Linear and nonlinear aspects of edge turbulence and transport in tokamaks

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Abstract. (1) The stability of ideal ballooning-peeling MHD modes is examined in the presence of external magnetic field perturbations. It is demonstrated that external field perturbation can increase the threshold of MHD modes. It is also shown that this effect should be much more important under ITER conditions than in the present devices like D-III D. (2) We also report a simple self-consistent theoretical model of multi-scale interaction of edge-localized modes (ELMs) such as the ideal ballooning-peeling modes interacting with zonal magnetic fields and zonal flows. The dynamics of self-consistent zonal flows in relaxation of ELMs is unimportant when beta exceeds beta critical and the edge pedestal is unstable to ideal mode. The secondary instabilities of zonal fields are used to estimate saturation level and energy flux induced by ballooning-peeling mode turbulence. (3) The linear instabilities of non-ideal curvature driven modes, including the influence of trapped electrons and electron inertia in the weakly collisional edge of hot reactor like plasmas, are investigated. Fluid theory for the weakly collisional edge tokamak plasma in the presence of trapped electrons has been used. Even when the plasma beta is less than its critical value, a robust non-ideal curvature driven instability persists in the presence of electron inertia and trapped electrons effects.

1. Introduction

This paper is naturally divided into three parts:
(I) Mitigation of ELMs by external magnetic field perturbations. **–** Edge Localized Modes of type-I (ELMs-I) are an intrinsic feature of the high confinement H-mode in tokamaks [1]. ELMs can lead to a dramatic increase of the impurity production and cause cyclic heat loads on divertor plates. Therefore, it is crucial to find a way to mitigate ELMs without negative consequences to the plasma properties. Recent experiments on the tokamak D-III D [2] have demonstrated that by applying external magnetic field perturbations from the so called I current coils large ELMs of type I can be effectively mitigated without any significant loss of confinement properties. This mode of operation is highly desirable for future fusion reactors and therefore it is very important to achieve an understanding of physical mechanism leading to ELMs mitigation through external field perturbations. Up to now the increase of transport in the barrier between ELMs is considered as the main cause of the ELMs mitigation by the perturbations from I coils [2]. This leads to a reduction of the pressure gradient under the threshold level for ballooning-peeling MHD modes, whose development causes, as it is widely believed, ELMs of type I [3]. In the present paper we demonstrate that, in addition to this mechanism, external perturbations can increase the threshold of MHD-modes through non-linear interaction with them. In order to qualitatively elucidate the importance of nonlinear effects character, we perform our analysis by using parametric instability technique [4], which allows an analytical treatment. In the linear part our approach is relatively simple, it provides the threshold of ballooning-peeling MHD modes in an agreement with well-known numerical codes, e.g., MISHKA. It is also shown that non-linear effect from external perturbations can be much more significant in ITER with larger dimensions and stronger magnetic field.

(II) Secondary instabilities of large scale magnetic fields in the background of short scales ideal ballooning mode turbulence. - The physics of Edge-Localized Modes (ELMs), characteristic excitations of the H-mode transport barrier, is a topic of great interest for confinement physics. Edge Localized Modes (ELMs) have a strong effect on the particle and energy losses from the edge transport barrier and also influence the global confinement behavior in H- mode plasmas. In order to assess this influence by means of transport modeling and to make predictions for the ELM effect in ITER and DEMO, one has to estimate the nonlinear saturation level of ideal ballooning-peeling instabilities responsible for the excitation of ELMs. Here, we present a simple self-consistent theoretical model of multi- scale interaction of ELMs governed by ideal ballooning mode with zonal magnetic fields and zonal flows. The influence of zonal fields and zonal flows on short scale ballooning turbulence (i.e. $q_\perp < k_\perp$ where $q_\perp$ and $k_\perp$ are perpendicular wave vector of long scale mode and short scale ballooning mode, respectively) is calculated from standard wave kinetic equation via modulation of linear growth and frequency of short scale ballooning mode. The equations for slow, long scale zonal field and zonal flows follow from fast time/space-averaged equations for parallel electron momentum and vorticity respectively. It is shown that when $\beta > \beta_c$, and the edge pedestal is unstable to ideal ballooning mode, the magnetic Reynolds stress completely suppresses the zonal flow growth in ballooning mode turbulence. Thus, the dynamics of self-consistent zonal flows in relaxation of ELMs is likely to be *unimportant.* We next concentrate on the nonlinear coupling to the zonal fields. As interest here lies in studying the saturation of ballooning mode by back reaction of zonal fields so first we present the instability of long-scale zonal magnetic field with $q_r \neq 0$ and $q_\theta, q_\parallel = 0$, ($q_r, q_\theta, q_\parallel$ are the radial, poloidal and parallel wave vectors associated with secondary instabilities). This can be viewed as a fast dynamo action by ideal ballooning instability. The interactions show possible excitation of secondary instabilities. In the end, a simple zero-dimensional model [5,11] will be presented to estimate the saturated amplitude of ballooning modes interacting with large-scale dynamo instabilities and some estimates of edge pedestal transport will also be made. The secondary instabilities of large-scale magnetic field with $q_r, q_\theta, q_\parallel \neq 0$; this can be viewed as an interaction of ideal ballooning mode with tearing instability will be published elsewhere.

(III) Collisionless ballooning instability in tokamak edge. - Large amplitude density and potential fluctuations are routinely observed in the edge region of tokamaks. If the edge temperature is low, then resistive ballooning mode [1] offers a reasonable explanation for the observed fluctuations. As one goes towards hotter tokamaks, the parallel connection length becomes shorter than the mean free path and so the resistive mechanisms are no longer adequate. Since beta is less than beta critical, the plasma is typically stable to ideal ballooning instabilities. Non-ideal effects associated with kinetic physics and/or electron inertia are then invoked to understand the basic driving mechanism of the instabilities. In this paper we include the effects due to trapped particles in considering the non-ideal curvature driven edge plasma instabilities. We give a multiple fluid description of the instability physics and ignore effects due to electron temperature fluctuations, ion trapping and wave particle resonant effects. Electromagnetic effects associated with the excited perturbations have been included as the inductive effect on the parallel motion of electrons is substantial in the collisionless case. Instability has been studied with analytical approximations as well as a numerical code. For low poloidal mode number, the instability growth rate is higher for the electromagnetic case. Trapped particle effects also become important at low poloidal mode numbers. A number of closely spaced eigenmodes appear all across the poloidal spectrum. Our conclusion is that even in the weakly collisional high temperature limit, a robust curvature driven non-ideal instability persists in the edge.

2. Linear Stability of Ballooning- Peeling MHD Modes

We consider the simplest model of ideal ballooning and peeling modes. The basic equations for normalized potential ($\tilde{\phi}$), parallel vector potential ($\tilde{A}_{//}$) and pressure ($\tilde{p}_{i,e}$) perturbations are:

$$(\partial_t - \tau\alpha_i\partial_y)\nabla_\perp^2\hat{\phi} + \varepsilon_n\partial_y(\hat{p}_e + \tau_i\hat{p}_i) - 0.5\beta_e\hat{J}'_{//0}\partial_y\hat{A}_{//} + \nabla_{//}\nabla_\perp^2\hat{A}_{//} = -[\hat{\phi},\nabla_\perp^2\hat{\phi}] + 0.5\beta_e[\hat{A}_{//},\nabla_\perp^2\hat{A}_{//}], \quad (1)$$

$$-\nabla_\perp^2\hat{A}_{//} + 0.5\beta_e\hat{\chi}_e(\partial_t + \alpha_e\partial_y)\hat{A}_{//} + \hat{\chi}_e\nabla_{//}(\hat{\phi} - \hat{p}_e) = -0.5\beta_e\hat{\chi}_e[(\hat{\phi} - \hat{p}_e), \hat{A}_{//}], \quad (2)$$

$$\partial_t\hat{p}_j + \alpha_j\partial_y\hat{\phi} = -[\hat{\phi},\hat{p}_j]. \quad (3)$$

Here, the parameters are normalized by $\hat{n} = (L_n/\rho_s)(\delta n/n_0)$, $\hat{\phi} = (L_n/\rho_s)(e\delta\phi/T_e)$, $t = tc_s/L_n$, $\hat{A}_{//} = (2c_sL_n/\rho_s c\beta)(e\delta A_{//}/T_e)$, $(x,y) = (x/\rho_s, y/\rho_s)$, $\nabla_{//} = L_n\nabla_{//}$, $\beta_e = 8\pi n_0T_e/B^2$, $c_e = (T_e/m_e)^{1/2}$, $\alpha_i = (1+\eta_i)$, $\alpha_e = (1+\eta_e)$, $\varepsilon_n = 2L_n/R$, $\eta_j = \partial_r \ln n_0/\partial_r \ln T_j$, $j = i,e, L_n = -(\partial_r \ln n_0)^{-1}$, $\tau_i = T_i/T_e$, $\hat{\chi}_e = (c_e/0.51\nu_{ei}L_n)(m_i/m_e)^{1/2}$, $L_n = -(\partial_r \ln n_0)^{-1}$, $\hat{J}'_{//0} = L_n\partial_r J_{//0}/en_0c_s$, and $[A,B] = \partial_x A\partial_y B - \partial_x B\partial_y A$ is the Poisson's Bracket. The main feature of ballooning modes (BM) can be described from local instability analysis. The linear dispersion relation of BM, including local current gradient (kink) and weak collisional effects, can be written as:

$$\hat{\omega}(\hat{\omega} + \tau\alpha_i\hat{\omega}_*) + (\alpha_e + \tau\alpha_i)\frac{\beta_e q^2 R}{L_n} + \left(\frac{3qR < J_{//0} >^{edge}}{2L_T e n_0 c_A}\frac{\Omega_i}{k_\perp c_A} - 1\right)\left[1 - i\left(\frac{0.51\nu_e}{\Omega_e}\right)\frac{qRk_\perp^2 c_A}{\Omega_i(\hat{\omega} - \alpha_e\hat{\omega}_*)}\right] = 0. \quad (4)$$

Here the frequencies are redefined by $\hat{\omega} = \omega qR/c_A$, $\hat{\omega}_* = qRk_y\rho_s c_s/L_n c_A$ and the local edge current gradient is evaluated from parallel component of Ohm's law i.e., $E_{//} = \eta_{//} < J_{//0} >^{edge}$ and thus $\partial_r < J_{//0} >^{edge} \sim 3 < J_{//0} >^{edge} /2L_{Te}$ and taking weak collisional limit i.e., $0.51\nu_e qRk_\perp^2 c_A/\Omega_e\Omega_i(\hat{\omega} - \alpha_e\hat{\omega}_*) < 1$. It is apparent from Eq. (4) that the peeling term reduces the β threshold and makes the ballooning mode more virulent. The stability condition with kink effects is $(1 + 1.22/k_\perp L_n\varepsilon^{1/2})\, q^2R\,|d\beta/dr| < 1$. The dissipative instabilities near the marginal point play a significant role since $(60-70)\%$ of input power is accounted by these instabilities.

Computer simulations and analytical work on stability of pedestal to the ideal ballooning with ion diamagnetic effects have demonstrated that the near marginal point, the typical poloidal wave number of ballooning mode is $k_\theta\delta_R \approx 1$, where $\delta_R = [\tau_i^2 R\rho_s^2/2(1+\tau_i)]^{1/3}$ [8]. Thus, for the plasma edge conditions with $R/r \approx 3-4$, $L_n 2-3cm$, $\rho_s \approx 0.1-0.15cm$, the relative contribution from the peeling effect is comparable with that from the ballooning one.

In the absence of diamagnetic effects ($\gamma > \omega_*$), the growth rate for the mode with near the marginal condition [i.e., $(\alpha_e + \tau_i\alpha_i)\beta_e q_a^2 R/L_n \approx 1 - 3q_a R < J_{//0} >^{edge} \Omega_i/2L_{Te}en_0k_\perp c_A^2$], the growth rate of dissipative BM is $\hat{\gamma} \approx [(\alpha_e + \tau_i\alpha_i)\beta_e q^2 R/L_n]^{1/3}[0.51\nu_e qRk_\perp^2 c_A/\Omega_e\Omega_i]^{1/3}$. This mode is also known as dissipative ballooning mode [9]. Note that even small fraction of dissipation can stimulate instability with significantly large growth rate. Near the marginal point and including diamagnetic effects with limiting case $\hat{\omega}_* > \omega$, we can recover purely growing dissipative mode, which develops on slow resistive time scale and independent of wave number [10]: $\gamma \approx \nu_e\beta_e^{-1}(2L_n m_e(\alpha_e + \tau_i\alpha_i))/Rm_i(\alpha_e\tau_i\alpha_i)$.

3. Suppression of MHD modes through nonlinear coupling with external field perturbations

We use the standard techniques of parametric instability in four-wave interaction process where pump is low frequency [4]. The external magnetic field perturbations are represented as oscillations of the vector potential $\tilde{A}_{0//} = A_0(r)\exp(-i\omega_0 t + ik_0 y)$ with the harmonic amplitude $\tilde{A}_0$ much larger than that of MHD fluctuations.. External perturbations can be considered as a pump wave which interacts with the MHD modes through the generation of side bands at the frequencies $\omega \pm \omega_0$, $\omega \pm 2\omega_0$ and so on. This interaction through side bands can suppress the main perturbations at the frequency ω. Henceforth, only two side bands with $\omega_{1,2} = \omega \pm \omega_0$, $\vec{k}_{1,2} = \vec{k} \pm \vec{k}_0$ will be taken into account. We neglect the potential and pressure perturbations on the scale of pump wave ($\omega_0, \vec{k}_0$). By accounting for non-linear terms on fluctuations in the main MHD mode ($\omega, \vec{k}$) with the accuracy of terms up to order $|A_0|^2$ in the right hand side of Eqs. (1-3), the nonlinear dispersion relation of Ballooning-peeling modes can be written as:

$$\varepsilon_k \approx \frac{\beta_e}{2} k_\perp^2 [(\hat{z}\times\vec{k})\cdot\vec{k}_0]^2 (1 - \frac{i2k_\perp^2}{\beta_e \hat{\chi}_e \omega}) | A_0 |^2 [(\frac{\omega^2}{\omega^2 - \omega_0^2}) + \frac{2}{\beta_e} k_{//}^2 k_\perp^2 (1 - \frac{k_{0\perp}^2}{k_\perp^2})\{(\frac{2}{\varepsilon_1} + \frac{2}{\varepsilon_2}) + (\frac{k_{0//}}{k_{//}} + \frac{\omega_0}{\omega})(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2})\}] \qquad (6)$$

Here ε_k is the linear dispersive function of ballooning-kink mode. For the experiments with I coils or ergodic divertor $\omega_0 < \omega$, $k_0 < k_\perp$ and by taking into account $\beta_e q_a^2 R / L_n >> (L_n / R)(\rho_s / \delta_R)^2$, which is relevant for the H-mode pedestal region, the growth rate of ballooning peeling modes:

$$\gamma_k = (c_A / qR)[q^2 R \mid d\beta / dr \mid (1 + 1.22\delta_R / L_n \varepsilon^{1/2}) - 1 - (qR / \delta_R)^2 (\delta B_r / B)^2]^{1/2} \qquad (7)$$

Physically, the reduction of the growth rate by the external perturbation is due to the fact that they bring additional field line bending. In order to overcome this, the unstable mode should generate side bands and this reduces the energy reservoir of the instability. It is interesting to note that this effect is similar to the magnetic Reynolds stresses role in stabilizing the streamers and zonal flows in drift wave driven turbulence through random magnetic shearing.

Eq. (7) predicts that the stabilizing effect of external perturbations becomes essential if $| \delta B_r / B | \geq b_{cr} \approx \delta_R / qR$. For the DIII-D conditions with $B = 1.6T$, $R = 1.72m$, $q_{95} = 3.7$ and the pedestal temperature of $250eV$ [2], this critical level is of $b_{cr} \geq 0.15\%$. It is interesting to note that since $b_{cr} \quad \alpha \quad (RB)^{-2/3} q^{-1}$ one can expect a significant decrease of the critical perturbation level in ITER with $B = 5.3T$ and $R = 6.2m$, by a factor of 5. Moreover, one should keep in mind that the considered effect works synergistically with the increase of transport by external perturbations between ELMs. Our calculations of the plasma parameters in the barrier, which will be published elsewhere, demonstrate that owing to this synergy ELM mitigation can be achieved simultaneously with a significant increase of the plasma pressure in the pedestal.

4. Secondary instabilities of large scale magnetic fields in the background of short scales ideal ballooning mode turbulence

Here we investigate the modulational instability of poloidally elongated zonal fields and zonal flows (i.e. $\partial / \partial x \neq 0$, $\partial / \partial y = \partial / \partial z = 0$ *or say* $q_x >> q_y, q_z$). Equations (1-3), the linear dispersion relation of short scale ideal ballooning mode gives; Real frequency: $\hat{\omega}_{rk}^{(0)} \approx -\alpha_i k_y / 2$, and growth rate: $\hat{\gamma}_k^{(0)} \approx (\alpha_i \varepsilon_n - 2\hat{k}_\parallel^2 / \beta_e)^{1/2}$. And near marginal point i.e., $\beta_e q^2 R / L_{pi} \approx 1$, the growth rate of short scale dissipative mode, accordingly that $\hat{\gamma}_k^{(1)} = (2\hat{\eta}\hat{k}_\perp^2 \hat{k}_\parallel^2 / \beta_e)^{1/3}$, where $\hat{\eta} = 2/(\beta_e \hat{\chi}_e)$. If we discard the equilibrium gradient in Eqs (1 – 3), these equations satisfied the conservation law,

$$(\partial / 2\partial t)\int dV\ [\ |\nabla_\perp \hat{\phi}|^2 + |\nabla_\perp \hat{A}_{//}|^2 + (\lambda_s^2/\rho_s^2)|\nabla_\perp^2 \hat{A}_{//}|^2 + |\hat{p}|^2\ ] = -\,\mathrm{Sink}$$

a. Large scale zonal field instability. - We assume that there is a sufficient non-axisymmetric in the spectrum, which separates long scale zonal magnetic fields ($\Omega_q, \vec{q}$) and small scale MHD mode ($\omega, \vec{k}$). The equation for slow, long scale zonal fields / zonal flows are the averaged over fast time and space scales the vorticity and parallel electron momentum equations,

$$\begin{aligned}\partial_t \nabla_\perp^2 \hat{\phi}_q &= -[(\partial_{yy} - \partial_{xx}) < \partial_x \hat{\phi}_k \partial_y \hat{\phi}_k > - \partial_{xy} < \partial_y \hat{\phi}_k)^2 - (\partial_x \hat{\phi}_k)^2 >] \\ &+ (\beta_e/2)[(\partial_{yy} - \partial_{xx}) < \partial_x \hat{A}_{\parallel k} \partial_y \hat{A}_{\parallel k} > - \partial_{xy} < (\partial_y \hat{A}_{\parallel k})^2 - (\partial_x \hat{A}_{\parallel k})^2 >]\end{aligned} \tag{8}$$

$$\begin{aligned}(1 - (\lambda_s^2/\rho_s^2)\nabla_\perp^2)\partial_t \hat{A}_{\parallel q} - \hat{\eta}\nabla_\perp^2 \hat{A}_{\parallel q} + (2/\beta_e)\nabla_\parallel \hat{\phi}_q &= \partial_x < \partial_y \hat{\phi}_k \hat{A}_{\parallel k} > - \partial_y < \partial_x \hat{\phi}_k \hat{A}_{\parallel k} > \\ - (\lambda_s^2/\rho_s^2)\partial_x < \partial_y \hat{\phi}_k \nabla_\perp^2 \hat{A}_{\parallel k} > + (\lambda_s^2/\rho_s^2)\partial_y < \partial_x \hat{\phi}_k \nabla_\perp^2 \hat{A}_{\parallel k} >\end{aligned} \tag{9}$$

The nonlinear terms in Eqs (8) and (9) can further be simplified via a quasilinear relation between $\hat{\phi}_k$ and $\hat{A}_{\parallel k}$, which is given by relation $\hat{A}_{\parallel k} \approx 2\hat{k}_\parallel(\hat{\omega}_{rk} - i\gamma_k)/\beta_e |\hat{\omega}_k|^2\ \hat{\phi}_k$. Neglecting electron inertia effects and assuming the wavelength of zonal fields $q_x c/\omega_{pe} < 1$, the equation of the equation of zonal flows, $\hat{\phi}_q$ and zonal field, $\hat{A}_{\parallel k}$ are:

$$\partial_t \hat{\phi}_q = \sum (1 - 2\hat{k}_\parallel^2/\beta_e |\omega_k|^2)\hat{k}_x \hat{k}_y |\hat{\phi}_k|^2 \tag{10}$$

$$\partial_t \hat{A}_{\parallel q} + \hat{\eta}\hat{q}_x^2 \hat{A}_{\parallel q} = -i\hat{q}_x \sum (2\hat{\gamma}_k \hat{k}_\parallel \hat{k}_y / \beta_e |\omega_k|^2)|\hat{\phi}_k|^2 \tag{11}$$

The response of zonal fields and zonal flows on turbulence can be calculated from standard wave kinetic equation,

$$\partial_t N_k + \partial_{\vec{k}}\omega_{nl} \cdot \partial_{\vec{x}} N_k - \partial_{\vec{x}}\omega_{nl} \cdot \partial_{\vec{k}} N_k = 2\gamma_{nl} N_k - \Delta\omega_k N_k^2 \tag{12}$$

Here ω_{nl} and γ_{nl} are the frequency and growth rate of the MHD mode and the steady state balance leads to mixing length saturation $N_k \sim 2\gamma_k/\Delta\omega_k$. In the limit $q_x < k_\perp$, the action density (N_k) of background short scale turbulence is conserved and defined as $N_k = E_k/\omega_k$; E_k is the energy of the underlying turbulence and N_k is defined as:

$$N_k = \Lambda_k |\hat{\phi}_k|^2, \text{ And } \Lambda_k \alpha_k \hat{k}_y = [k_\perp^2 + (2\hat{k}_\parallel \hat{k}_\perp / \beta_e |\omega_k|)^2 + (\alpha_i \hat{k}_y / \tau_i |\omega_k|)^2] \tag{13}$$

The action density (N_k), $\hat{\omega}_{rk}$, and $\hat{\gamma}_k$ fluctuate due to slowly varying fields. Here the linear frequency is modified due to slow variation of Doppler shift and the growth is modified due to slow variation of parallel wave vector,

$$\hat{\omega}_k = \hat{\omega}_{rk}^{(0)} - \hat{k}_y \partial_x \hat{\phi}_q \text{, and } \hat{\gamma}_k \approx \hat{\gamma}_k^{(0)} + \hat{\gamma}_k^{(1)} - (\hat{k}_y \beta_e / 2)(\partial \hat{\gamma}_k^{(0)} / \partial \hat{k}_\parallel + \partial \hat{\gamma}_k^{(1)} / \partial \hat{k}_\parallel) \partial_x \hat{A}_{\parallel q} \quad (14)$$

Where $\partial \hat{\gamma}_k^{(0)} / \partial \hat{k}_\parallel = -2\hat{k}_\parallel / \beta_e \hat{\gamma}_k^{(0)}$, $\partial \gamma_k^{(1)} / \partial \hat{k}_\parallel = 2\gamma_k^{(1)} / 3\hat{k}_\parallel$. It is interesting to note from equations (14) that the both electrostatic potential and magnetic field of long scales (i.e., $q_x < k_\perp$) are generated in the MHD turbulence. Introducing the Fourier transform in equation (12), the response of slow varying fields can be written as

$$\overline{\Omega}_{q,k} \delta N_k = -i\hat{q}_x^2 k_y \partial_{k_x} \overline{N}_k \hat{\phi}_q + \beta_e \hat{q}_x \hat{k}_y (\partial \gamma_k^{(0)} / \partial \hat{k}_\parallel + \partial \gamma_k^{(1)} / \partial \hat{k}_\parallel) \overline{N}_k \hat{A}_{\parallel q} ; \overline{\Omega}_{q,k} = \Omega_q - \hat{q}_x V_{gx} + 2i\hat{\gamma}_k \quad (15)$$

The evolution of zonal flows (10) and magnetic field (11) are closed via response of $|\hat{\phi}_k|^2$ and through quasilinear approach, from equation (13) it can be expressed as, $\delta N_k = \Lambda_k \delta |\hat{\phi}_k|^2$. From equations (10-15) the growth rate of zonal flows and zonal fields recognized as:

$$\hat{\gamma}_q^{\phi} \approx -\nu_\phi + \hat{q}_x^2 \sum (1 - 2\hat{k}_\parallel^2 / \beta_e |\hat{\omega}_k|^2)(\hat{k}_y^2 \hat{k}_x / 2\gamma_k^{(0)} \Lambda_k)(-\partial \overline{N}_k / \partial \hat{k}_x) , \quad (16)$$

$$\hat{\gamma}_q^{A_\parallel} = -\hat{\eta} \hat{q}_x{}^2 + \hat{q}_x^2 \sum (\hat{k}_\parallel \hat{k}_y^2 / |\omega_k|^2)[(2\hat{k}_\parallel / \beta_e \hat{\gamma}_k^{(0)}) - (2\hat{\gamma}_k^{(1)} / 3\hat{k}_\parallel)](\overline{N}_k / \Lambda_k) . \quad (17)$$

Here ν_ϕ is the neoclassical collisional damping of zonal flow, and whereas $\hat{\eta}\hat{k}_\perp^2$ is the resistive damping of zonal fields.

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We now compare the growth rates of linear MHD modes, large scale zonal flows and magnetic instability with Alfven time scale for that we write the growth rates of above instabilities in un-normalized form; (1) The growth rate of ideal mode: $\gamma_k^{(0)} \approx (c_A / qR)[\beta_c - 1]^{1/2}$, (2) the zonal flows and zonal field growth rates are:

$$\gamma_q^{\phi} \approx -\nu_i + k_\parallel c_A (q_x \rho_s)^2 [1 - \frac{1}{(\beta_c - 1)}](\frac{k_y L_{pi}}{2})^2 (\frac{R}{L_{pi}}) \frac{\beta_c}{(\beta_c - 1)^{1/2}} |\frac{e\phi}{T_e}|^2 \, \Theta(\beta_c - 1) , \quad (18)$$

$$\gamma_q^{A_\parallel} \approx -\frac{\nu_e}{\beta_e} \frac{m_e}{m_i} (q_x \rho_s)^2 + k_\parallel C_A (q_x \rho_s)^2 (\frac{k_y L_{pi}}{2})^2 (\frac{R}{L_{pi}}) [\frac{2\beta_c}{(\beta_c - 1)^{3/2}} \Theta(\beta_c - 1) - \frac{\beta_c}{f(\nu_e)}] |\frac{e\phi}{T_e}|^2 \quad (19)$$

Here $\beta_c = \beta q^2 R / L_{pi}$, $T_i = T_e$, the parameter $f(\nu_e) \approx [(0.51\nu_e / \Omega_e)(qRk_\perp^2 c_A / \Omega_i)]^{1/3}$ and $k_\parallel = 1/qR$, $\Theta(\beta_c - 1)$ is the Heavy-side function. It is important to note that when $\beta_c \geq 1$ the ideal ballooning mode is unstable, the magnetic Reynolds stress effects completely suppresses the zonal flow growth. *Thus, the dynamics of zonal flows in ELMs relaxation is unimportant when* $\beta_c \geq 1$.

b. Estimations of transport in pedestal. - The turbulent radial heat flux is given by

$$\Gamma_Q = < \delta p_i \delta V_r > \approx p_0 c_s (k_y \rho_s) \eta_i \omega_* \gamma_k / |\omega_k|^2 | e\phi / T_e |^2 \quad (19)$$

For $\omega_{rk} < \gamma_k$, the Fick's law $\Gamma_Q = -\chi_i dp_i / dx = p_0 \chi_i / L_p$ yields the turbulent heat diffusivity,

$$\chi_i = \chi_i^{GB} (k_y L_p)^2 (R / 2L_p)^{1/2} [\beta_c / (\beta_c - 1)]]^{1/2} | e\phi_k / T_e |^2 ; \; \chi_i^{GB} = c_s \rho_s^2 / L_p$$

To estimate the saturated level of ELMs, we write the coupled system of zonal fields and ballooning mode that are like 'Predator-prey' type equations for $<\Phi_k>$ evolution equation of ballooning mode and $A_{\parallel q}$, driven zonal field equations [11],

$$\partial_t <\Phi_k>=\gamma_k <\Phi_k> -\alpha_0 \hat{A}_{\parallel q} <\Phi_k> -\Delta\omega <\Phi_k>^2 \tag{20}$$

$$\partial_t \hat{A}_{\parallel q} = \alpha_0 \hat{A}_{\parallel q} <\Phi_k> -\eta_0 \hat{A}_{\parallel q} \tag{21}$$

Here $<\Phi_k>=| e\phi_k / T_e |^2$; $\eta_0 = (\nu_e m_e / \beta_e m_i)(q_x \rho_s)^2$; $\alpha_0 = k_\parallel C_A (q_x \rho_s)^2 (k_y L_{pi})^2 (R/4L_{pi})(2\beta_c/(\beta_c-1))^{3/2}$. In steady state, balancing the growth of zonal fields and damping due to resistivity sets the turbulent level, and that yields $<\Phi_k>=\eta_0/\alpha_0$. Then, the turbulent thermal diffusion coefficient for zonal magnetic field saturation is: $\chi_i = 1.42\chi_i^{GB} \beta_e^{-3/2} (\nu_e L_p m_e / c_A m_i)(\beta_c - 1)$.

5. Collisionless ballooning instability in tokamak edge

We study the linear instabilities of non-ideal curvature driven modes using fluid models, including the influence of trapped electrons and electron inertia in the weakly collisional edge. This fluid model is a reduced formulation of the standard Weiland model extended with a trapped fluid treatment [12]. The model equations consist of ions, trapped and untrapped electrons continuity equations, and parallel electron momentum equations for untrapped electrons. The ion and trapped electron mode fluid model reasonably symmetric except for the FLR terms in the ion fluid and collision between passing and trapped particles terms in trapped electron fluid. We have further simplified our model assuming uniform temperature for all species and for $\omega \sim c_e / qR$; the wave particle resonant effect has been neglected for the simplicity. These equations are:

$$\omega\tilde{n}_i - \omega_{*n}\tilde{\phi} + \frac{\omega_c}{2}(\tilde{n}_i + \tilde{\phi}) + k_\perp{}^2\rho^2(\omega+\omega_{*n})\tilde{\phi} = 0, \quad \omega\tilde{n}_t - \omega_{*n}\sqrt{\varepsilon}\tilde{\phi} + \frac{\omega_c}{2}(\sqrt{\varepsilon}\tilde{\phi} - \tilde{n}_t) = -i\nu_{eff}(\tilde{n}_t - \sqrt{\varepsilon}\tilde{n}_u), \tag{22}$$

$$\omega\tilde{n}_u - \omega_{*n}(1-\sqrt{\varepsilon})\tilde{\phi} + \frac{\omega_c}{2}\tilde{\psi} + i(1-\sqrt{\varepsilon})\nabla_\parallel \tilde{v}_{\parallel e} = 0, \quad \tilde{\psi} = (1-\sqrt{\varepsilon})\tilde{\phi} - \tilde{n}_u \tag{23}$$

$$(-k_\perp{}^2 \tilde{A}_\parallel = i\beta m_i [2m_e qR(\omega + i0.51\nu_{ei})(1-\sqrt{\varepsilon})\rho^2]^{-1}[\partial_\theta \tilde{\psi} - i\{\omega_{*n} - \omega(1-\sqrt{\varepsilon})\}]\tilde{A}_\parallel . \tag{24}$$

Combining Eqs (22-23) yields

$$d_{\theta\theta}\tilde{\psi} + c(\beta)(\omega + i0.51\nu_{ei})[\omega - \omega_c/2 + \{(\omega_n - \omega)(1-\sqrt{\varepsilon})/F(\theta)\}\{\omega - \omega_c/2 + i\nu_{eff}(1+\sqrt{\varepsilon})\}/\{\omega - \omega_c/2 + i\nu_{eff}\}]\tilde{\psi} = 0 \tag{25}$$

Here $c(\beta) = 1 + \beta m_i(\omega[1-\sqrt{\varepsilon}] - \omega_{*n})/(2m_e[\omega + i0.51\nu_{ei}](1-\sqrt{\varepsilon})k_\perp{}^2\rho^2)$, $\omega_c = 2\varepsilon_n\omega_n(\cos\theta + s\theta\sin\theta - \varepsilon)$
$F(\theta,\omega) = \sqrt{\varepsilon}[(\omega_n - \omega - i\nu_{eff}\sqrt{\varepsilon})/(\omega - \omega_c/2 + i\nu_{eff})] + [\{\omega - \omega_n + \omega_c + k_\perp^2\rho_s^2(\omega+\omega_n)\}/(\omega + \omega_c/2)]$,
$k_\perp^2 = k_\theta^2(1+s^2\theta^2)$. We now solve the Eq. (25) for hotter plasma edge parameters: $T_e = T_i = 300eV$, $n = 8\times10^{12}$, $a = 0.8m$, $R = 2.6m$, $B_T = 4T$, $s = 0.1-3.5$, $L_n = (0.02-0.1)m$, and $\lambda_f > qR$. We use a finite difference code with boundary condition $\psi = 0$ at $\theta = \infty$ and $\psi' = 0$ at $\theta = 0$ and frequencies are normalized with electron transit frequency ($\omega_{te} = c_e / qR$). As it is shown in Fig. 1(a) that the growth rate of electrostatic mode (dashed) is dominate over electromagnetic mode for

$m > 200$. These eigen-modes having maximum growth at different value of m, which are well localized in θ-space. Electrostatic mode has maximum growth rate at $m \sim 750$ (that gives $k_\theta \rho_s \approx 0.4$) for small value of L_n / R. Fig. 1(b) shows the growth of electrostatic (dashed) and electromagnetic (solid) modes. Both perturbations show maximum growth rate at $m = 600$. It is to be noted that the growth rate of these modes with trapped electron effects is higher than without trapped particle effects. We also observe the magnetic shear destabilization of short wave the mode ($m > 200$).

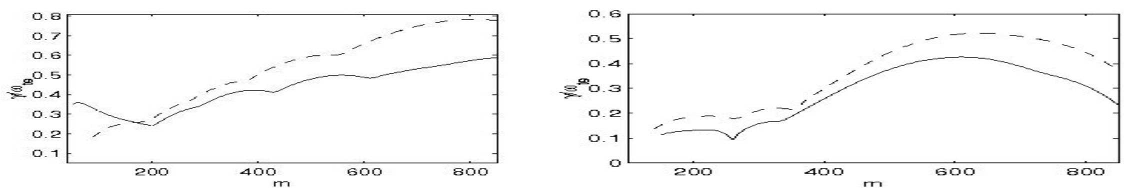

FIG. 1 (a) the growth rate versus m for electrostatic (dashed line) and electromagnetic (solid line); (b) growth rate versus m without trapped electron effects.

A detailed understanding of particle and energy transports in the presence of external magnetic perturbations is still an open challenging topic to investigate. Here we have proposed a theoretical model for mitigation of ELMs due to stochastic field lines as observed in DIII-D. We have elucidated the physics of various modes as potential candidate for explaining the anomalous transport in the edge pedestal. The linear stability of ballooning-peeling analysis shows that the kink effect significantly modifies the threshold of ballooning mode. In nonlinear instability analysis, we have carried out the effect of external field perturbations on the coupled ballooning-kink mode by using the technique of parametric instability calculations. It is shown that the back reaction of external magnetic field perturbations can drastically modify the threshold of the ballooning-kink mode if $| b_{cr} > \delta_R / qR \approx 0.15\%$ in D-IIID. We also report a simple self-consistent theoretical model of multi-scale interaction of edge-localized modes (ELMs) such as the ideal ballooning-peeling modes interacting with zonal magnetic fields and zonal flows. The secondary instabilities of zonal fields are used to estimate saturation level and energy flux induced by ballooning-peeling mode turbulence. Finally, the linear instabilities of non-ideal curvature driven modes, including the influence of trapped electrons and electron inertia has also been investigated using simple fluid model. Even when $\beta < \beta_c$, electron inertia and trapped effects can derive a robust non-ideal curvature driven instability.

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