Interpretation of Mode Frequency Sweeping in JET and NSTX

H. L. Berk 1), C. J. Boswell 2), D. Borba 3,4) , B. N. Breizman 1), A. C. A. Figueiredo 3), E. D. Fredrickson 5), N. N. Gorelenkov 5), R. W. Harvey 6), W. W. Heidbrink 7) T. Johnson 8), S. S. Medley 5), M. F. F. Nave 3), S. D. Pinches 4), E. Ruskov 7), S. E. Sharapov 4), R. G. L. Vann 9), and JET EFDA contributors 10)

1) Institute for Fusion Studies, University of Texas, Austin, TX 78712
2) Naval Surface Warfare Center, Indianhead Division, Indianhead, MD 20640
3) Centro de Fusão Nuclear, Associação EURATOM/IST, Instituto Superior Tcnico, Lisboa, Portugal
4) Euratom/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX143DB, UK
5) Princeton Plasma Physics Laboratory, Princeton, NJ 08543-0451
6) Comp X, Del Mar, CA 92014-5672
7) University of California, Irvine; Irvine, CA 92623-9557
8) Alfvén Laboratory, KTH, Euratom-VR Association, Sweden
9) University of York, Heslington, York, YO105DD, UK
10) See annex of Watkins M.L., et al, 2006 Proc. 21st Int. Conf. on FusionEnergy 2006 (Chengdu 2006) (Vienna: IAEA) OV/1-3

e-mail contact of main author: hberk@mail.utexas.edu

1. Introduction

Rapid spontaneous frequency sweeping has been observed in many different fusion related plasma physics experiments [1 – 7] for such modes as the Fishbone, TAE and Hot Electron Interchange Mode. This phenomenon been attributed to the formation of phase space structures, that together with plasma dissipation from the background plasma, forces the frequency to sweep [8-11]. These phase space structures take the form of holes and clumps where the distribution function of the holes (clumps) is lower (higher) than the ambient background distribution surrounding these structures in phase space. The sharp phase space gradient is maintained by wave trapping of particles trapped by a finite amplitude wave. A quantitative description of this process is described in [11] for the bump-on-tail instability. The theory was extended to the TAE mode in [12] and a theoretical outline of the general case valid for waves in nearly any configuration is given in [13]. The frequency sweeping of modes is expected to take place near the marginal conditions for the onset of instability, when there is a balance between dissipation mechanisms and the kinetic drive from fast particles. Frequency sweeping may facilitate the overlap of Alfvénic modes that can lead to undesirable global transport of alpha particles in a burning plasma. Here we shall also speculate how frequency sweeping may prevent parasitic alpha particle loss. Hence, it is of intrinsic interest for fusion confinement to thoroughly understand the frequency sweeping process.

Recently, pronounced frequency sweeping (~20% of the original frequency) has been reported for two new modes in NSTX [14] and JET [7], respectively. In the NSTX experiment, the mode that chirps has characteristics compatible with the compressional Alfvén wave (CAE) (or perhaps a global Alfvén wave). The destabilizing resonance is attributed to anisotropy in the injected energetic neutral beam exciting the CAE through the ion cyclotron resonance of Doppler shifted fast particles, yet another type of resonant particle interaction in fusion plasmas that leads to the formation of phase space structures. In JET a sweeping $n = 0$ mode, first reported in [15], has recently been identified [7, 16] as a geodesic acoustic mode (GAM) [17]. It is significant in that this mode couples an intrinsic kinetic process, the formation of phase space structures, with the mode that regulates fluid-like plasma turbulence [18].

The existence of phase space structures should be strongly dependent on dynamical stochastic processes that can potentially destroy phase space structures as is suggested theoretically [11,13] and has been demonstrated experimentally and theoretically on Terella [6] where the observed frequency chirping, attributed to phase structures, was quenched. A similar experiment was attempted on NSTX [19] and results will be reported below.

Numerical simulations of the electrostatic two-stream instability by Vann [20] have recently been performed with a source and sink (as well as background dissipation) that would produce a highly unstable equilibrium. What is found is that a plasma state forms that is kept near marginal stability by pronounced frequency chirping. Here we also study a speculation in [7,16], concerning whether a similar situation can be established using energetic particles interacting with the GAM mode.

2. Theory of Evolution of Phase Space Structures

Frequency chirping is explained as due to the spontaneous formation of phase space structures that can arise when the resonant particles have enough free energy to drive a wave of the plasma unstable. A perturbative theoretical description requires that the mode of interest exist, with a frequency ω_0, without energetic particles present, damp at a rate γ_d, with $\gamma_d/\omega_0 \ll 1$. The energetic particle drive produces a linear growth rate γ_L when dissipation is absent, while with dissipation, phase space structures can emerge when there is a near balance between the damping rate, γ_d, and the growth rate γ_L. This is a natural condition in any experimental plasma under steady conditions, as then the plasma hovers near marginal stability. In addition the instability needs to be discrete and of low enough amplitude so that particle-wave resonances do not overlap. The theory then predicts that there are intrinsic relationships of the saturation level, expressed in terms of the wave trapping frequency, $\omega_b \propto |C|^{1/2}$ (C is the amplitude of the mode) and the sweeping rate, $d\omega/dt$, to γ_L and a 'resonant power factor' that will be defined below.

In absence of drive and dissipative mechanisms a mode is characterized by an eigenfunction for the electric field here taken as $\vec{E}(\vec{r},t) = C(t)\mathbf{e}(\vec{r},\omega_0)e^{-i\omega_0 t} + cc$, where it is assumed that the mode amplitude varies slowly in time and the spatial structure does not change in time. The complex mode amplitude can be shown to be driven by resonant particle currents, $\vec{j}_{res}(\vec{r},t)$, projected onto the eigenfunction through the equation,

$$G_{\omega_0}\left(\partial C(t)/\partial t + \gamma_d C\right) = -i\int d\vec{r}\,\vec{j}_{res}(\vec{r},t)\cdot\mathbf{e}(\vec{r})e^{i\omega_0 t} = -i\int d\Gamma e f_{res}(\vec{r},\vec{p},t)\vec{\mathrm{v}}\cdot\mathbf{e}(\vec{r})e^{i\omega_0 t} \quad (1)$$

where $G_{\omega_0}|C|^2$ is the wave energy of the mode, $d\Gamma$ the increment of six dimensional phase space, $f_{res}(\vec{r},\vec{p},t)$ the distribution of resonant and nearly resonant particles. In the absence of perturbed fields in a tokamak, the trajectories of the particles undergo multi-periodic motion with each trajectory determined by constants of motion: the energy, H, the magnetic moment, μ, and the toroidal canonical momentum, P_ϕ. Of particular importance is the function, $e\vec{\mathrm{v}}\cdot\mathbf{e}(\vec{r})$ (e is particle charge) which when followed along is unperturbed trajectory, can be written as a the series,

$$e\vec{\mathrm{v}}(\mathrm{t})\cdot\mathbf{e}(\vec{r}(t)) = \sum_{p,l}\exp\left(i\Omega_{p,l.n}(H,\mu,P_\phi)t\right) < e\vec{\mathrm{v}}\cdot\vec{\mathbf{e}} >_{p,l.n} \quad (2)$$

where $\Omega_{p.l.n} = p\bar{\omega}_\theta + l\bar{\omega}_{ci} + n\bar{\omega}_\phi$, $< e\vec{\mathrm{v}}\cdot\mathbf{e} >_{p,l.n} = \int_{-\pi}^{\pi}\omega_\theta dt\exp(-i\Omega_{p,l.n}t)e\vec{\mathrm{v}}(\mathrm{t})\cdot\mathbf{e}(\vec{r}(t)/2\pi$, the upper bars refer to the time average over a poloidal bounce oscillation, θ and ϕ refer to poloidal and toroidal angle and p, l, n are integers. One finds that γ_L, the linear growth rate in absence of dissipation can be expressed in terms of the resonant power factor $e < \vec{\mathrm{v}}\cdot\mathbf{e} >_{p,l.n}$, viz.

$$\gamma_L = \int d\Gamma \sum_{p.l} \tilde{\gamma}_{p,l,n} \delta\left(\omega_0 - \Omega_{p,l,n}(H,\mu,P_\phi)\right), \text{ where}$$

$$\tilde{\gamma}_{p,l,n} = \frac{\pi}{G_\omega} |< e\vec{v}\cdot\mathbf{e} >_{p.l,n}|^2 \left(\frac{\partial F}{\partial H} + \frac{l\omega_c}{\omega_0 B}\frac{\partial F}{\partial \mu} + \frac{n}{\omega_0}\frac{\partial F}{\partial P_\phi} \right) \quad (3)$$

In this expression the $F(H,\mu,P_\phi)$ is the equilibrium distribution function.

When phase space structures form, the fields of the waves will trap particles whose particle oscillation frequency, $\Omega_{p,l,n}$, lies near the mode frequency ω. The square of the trapping frequency of the deepest trapped particle is found to be,

$$\omega^2_{b;p,l,n}(\Gamma) = |C\hat{\omega}^2_{b;p,l,n}(\Gamma)|; \quad \hat{\omega}^2_{b;p,l,n}(\Gamma) = 2\left|\left(\frac{\partial \Omega_{p,l,n}}{\partial H} + \frac{l\omega_{ci}}{B\omega_0}\frac{\partial \Omega_{p,l,n}}{\partial \mu} + \frac{n}{\omega_0}\frac{\partial \Omega_{p,l,n}}{\partial P_\phi} \right) e < \vec{v}\cdot\vec{\mathbf{e}} >_{p,l.n}\right| \quad (4)$$

If the frequency changes slowly enough, the particles trapped by the wave move in synchronism with the changing frequency into phase space regions where the value of its distribution differs significantly from the distribution of the surrounding passing particles. In particular the trapped particles have the value of the equilibrium distribution, $F(E_0, \mu_0, P_{\phi 0})$, at the original resonant frequency $\Omega_{p,l,n}(E_0, \mu_0, P_{\phi 0}) = \omega_0$, while the phase space coordinates E, μ, P_ϕ of the ambient distribution are shifted from resonance,

$$\Delta E = \frac{\omega_0 B}{l\omega_{ci}}\Delta\mu = \frac{\omega_0}{n}\Delta P_\phi = \Delta\omega \Big/ \left(\frac{\partial \Omega_{p,l,n}}{\partial E} + \frac{l\omega_{ci}}{\omega_0 B}\frac{\partial \Omega_{p,l,n}}{\partial \mu} + \frac{n}{\omega_0}\frac{\partial \Omega_{p,l,n}}{\partial P_\phi} \right). \quad (5)$$

Thus, with a moderate frequency shift, the trapping region's distribution differs from the ambient one by,

$$\Delta F = \Delta\omega \left(\frac{\partial F}{\partial E} + \frac{l\omega_{ci}}{\omega_0 B}\frac{\partial F}{\partial \mu} + \frac{n}{\omega_0}\frac{\partial F}{\partial P_\phi} \right) \Big/ \left(\frac{\partial \Omega_{p,l,n}}{\partial E} + \frac{l\omega_{ci}}{\omega_0 B}\frac{\partial \Omega_{p,l,n}}{\partial \mu} + \frac{n}{\omega_0}\frac{\partial \Omega_{p,l,n}}{\partial P_\phi} \right) \quad (6)$$

Using this result in Eq. (1), determines the magnitude |C| of the nonlinear wave,

$$|C|^{1/2} = \frac{16}{3\pi^2} < \frac{\tilde{\gamma}}{\hat{\omega}_b(\Gamma)} > \quad \text{, where } < \frac{\tilde{\gamma}}{\hat{\omega}^s_b(\Gamma)} > = \int d\Gamma \sum_{p,l} \delta(\omega_0 - \Omega_{p,l,n}) \frac{\tilde{\gamma}_{p,l,n}}{\hat{\omega}^s_b(\Gamma)}; \; < \frac{\tilde{\gamma}}{\hat{\omega}^0_b(\Gamma)} > = \gamma_L . \quad (7)$$

After phase space structures form, background dissipation forces frequency shifts to regions that cause the trapped particles to only move to phase space regions where the resulting energetic particle distribution reduces its energy. The rate of the frequency shift can be calculated by balancing the energy released by the phase space structures with the energy dissipated to the background plasma. A straightforward generalization of the calculation of a single resonance, first described in [11], gives that the frequency shifts in time as,

$$\omega - \omega_0 = \pm \left(\pi^2 \gamma_d |C|^{3/2} t \Big/ 8S \right)^{1/2} \equiv \frac{\pm 16\sqrt{2}\gamma_L \left(\sigma\gamma_d t\right)^{1/2}}{3\pi^2\sqrt{3}}; \quad \begin{matrix} S = < \dfrac{\tilde{\gamma}}{\hat{\omega}^3_b} > \\ \sigma = < \dfrac{\tilde{\gamma}}{\hat{\omega}_b} >^3 / < \dfrac{\tilde{\gamma}}{\hat{\omega}^0_b} >^2 < \dfrac{\tilde{\gamma}}{\hat{\omega}^3_b} > \end{matrix} . \quad (8)$$

The parameter σ is unity for a single resonance but in general differs somewhat from unity due to, differing weightings of the resonant power factor, when integrating over phase space. Note that the frequency shifts both upwards and downwards as $t^{1/2}$, a result seen in many tokamak experiments.

3. CAE mode in NSTX

Figure 1a shows that repetitive up-down frequency chirps observed in NSTX of a mode whose characteristics are compatible with a CAE as we shall assume here (however, there is flexibility in satisfying the resonance and dispersion relation conditions so the possibility of the wave being a global Alfvén wave can not be excluded). The mode arises due to the neutral beam injection of 90keV deuterons. Successive sweeping pulses generally exhibit this up-down symmetry though there are some exceptions. The time scale of each successive pulse is repetitive and the repetition rate of the pulses tend to be proportional to the applied beam power. Figure 1b shows a blow-up of a single chirp. In Fig. 1b we see that the chirp sweeps up and down and the frequency sweeping signal fits the frequency sweeping formula [Eq. (8)] which predicts that the frequency is proportional to $|t|^{1/2}$. In Fig. 1b shows a single frequency sweep signal fitted to the formula in Eq. (8), that predicts frequency shifts up and down proportional to $t^{1/2}$. Under the assumption $\gamma_d = \gamma_L$ the growth rate γ_L can be predicted once the parameter σ is determined.

We now attempt to make the prediction for what the growth rate ought to be given a reasonable model for the distribution function and compare the growth rate calculated from Eq. (3) with the growth rate inferred from sweeping. This technique also predicts the internal saturated field. A similar technique was previously use in [21] for a TAE mode. It is measured that the toroidal wave number is $n = 5$. The plasma is rotating in the direction of the beam and the wave propagates in the opposite direction to the beam. As result, there is a Doppler shift to produce the 394MHz observed in the laboratory frame

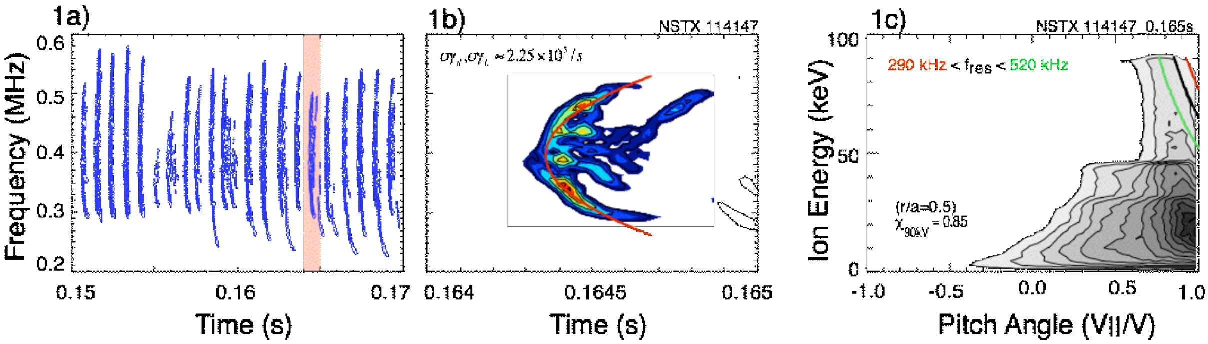


FIG. 1(a) Repetition of frequency sweeping pulses. (b) Spectral frequency content of a single pulse vs. time (c)Resonance regions of energetic particle distribution function, with sweeping signal

while the frequency in the plasma frame is 420 kHz, if the mode is assumed to be localized at $r_{\text{mode}}/a \cong 0.5$. For the fitting we assume that a CAE mode is being observed which is known to appear as an eigenmode of the torus [22,23]. As a rough first approach we estimate the growth rate as if the CAE were in an infinite medium propagating with a wavenumber $\vec{k} = \vec{k}_\perp + \vec{b}k_\parallel$. We use the local dispersion relation $\omega_{CAE}^2 = k^2 \mathrm{v}_A^2 \cong k_\perp^2 \mathrm{v}_A^2$ (in the NSTX experiment the Alfvén speed, v_A, is found to be $\cong 5.5 \times 10^7$ cm/s). From this relation we can determine the m number assuming we know the mode position (a large uncertainty in this experiment). With m determined, one uses $k_\parallel = \left(n - m / q(r_{\text{mode}})\right) / R_{\text{mode}}$ and finds that there is a particularly strong resonance interaction with the $l = 1$ cyclotron resonance where $\omega = k_\parallel \mathrm{v}_\parallel + \omega_{ci}$ when $m = -2$. For this wavenumber one finds $k_\parallel \mathrm{v}_b = -.83\omega_{ci}$ where v_b is the injection beam speed. Now using Eq. (3) for the growth rate and taking a plane wave eigenfunction we find that the growth rate is given by:

$$\gamma_L = \frac{2\pi^2 e^2 \omega^2}{|\delta B_\parallel|^2} \int d^3p\,\delta(\omega - k_\parallel \mathrm{v}_\parallel - \omega_{ci}) \left(\frac{\partial F}{\partial H} + \frac{\omega_{ci}}{\omega} \frac{\partial F}{B \partial \mu} \right) |\,\delta B_\parallel \left(\frac{\mathrm{v}_\perp}{k_\perp c} J_1'(z) - \frac{\omega^2}{\omega_{ci} k_\perp^2 c} J_1(z) \right) |^2 . \quad (9)$$

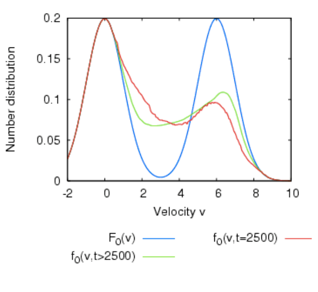
To evaluate the growth rate integral we use a slowing down distribution function of the form,

$F(\mathrm{v}, \lambda) = \dfrac{\beta_H B^2}{4\pi^2 m^5 \mathrm{v}_c^2} \delta(\lambda - \lambda_0) \dfrac{\theta(\mathrm{v}_c - \mathrm{v})}{\mathrm{v}^3}$ where $\lambda = \mathrm{v}_\parallel / \mathrm{v}$, β_H is the ratio of kinetic energy to magnetic energy of the hot particle distribution function at the mode location where from TRANSP we find $\beta_H = .058$, and $z = k_\perp v_\perp/\omega_{ci}$. We have ignored the width of this distribution, and this can be justified because we are interested in waves interacting with particles somewhat near the injection energy, where the angular distribution of energetic particles is well defined (the angular width primarily comes from the varying pitch angle of the beam due to the different directions of the magnetic field in the region where the mode is localized). Evaluating this integral with the choice, $\lambda_0 = 0.8$ we find $\gamma_L/\omega_0 \cong .040$. On the other hand, to infer γ_L/ω_0 from the frequency sweep we need to determine the parameter σ which can now found be to be 1.6 using our theoretical model distribution function in the equation just after Eq. (8). Then from the sweeping data, under the assumption $\gamma_d = \gamma_L$, the sweeping data infers that $\gamma_L/\omega_0 \cong .053$. Thus our initial results have achieved a reasonable match. However, more systematic studies and more realistic calculations still need to be performed before the predictive capability of this procedure is fully confirmed.

It is also interesting to see how the sweeping range of frequency matches the resonance conditions of a distribution function determined by TRANSP using a Monte Carlo calculation. Figure 1c shows that the entire frequency sweep range resonates with particles that have a significant value for the distribution function at energies somewhat below the highest injection energy. For this instability it is the clumps that chirp down in frequency and the holes that chirp up in frequency as will be explained below. We see in Fig. 1c that the upward frequency branch moves the resonant surface into the region where the distribution is increasing, which makes it easier for holes to deepen as is the requirement for the sweeping theory.

The logic for determining the direction holes and clumps sweep is as follows. From Eq. (5) and the definition of λ, the shift of the pitch angle, $\delta\lambda$, of a resonant bucket, must satisfy $\delta\lambda = \frac{\delta E}{2\lambda_0 E}\left(-\frac{\omega_c}{\omega} + (1-\lambda^2)\right) \approx -\frac{\delta E}{2\lambda_0 E}\frac{\omega_c}{\omega}$. The frequency shift caused by the background dissipation causes the phase space bucket to release energy. Thus, as the trapped particles in a clump (hole) must lose (gain) energy, it is required that there be a decrease (increase) of δE for the particles trapped in the bucket, therefore $\delta\lambda$ increases (decreases). Finally, as the chirping condition requires $\omega = \omega_{ci} - |k_\parallel| v\lambda$, Thus, $\delta\omega \cong -|k_\parallel| v\delta\lambda$, so that as $\delta\lambda$ increases (decreases) for the clump (hole), the frequency decreases (increases).

4. Simulation of chirped sustained equilibrium



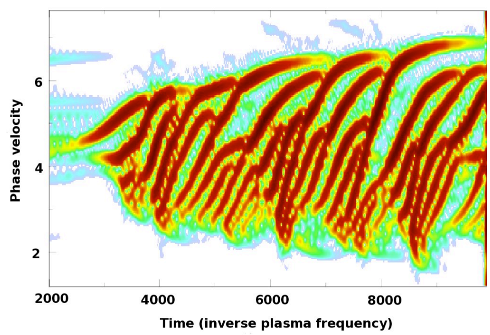


FIG. 2 (a) Distribution function in two-stream source-sink problem. Blue curve, predicted distribution function in absence of wave excitation; green curve, distribution function at onset of sweeping, red curve, distribution function late in the run. (b) Frequency sweeping spectrum. Time unit normalized to plasma frequency.

Recently a Vlasov simulation of the bump-on-tail instability with sources and sinks [20] has demonstrated that frequency sweeping can sustain an equilibrium near a marginal stability point. In this simulation the periodic boundary condition restricts the linearly unstable modes to only one mode. The results are illustrated in Figs. 2a and 2b. The solid blue curve in Fig. 2b is the steady state distribution that would be achieved in the system in the absence of wave perturbations. Such a distribution is very unstable to the two-stream instability. In Fig. 2b we see that after chirping instabilities arises, the distribution function remains nearly constant in time (compare the distribution at t = 2500, after the onset of the chirping events, with the distribution towards the end of the run).

We interpret this result as indicating that the chirping allows the distribution to relax to a marginal state and then not deviate much from this state. At a given frequency, the amplitude of the wave is quite moderate, and the distribution of energetic particles off-resonance should not transport in phase space. However, the chirping "democratizes" the wave-particle interaction so that a resonant interaction arises for nearly all of the energetic particles, and apparently allows the global distribution to reach a marginal state. This marginal state appears close to the one analyzed by Penrose [22], where the resonant velocity of the linear wave is at the minimum of the distribution function. In that case, the only way for phase space structures to release energy is to form holes and sweep up. This observation is compatible with the observed solely upward phase space chirping in Fig. 2b.

5. n=0 chirping in JET

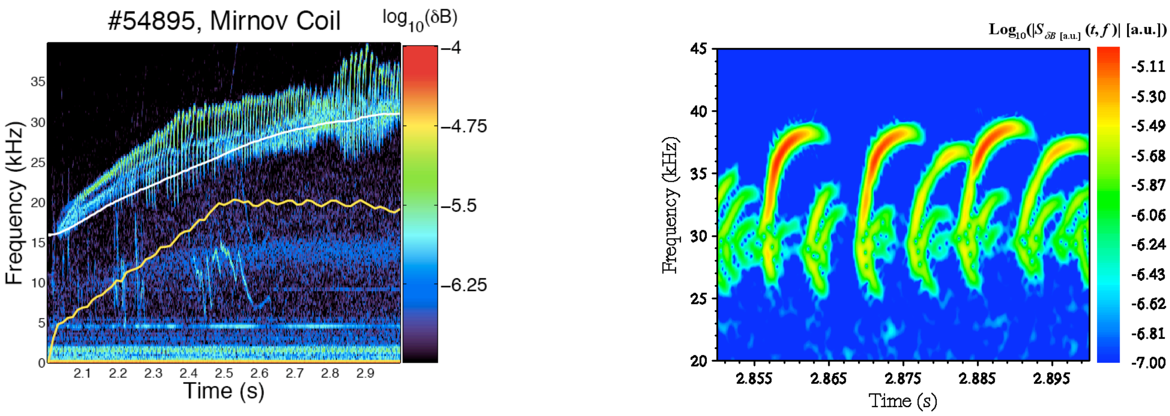


FIG. 3(a) Sustained chirping of the n=0 (GAM) that emerges after the onset of off-axis ICRF power (brown curve) with the onset frequency proportional to $T_e^{1/2}$ (white curve). (b) Zoom of the frequency chirp signal. The upward (downward) sweep can be attributed to the formation of clumps (holes) due to an effective negative mass effect in mirror trapped particles(ref.).

A significant aspect of the two-stream simulation may be applicable to a fusion plasma where we wish the power production of energetic alpha particles to be used to heat the thermal particles. In the simulation, there is a fixed power input, and this power is being transferred through a wave intermediary to the plasma background in the form of dissipation. Thus, the instability does not limit plasma heating, although it does lower the stored energy of the energetic particle distribution. This reduction is a favorable result, as it might avoid the excitation of additional instabilities that could cause instability loss. The question arises whether the simulation mechanism can be replicated in a fusion plasma.

In a tokamak, a promising mode that might be used to extract energy relatively rapidly from an inverted population of energetic particles, is the n = 0 geodesic acoustic mode (GAM), as has been suggested in [7,16]. When this mode is excited it can produce pronounced frequency chirping, as is observed in Fig. 3. The energy of resonant energetic particles diffuses, but because it is low frequency the magnetic moment is conserved and because the mode is an n = 0 mode the canonical toroidal angular momentum is conserved. Consequently, there will be no direct energetic particle loss due to radial diffusion and the background dissipation would then allow the energetic particle energy to be transferred to the background plasma. If this energy transfer rate from the energetic particle component to the background plasma, competes or is faster than the transfer rate from classical processes (typically electron drag) one would lower the expected stored energetic particle energy density but not lower the heating of the plasma. Then perhaps one might ameliorate or even eliminate the excitation of other Alfvénic instabilities that can cause parasitic loss.

In Fig. 3 we see an example of the chirping in JET data due to the self- excitation of the GAM $n = 0$ mode which then forms phase space structures that chirp by the mechanism discussed in this paper. To make an assessment whether chirping can be a mechanism for "channeling" energy [25], we combine elements of the analytic theory and experimental observation. We see from the figure that initially the frequency changes by 5kHz in 1 ms. With this information, we can then use the formulas in Sec. 2 to estimate the power the waves transfer to the plasma. For simple estimation purposes, we set $\sigma = 1$ in Eq. (8). We can then infer, using from Eq. (7) that $\gamma_L \approx 0.09\omega$ with $\omega \approx 2\pi \cdot 30$ kHz $\approx 190 \times 10^3$ rad/s. We see that the mode (especially the frequency upshift branch) is nearly always present during the discharge with only a moderate amount of multiplicity of modes being excited simultaneously. Thus we will assume that on average there is a constant electric field amplitude present from a single GAM mode, and the rate at which the mode dissipates power is $P_{\mathrm{diss}} = 2\gamma_L W_{md}$, where we assume $\gamma_d \approx \gamma_L$, and W_{md} is the wave energy of the mode. We note that the geodesic acoustic mode is an electrostatic mode where the oscillation energy is an equal balance of the cross-field kinetic energy and the parallel flow energy. Then the wave energy is twice the time average of the cross-field energy density, $W_{md} = 2n_p m_p \dfrac{c^2 |<\nabla\phi|^2>}{2B^2} V_{md} \approx n_p m_p \dfrac{c^2\phi_0^2}{B^2\Delta_m^2} V_{md}$, with V_{md} the volume occupied by the mode and Δ_m the radial mode width of the mode. Thus, we need to estimate the mode amplitude, ϕ_0, which can be expressed in terms of the wave trapping frequency ω_b. From Eq. (8) we have $\omega_b \approx 0.5\gamma_L$. Then to extract ϕ_0 from this relation we need to evaluate $\partial\Omega/\partial H$. We note resonant particle resonance, Ω, is due to mirror trapped particles, which can be expressed as $\Omega = \sqrt{\dfrac{\mu B_0 r}{q^2 R^3}}\left(1 - \dfrac{(H - \mu B_0)R}{8\mu B_0 r}\right)$ .Thus, $\left|\dfrac{\partial\Omega}{\partial H}\right| = \sqrt{\dfrac{\mu B_0 r}{q^2 R^3}}\dfrac{R}{8\mu B_0 r} \approx \omega \dfrac{R}{8\mu B_0 r}$. For the resonant power factor we find $|< e\vec{\mathrm{v}}\cdot\mathbf{e} >| \approx e\omega\varphi_0$ if the energetic particle Larmor radius is smaller than the mode width. From these relations follows $e\phi_0 \approx \left(\dfrac{\gamma_L}{\omega}\right)^2 \mu B \dfrac{r}{R}$ and we find that the power dissipated is given by, $P_{\mathrm{diss}} = \gamma_L(\gamma_L/\omega)^4(ra_r/R\triangle_m)^2 E_r n_p V_{md}$, where E_r is the energy of a wave trapped particle, $a_r^2 = 2E_r/(eB/c)^2$ is the square of Larmor radius of resonant particle and V_{md} the volume occupied by the mode. This expression can be rewritten as: $P_{diss} = 0.5MW \dfrac{a_r^2}{r_m\Delta_m}\dfrac{2.5r_m}{a}\dfrac{4a}{R}\dfrac{n}{10^{19}}\left(\dfrac{R}{3}\right)^3$, where n is the density in m^{-3}, R is *m*, $\triangle_m$ is the mode width and the numbers in the numerators and denominators are the nominal choices for describing the JET experiment. We see that the local heating at the location of the mode is considerably less than *0.5 MW* as $\dfrac{a_r^2}{r_m\Delta_m} \leq 0.1$ The ICRF heating is about *5 MW* and probably concentrated in the region of the mode location. It appears that the proposed relaxation process is not competitive with the heating process in the JET experiment.

6. Effect of stochasticity on phase space structures

Stochastic processes, such as particle collisions or orbital chaos induced by rf heating, may have a significant impact on the formation and persistence of phase structures, far more so than in linear theory. For example, the validity of collision-less Landau damping requires $\nu_{St}/\triangle\Omega_{p,l,n} << 1$ where ν_{St} is the 90° relaxation rate, $\triangle\Omega_{p,l,n}$ is the spread of the orbital frequencies, $\Omega_{p,l,n}$, over the energetic particle distribution function. In contrast, the collision-less frequency sweeping phenomenon presented here requires a far more stringent condition on the magnitude of ν_{St}. Specifically, the theory in [11] requires $(\nu_{St}\Omega_{p.l.n}^2)^{1/3}/\gamma_L \leq 1$ to initially form a phase space structure, and $(\omega - \omega_L)\nu_{St}\Omega_{p.l.n}^2/\gamma_L^4 \leq 1$ to achieve a frequency shift, $\omega - \omega_L$. Thus as long as $1 >> \gamma_L/\omega \cong \gamma_L/\Omega_{p,l,n}$ (this inequality is typically satisfied in experiment) the nonlinear sweeping process is far more sensitive to small stochastic effects than the linear process. In all three cases high harmonic fast wave (HHFW) heating was used to induce stochasticity effects. The results were in accord with our expectations only for some CAE observations. Figure 4 illustrates one of these examples. In Fig. 4c we see that prior to HHFW heating that continual chirp events arise, from 0.125-0.160s, due to the excitation of CAE modes. A typical up-down chirp pulse is shown in Fig. 4a. At 0.16s, 22MW of rf heating pulse is suddenly applied for 0.015s. An immediate response of the CAE signal is seen in Fig. 4c. Indeed the blow-up figure 4b shows that the extent of chirping is much reduced and chirping is only downward. However, just before the end of the heating there is the onset of another symmetric chirp signal, perhaps an indication that extra rf power allows a change of equilibrium parameters that lead to an altered character to the chirping pulses. After the rf is turned off, the character of the post heating chirp events, shown if Fig. 4c, differs from the pre-heating chirp events (there is a larger time interval between chirp events. This experimentally clear example was not always for the CAE chirping events and never the case when HHFW pulse was applied to the TAE or the Fishbone. With rf heating, sometimes the TAE was completely stabilized (apparently due to a dilution of the phase space density in the wave-particle resonance region), and sometimes the chirp was unaffected by heating (in those cases the mode was likely to have been an Energetic Particle Mode [24]). More study is needed to obtain a clear reason for why these responses differ from our initial expectations. Possible reasons include: there was not enough heating power present (rough calculations indicate that the heating power is at the margin for having an effect), the resonance regions for Rf heating doesn't overlap with the resonance regions that drive the chirping instabilities and that the system chirp for some cases may be due to an entirely different mechanism [24].

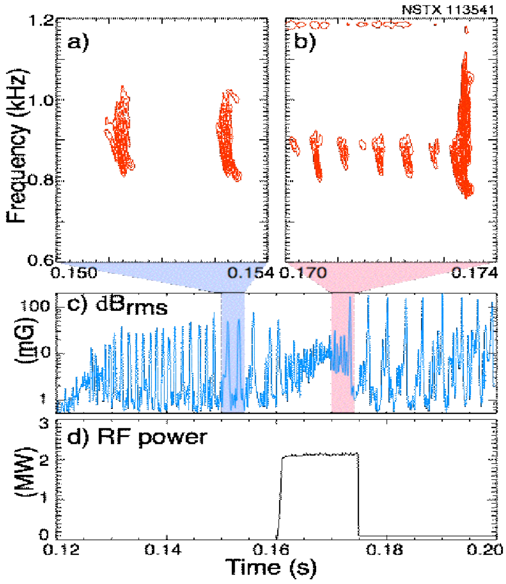


Fig. 4 *Effect of HHFW of CAE chirping signal. (a) Chirp signal prior to imposing HHFW rf pulse. (b) Chirp signal towards the end of rf heating pulse (c) Overview of CAE mode activity over an extended period of time (d) HHFW pulse power vs. time.*

7. Summary

A theory for frequency sweeping of discrete waves, applicable to a wide variety of plasma waves and systems, has been applied to the interpretation of fast frequency sweeping signals observed in the NSTX and JET experiment. In NSTX good correlation was obtained from the inferred intrinsic growth rate of a CAE mode, with the growth rate directly calculated from instability theory. Uncertainties in the choice of parameters still need to be assessed to ensure validation of the comparison. The sweeping theory was also used to assess whether the observed chirping of n=0 GAM oscillations in JET could be locally transferring power to the plasma at a rate comparable to the heating rate. Such an effect is achieved in a toy simulation of the bump-on-tail instability. However for JET, the experimental signals together with sweeping theory, indicates that it is not likely that the that the local energy transfer rate is competitive. An attempt was made to demonstrate the sensitivity of sweeping due to stochasticity effects induced by HHFW heating. Partial success was achieved for altering characteristics of the CAE mode. However, the sweeping remained robust for the fishbones, and sometimes completely stabilized the TAE mode. More study is clearly needed.

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