.

The studies of the dynamics of packets of nonmodal drift waves, have shown [4], that such wave packets are stagnated or reflected by the shear flow and the component of the group velocity along the flow shear rapidly vanish with time as $(v_0' t)^{-3}$. That implicitly suggests that in the regions with shear flow transport barriers are formed.

The analysis of the equation for the energy of the nonmodal drift disturbances in plasma shear flow,

$$\frac{1}{2}\frac{d}{dt}\int \left[\frac{1}{\rho_s^2} n^2 + (\nabla\phi)^2\right] dV = -\frac{v_*}{\rho_s^2}\int n \frac{\partial \phi}{\partial y} dV, \tag{10}$$

shows the decreasing the energy density with time as

$$\begin{aligned}\rho_s^2 \frac{\partial}{\partial t}(\nabla\phi)^2 = \iiint dkdldk_z\, 2\phi^2(t, k_\perp, l, k_z)\, v_0' \Big[v_0' t\, l^2 \rho_s^2 - \left(l^2 \rho_s^2 \left(1 + (v_0' t)^2\right) + k_z^2 \rho_s^2\right) \\ \times \left(2 v_0' t\, l^2 \rho_s^2 + i \frac{S}{(1 + l^2 \rho_s^2)\left(1 + l^2 \rho_s^2 (v_0' t)^2\right)}\right)\Big] + \mathcal{O}\left(\frac{1}{C}\right) \sim -\frac{1}{v_0' t}\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial n}{\partial t} = -\iiint dkdldk_z\, 2\phi^2(t, k_\perp, l, k_z)\, v_0' \\ \times \left(2 v_0' t\, l^2 \rho_s^2 + i \frac{S}{(1 + l^2 \rho_s^2)\left(1 + l^2 \rho_s^2 (v_0' t)^2\right)}\right) + \mathcal{O}\left(\frac{1}{C}\right) \sim -\frac{1}{(v_0' t)^3}\end{aligned} \tag{12}$$

The anomalous transport of ions, resulting from the non–modal drift turbulence decreases rapidly with time as $(v_0' t)^{-4}$,

$$\begin{aligned}\Gamma_x = \langle nv \rangle_x \simeq -\frac{c}{B_0} \iint_{-\infty}^{\infty} dkdl\ (\phi_0)^2 \frac{cl}{B_0}\frac{S}{C} \\ \times \left[\frac{1 + \rho_s^2 (l^2 + k^2)}{1 + \rho_s^2 l^2 \left(1 + (v_0' t)^2\right)}\right]^2 \left(\frac{l\rho_s \left(1 + (v_0' t)^2\right)}{1 + \rho_s^2 l^2 \left(1 + (v_0' t)^2\right)}\right) \sim -\frac{1}{(v_0' t)^4}\end{aligned} \tag{13}$$

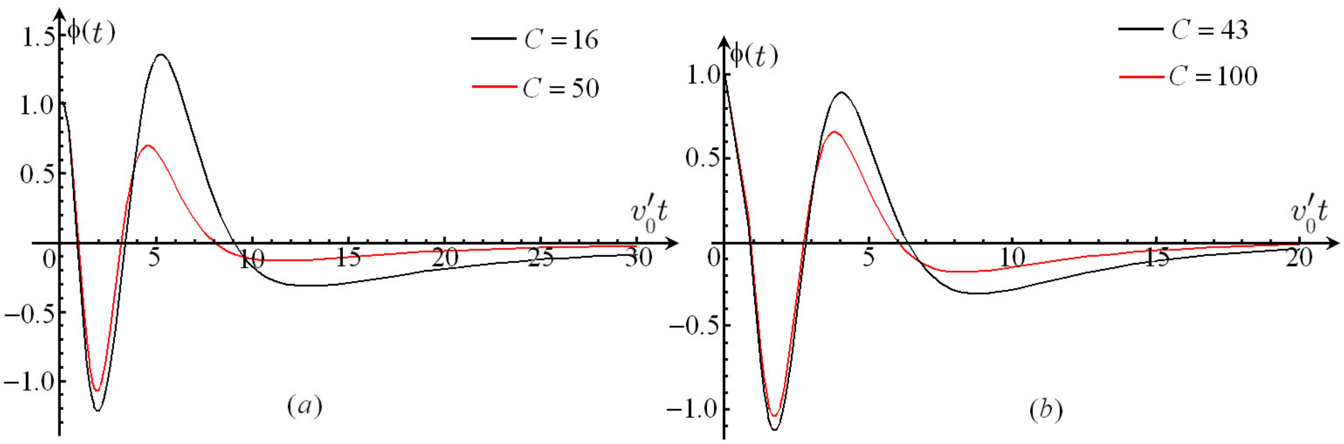

Figure 1: Temporal evolution of the electrostatic potential of drift waves for the conditions of the tokamak TEXT(a) and torsatron U-3M (b).

Power-like non-modal solutions[7] in the case of the Rayleigh-Taylor instability for the perturbed electrostatic potential and electron density,

$$\phi\left(t\right)\approx C_1\left(v_0't\right)^{\nu-2},\qquad n\left(t\right)\approx -i\frac{cv_0'l}{B_0\omega_{ci}v_{Re}}\nu C_1\left(v_0't\right)^{\nu},$$

where

$$\nu=\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{\gamma_0}{v_0'}\right)^2},\qquad \gamma_0=\frac{\sqrt{v_{Re}v_{de}}}{\rho_s},$$

is settled in time $tv_0'\geq 1$. For shear flow with $\frac{1}{\sqrt{2}}\gamma_0\leq|v_0'|$ such non-modal solution is settled in time of the order of the inverse growth rate. In that case the Rayleigh-Taylor instability is suppressed by shear flow in the time less then the inverse growth rate. For this case any nonlinear processes with modal solutions can't develop. In that case, as it is in the case of the above considered nonmodal drift waves, the energy density E and radial flux Γ_x of electron density are reduced with time,

$$\frac{\partial E}{\partial t}\approx\frac{\partial}{\partial t}\int dV\left(\nabla\phi\right)^2\sim\left(v_0't\right)^{\nu-1}C_1,\qquad 1\leq v_0't\ll\left(k\rho_s\right)^{-1}$$

$$\Gamma_x=\langle nv_x\rangle\simeq-\frac{c^2}{B_0^2\omega_{ci}}\frac{v_0'}{v_*}\iint\limits_{-\infty}^{\infty}dkdl\ l^2\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{\gamma_0}{v_0'}\right)^2}\right)C_1^2\left(v_0't\right)^{-2+\sqrt{4+\left(\frac{\gamma_0}{v_0'}\right)^2}}$$

All above presented results proves that the character of fluctuations of the drift and interchange type in plasma flows with sufficiently strong flow shear is dominated by the sheared flow. These mode specific nonmodal solutions, not the conventional modal solution, have to be used in the development of the theory of plasma turbulence and anomalous transport of the plasma shear flow as well as in the development of the theory of the improved confinement and formation of transport barriers.

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