Theory of Alfvén waves and energetic particle physics in burning plasmas

L. Chen 1) and F. Zonca 2)
1) Dept. of Physics and Astronomy, Univ. of California, Irvine CA 92697-4575, U.S.A.
2) Associazione Euratom-ENEA sulla Fusione, C.P. 65 - I-00044 - Frascati, Rome, Italy

e-mail contact of main author: liuchen@uci.edu

Abstract. We present an overview on one issue of practical interest for burning plasmas; i.e., whether fast ions and charged fusion products are sufficiently well confined that they transfer their energy and/or momentum to the thermal plasma without appreciable degradation due to collective modes. In the present work, we analyze this problem starting from its first principle theoretical grounds, i.e. the identification of key physics in burning plasma stability properties as well as the non-linear dynamics above threshold. We also discuss the investigations of such processes via computer simulations as well as the importance of benchmarking with existing or future experimental observations.

1. Introduction

A burning plasma is a self-organized system, where the energy density of fast ions (MeV energies) and charged fusion products is a significant fraction of the total plasma energy density. Generally, one can address two main issues of practical interest concerning these plasmas: (i) whether fast ions and charged fusion products are sufficiently well confined that they transfer their energy and/or momentum to the thermal plasma without appreciable degradation due to collective modes; (ii) whether mutual interactions between collective modes and energetic ion dynamics on the one side and drift wave turbulence and turbulent transport on the other side, may decrease, on long time scales, the thermonuclear efficiency of the considered system. In terms of their consequences, these two issues have different implications: the first one has direct impact on the operation scenarios and boundaries, since energy and momentum fluxes due to collective losses may lead to significant wall loading and damaging of plasma facing materials; while the second one poses soft limits in the operation space. In the framework of plasma theory, meanwhile, the first point is connected with the identification of burning plasma stability boundaries with respect to collective mode excitations by fast ions and charged fusion products as well as with the non-linear dynamics above the stability thresholds, whereas the second issue is associated with long time-scale non-linear behaviors typical of self-organized complex systems. In the present work, we analyze the first aspect starting from its first principle theoretical grounds and discuss the possibilities of investigating such processes via computer simulations as well as the importance of benchmarking with existing or future experimental observations. The second issue is much more challenging and will be mentioned only in the Discussion Section.

Shear Alfvén (SA) wave excitations by energetic ($E \approx 1MeV$) ions are investigated as the primary candidate for causing fluctuation induced transport of fast ions and charged fusion products (from now on referred to as fast or energetic ions) in burning tokamak plasmas. This subject is a clear example of the successful and positive feedback between theory and experiment in the recent years of fusion plasma physics research. The resonant excitations of Alfvén Modes in toroidal plasmas by fast ions is considered in the present work, revisiting previous analyses that demonstrated in general the existence of two types of modes; *i.e.*, a discrete shear Alfvén gap mode, or Alfvén Eigenmode (AE), and an Energetic Particle continuum Mode

(EPM) [1]. It can be consistently shown that the resonant fast ion dynamics in toroidal plasmas enters mainly via the magnetic drift curvature coupling in the vorticity equation, while the non-resonant contributions depend on the ratio of the characteristic fast ion orbit width, ρ_{EP}, to the perpendicular mode wavelength, $\lambda_\perp$. The optimal wavelength ordering for analyzing energetic ion transport in burning plasmas, meanwhile, is typically $\lambda_\perp \gtrsim \rho_{EP}$. As a general result, we show that a general *fishbone*-like dispersion relation [1, 2] not only addresses the excitation of the SA frequency spectrum by energetic ions *precession*, *precession-bounce* and *transit* resonances in the whole frequency range from Kinetic Ballooning Modes (KBM)[3]/Beta induced AE (BAE)[4] to Toroidal AE (TAE)[5]. It can also describe Alfvén Cascades (AC) [6] as well as the transition to Alfvénic Ion Temperature Gradient (AITG) [7]. Alfvén mode excitation by both energetic ions (at the longest wavelengths) as well as by thermal ions (at the shortest wavelengths) on a wide range of mode numbers were recently observed in DIII-D and confirmed by numerical stability calculations [8].

Non-linear evolutions of the energetic ion distribution function are affected by formation of phase space-structures. Noticeable examples of such structures are the pitchfork splitting of TAE spectral lines observed on JET [9] and explained in terms of hole-clump pair formations in phase space, near marginal stability [10]. Simulation results indicate that, above threshold for the onset of resonant EPM [1], strong fast ion transport occurs in "avalanches"[11]. Such strong transport events occur on time scales of a few inverse linear growth rates (generally, $100 \div 200$ Alfvén times) and have a ballistic character [12] that fundamentally differentiates them from the diffusive and local nature of weak transport. Recent experimental observations on the JT-60U tokamak have also shown macroscopic and rapid (in the sense discussed above) energetic particle radial redistributions in connection with the so called Abrupt Large amplitude Events (ALE) [13].Therefore, it suggests the importance of theoretically assessing the potential impact of fusion product avalanches due to the hard limit that these may pose on burning plasma operations. Here, we will explore and analyze these issues based on recent numerical simulations as well as lower-dimensional analytic non-linear dynamic models [11].

2. Linear Shear Alfvén Waves and Energetic Particle physics

The significance of energetic particle (E.-P.) physics in burning plasmas (B.-P.) is profoundly connected with the fact that E.-P. are an intrinsic component of B.-P. experiments such as ITER. In the present work, we focus on the roles played by fast ions (MeV energies), produced by additional heating and/or current drive systems, and charged fusion products, *e.g.* α-particles in D-T plasmas, equivalently referring to these as either E.-P. or fusion α's; however, the role of supra-thermal electrons can be important as well for very similar aspects [14].

Shear Alfvén Waves (SAW) are anisotropic electromagnetic (e.m.) waves in magnetically confined plasmas, satisfying the dispersion relation

$$\omega^2 = k_\parallel^2 v_A^2 = \omega_A^2 \quad , \tag{1}$$

with $v_A = B/(4\pi\varrho)^{1/2}$ the Alfvén velocity, B the equilibrium magnetic field strength, ϱ the plasma mass density and $k_\parallel$ the parallel (to $\mathbf{B}$) wave vector. Shear Alfvén Waves are fundamental oscillations in laboratory as well as space plasmas, *e.g.* solar, interstellar and magnetosphere plasmas [15]. For example, SAW have important dynamic roles in solar corona heating [16] and accelerating auroral electrons [17]. Collective excitations of the SAW spectrum by E.-P. are possible because of the resonant condition, $v_\alpha \simeq v_A$, between the α-particle characteristic speed, v_α, and the Alfvén velocity, which is easily satisfied in a B.-P. [18, 19]. Furthermore, the resonance condition is efficiently maintained since the SAW group velocity is directed along $\mathbf{B}$ and has magnitude v_A, $\mathbf{v}_g = (\partial\omega/\partial k_\parallel) = \pm(\mathbf{B}/B)v_A$, and particles are tight to a given field line while freely moving along $\mathbf{B}$ in a strongly magnetized system.

In non-uniform plasmas, gradients across the magnetic surfaces cause the SAW frequency to become dependent on the spatial location. In the simplest one dimensional case , $\omega_A^2 = \omega_A^2(r)$, as predicted by Eq. (1), with r the radial coordinate across the plasma cylinder: thus, SAW are characterized by a continuous spectrum. As a consequence of the SAW continuum, initial perturbations decay via phase mixing on time scales $\approx (\Delta r(d/dr)\omega_A(r))^{-1}$, with Δr the perturbation radial extent. Meanwhile, a driven perturbation with frequency ω_0 is *singularly* absorbed at the *resonant layer*, where $\omega_0^2 = \omega_A^2(r_0)$: the resonant absorption (*continuum damping*) rate scales as $\propto (d/dr)\omega_A(r_0)$ [20]. The exact analogy of phase mixing and singular absorption with collisionless dissipation in Vlasov plasmas was emphasized by Grad [21]. In higher dimensionality systems, such as nearly two-dimensional or three-dimensional toroidal devices, the main additional complication is due to the *modulations* of v_A along $\mathbf{B}$. This causes the loss of translational symmetry for SAW traveling along $\mathbf{B}$ and sampling regions of periodically varying v_A. Similarly to electron wave packets traveling in a one-dimensional periodic lattice, SAW in toroidal systems are characterized by gaps in the SA continuous spectrum, corresponding to the formation of standing waves at the Bragg reflection condition. Continuing the analogy with the one-dimensional periodic lattice case, discrete SA eigenmodes (AE) can be *localized* in these gaps due to MHD and/or E.-P. effects (i.e., the *defects*). The AE are weakly affected by continuum damping and the consequences of this important fact are discussed in detail in Section 2.1.

Fluctuations of the SAW spectrum are characterized by frequencies, $|\omega_A| \ll \omega_c$, much lower than the ion cyclotron frequency. The magnetic moment, μ, is thus well conserved, while both second (action) invariant, $J = M\oint v_\parallel dl$, and toroidal canonical angular momentum, $P_\phi = M\left(Rv_\parallel - \omega_c\psi/B\right)$, are broken by wave-particle interactions in the presence of SAW. Here, M is the particle mass, $v_\parallel$ it the particle velocity along the equilibrium magnetic field, l is the arc length along $\mathbf{B}$, ψ is the poloidal magnetic flux defined such that $\mathbf{B} = F(\psi)\nabla\phi + \nabla\phi\times\nabla\psi$ and we have used cylindrical coordinates (R, Z, ϕ). The breaking of two of the invariants of particle motion in the presence of SAW, implies that we must expect fluctuation induced particle transports (redistribution) in phase space. In strongly magnetized plasmas, such as those of fusion interest, $P_\phi \simeq -M\omega_c\psi/B$ and fluctuation induced (*anomalous*) particle transports are conveniently (and equivalently) represented in the $(\mathcal{E} = v^2/2, r)$ rather than the (J, P_ϕ) phase space, where r is a radial-like flux coordinate in the standard representation of toroidal flux coordinates, (r, θ, ϕ), with θ the poloidal angle. The various effects and implications of anomalous E.-P. transports in B.-P. will be analyzed in Section 3.

For SAW in $\beta = 8\pi P/B^2 \approx 10^{-2} \ll 1$ toroidal plasmas, with P the total plasma pressure, magnetic drifts, $\mathbf{v}_d$, play a fundamental role for wave-particle interactions and collective wave excitations [22]. In these conditions, in fact, SAW are characterized by negligible fluctuations of both electric and magnetic fields along $\mathbf{B}$; i.e., $\delta E_\parallel \approx 0$ and $\delta B_\parallel \approx 0$. Thus, particles experience the $(\mathbf{v}_d \times \delta\mathbf{B}_\perp)\cdot\mathbf{b}$ force and exchange energy with SAW via $\mathbf{v}_d\cdot\delta\mathbf{E}_\perp$, where $\mathbf{b} = \mathbf{B}/B$ and the other notation is standard. The wave-particle resonance conditions, meanwhile, are different for circulating particles (which move along $\mathbf{B}$ without changing sign of $v_\parallel$) and those that are trapped between magnetic mirror points. For the first particle class, the resonance condition is

$$\omega - k_\parallel v_\parallel - p\omega_t = 0 \ , \qquad p \in \mathbb{Z} \ , \tag{2}$$

with $\omega_t \approx v_\parallel/(qR)$ the particle transit frequency around the torus and q the safety factor; meanwhile, Eq. (2) for trapped particles becomes

$$\omega - \bar{\omega}_d - p\omega_b = 0 \ , \qquad p \in \mathbb{Z} \ , \tag{3}$$

with $\bar{\omega}_d$ and ω_b, respectively, the toroidal precession and bounce frequencies.

2.1. Shear Alfvén Wave spectrum: continuous and discrete spectra

As discussed in Section 2, the various SA modes that can be excited in the presence of the energetic ion free energy source are strongly influenced by the presence of the SA continuous spectrum, which is characterized by gaps. The frequency gap at $v_A/(2qR_0)$ (R_0 being the plasma major radius and q the safety factor) is due to the finite toroidicity of the system [23], but other gaps generally exist at $\omega = \ell v_A/(2qR_0)$, due to either non-circularity of the magnetic flux surfaces ($\ell = 2, 3, \ldots$) [24], to anisotropic trapped energetic ion population ($\ell = 1, 2, 3, \ldots$) [25] or to finite-β (mainly $\ell = 2$) [26]. A low-frequency gap also exists because of finite plasma compressibility [27].

In order to nullify or minimize continuum damping, discrete SA eigenmodes (AE) must be localized in the SA continuum frequency gaps and/or around radial positions where $(d/dr)\omega_A(r) = 0$. The degeneracy of AE mode frequency with the SA continuous spectrum is removed by equilibrium non-uniformities, which make it possible for these fluctuations to exist as discrete modes. In particular, AE collective excitations by E.-P. are possible due to the coupling of the E.-P. pressure perturbation with the SAW vorticity equation via magnetic curvature drifts [22]. The discrete AE existing in the various frequency gaps and have been given different names accordingly; *e.g.*, Beta induced AE (BAE) [4, 27] for $\omega \simeq \beta_i^{1/2}(7/4 + T_e/T_i)^{1/2} v_A/R_0$ [28], with β_i the thermal (core) ion β and T_e (T_i) the core electron (ion) temperature; Toroidal AE (TAE) [5] for $\omega \simeq v_A/(2qR_0)$; Ellipticity induced AE (EAE) [24] for $\omega \simeq v_A/(qR_0)$, etc.. In addition, a variety of kinetic counterparts of the corresponding ideal AE also exists when, *e.g.* finite resistivity [5] or finite Larmor radius (FLR) are accounted for, as in [29] for the Kinetic TAE (KTAE). Similarly, and consistently with the fact that removal of the frequency degeneracy of AE with the SA continuum depends on equilibrium non-uniformities, peculiar behaviors of local plasma profiles can produce variants of the AE mentioned above, as in the case of TAE mode structure and existence condition, which are modified at low magnetic shear values, $s = (r/q)(dq/dr) \ll 1$, typical of the plasma near the magnetic axis and have been dubbed core-localized TAE [30].

Global AE (GAE) [31] may also exist in a non-uniform cylindrical plasma equilibrium and are localized in both frequency and radial position near an extremum of the SA continuous spectrum. A special case of $(d/dr)\omega_A(r) = 0$ is given by tokamak plasma equilibria with for hollow-q profiles, characterized by negative magnetic shear, $s < 0$, inside the the minimum-q surface. For these equilibria, a frequency gap is formed in the local SA continuous spectrum, where AE can be excited [32], yielding the so called Alfvén Cascades (AC) [6] or Reversed Shear AE (RSAE) [33].

In the next Section, we discuss how all these modes can be described by the same dispersion relation, which is written in one single general form.

2.2. Stability properties: general fishbone-like dispersion relation

Perturbations of the SAW spectrum generally consist of singular (inertial) and regular (ideal MHD) structures. For this reason, it is always possible to derive a general *fishbone-like* dispersion relation in the form [1, 22]

$$i\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k \quad . \tag{4}$$

Here, the left hand side (LHS) is the inertial layer contribution due to thermal ions, while the right hand side (RHS) comes from background MHD and E.-P. contributions in the regular ideal regions. On the basis of Eq. (4), two types of modes exist: a discrete AE, for $\mathbb{R}\mathrm{e}\Lambda^2 < 0$; and an Energetic Particle continuum Mode (EPM) [1] for $\mathbb{R}\mathrm{e}\Lambda^2 > 0$. For AE, the combined effect of $\delta\hat{W}_f$ and the non-resonant fast ion response, $\mathbb{R}\mathrm{e}\delta\hat{W}_k$, provides a real frequency shift, which removes the degeneracy with the SA continuum accumulation point and makes the mode weakly damped [5]. Meanwhile, the resonant wave-particle interaction, $\mathbb{I}\mathrm{m}\delta\hat{W}_k$, gives the mode drive, which is necessary to overcome the small but finite damping due to the core plasma component. In the case of EPM, ω is set by the relevant energetic ion characteristic frequency and mode excitation requires the drive exceeding a threshold due to continuum damping [20, 34, 35, 36]; i.e., $\mathbb{I}\mathrm{m}\delta\hat{W}_k > \mathbb{R}\mathrm{e}\Lambda$ in Eq. (4). However, the non-resonant fast ion response is crucially important for EPM excitation as well, since it provides the compression effect that is necessary for balancing the generally positive MHD potential energy of the wave [1, 2], $\mathbb{R}\mathrm{e}\delta\hat{W}_k + \delta\hat{W}_f \simeq 0$. A celebrated example of EPM is the *fishbone* instability [2, 37], where

$$i|s|\left[\left(R_0^2/v_A^2\right)\omega\left(\omega-\omega_{*pi}\right)(1+\Delta)\right]^{1/2} = \delta\hat{W}_f + \delta\hat{W}_k \quad , \tag{5}$$

where ω_{*pi} is the core ion diamagnetic frequency and $\Delta \propto q^2$ is the enhancement of plasma inertia due to geodesic curvature [38, 39].

The combined effect of $\delta\hat{W}_f$ and $\mathbb{R}\mathrm{e}\delta\hat{W}_k$, which determines the existence conditions of AE by removing the degeneracy with the SA accumulation point, depends on the plasma equilibrium profiles. Thus, various effects in $\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k$ can lead to AE localization in various gaps, i.e. to different species of AE, as described in Section 2.1. This AE Zoology [40] is consistently described by the single and general dispersion relation Eq. (4). A simple example is the TAE near the lower accumulation point, ω_ℓ, of the toroidicity induced frequency gap in the SA continuum [23]. In this case, $\Lambda^2 v_A^2/(q^2R_0^2) = \omega_\ell^2 - \omega^2$ and the TAE mode existence condition reads $\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k > 0$ [5]. Similar pictures could be easily extended to TAE localized near the upper SA accumulation point, and to AC or RSAE. In this latter case, *e.g.*, $\Lambda^2 v_A^2/(q^2R_0^2) = (\omega^2 - k_\parallel^2 v_A^2)/(k_\parallel q R_0)$ [22] and the gap mode existence condition, $\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k > 0$, is favored by the Mercier stability condition at $s = 0$ [41]. The effect of plasma compressibility on ACs was also analyzed recently [41, 42], when the mode frequency becomes comparable with that of the low-frequency SA accumulation point $\omega \simeq \beta_i^{1/2}(7/4 + T_e/T_i)^{1/2} v_A/R_0$ [27, 28].

The low frequency SA continuum frequency gap deserves a special note since, in this case, the mode frequency can be comparable with thermal (core) ion diamagnetic and/or transit frequencies; i.e., $|\omega| \approx \omega_{*pi} \approx \omega_{ti}$. We can generally refer to this SA frequency gap as the Kinetic Thermal Ion (KTI) gap. In fact, the ideal MHD accumulation point ($\omega = 0$ at $k_\parallel = 0$) is shifted by either the ion diamagnetic drift (as in the KBM case [3]), or by parallel and perpendicular ion compressibility (as for BAE [4]), or, more generally, by the combined effects of finite

∇T_i and wave-particle resonances with thermal (core) ions (as for AITG [7]) in the inertial layer. For the AITG, the SA continuum accumulation point becomes unstable for modes with sufficiently short wavelength ($\lambda_\perp \gtrsim \rho_i$, the ion Larmor radius) and the mode "localization" condition corresponds to the excitation of an unstable discrete AITG independently of the E.-P. drive. Obviously, similar physics considerations and results will be applicable to RSAE/AC.

2.3. Experimental verification of linear Alfvén Eigenmodes and stability predictions in burning plasmas

The many experimental observations of AE and EPM are well documented [40, 43] and reporting them in detail is beyond the scope of the present work. Here, we illustrate the successful and positive feedback between theory and experiment in this area by the recent observation in DIII-D of AC/RSAE excitation on a wide range of mode numbers [8]. The theoretical mode frequency, $\omega \simeq |n - m/q_{min}| (v_A/R_0)$ (m/n being the poloidal/toroidal mode number), for mode excited near the minimum-q surface, is well confirmed by experimental measurements. Meanwhile, the observation of mode numbers up to $n \sim \mathrm{O}(40)$ demonstrates AC/RSAE excitation by both energetic ions (at the longest wavelengths) as well as via the AITG mechanism (at the shortest wavelengths), as confirmed by numerical stability calculations [8].

Numerically calculated AE damping rates using non-perturbative kinetic models agree qualitatively with experiments, although details of damping mechanisms are still being debated [44, 45, 46, 47]. Precise comparisons with measured damping rates depend on plasma edge boundary conditions [46], as it was recently confirmed by global gyrokinetic code simulations [47]. This suggests that future developments in numerical stability analyses of B.-P. in realistic conditions will need to incorporate accurate models of the Scrape Off Layer (SOL) and of the mode structure outside the last closed magnetic surface in divertor configurations. Another critical aspect is the accurate modeling of the mode conversion of long wavelength MHD-like modes to shorter wavelengths, typical of Kinetic Alfvén Waves (KAW) [48]. Differences in the wave propagation properties are the explanation of different predictions on the AE kinetic damping rates in the plasma interior, which still require some efforts in code benchmarking among themselves and vs. known analytical results. Finally, reliable numerical stability calculations for EPMs must account for self-consistent energetic ion physics, since the fast ion free energy source may modify the wave field structure itself [1].

As all instabilities that tap the expansion free-energy from energetic particle spatial gradients, AE and EPM have both linear growth as well as transport rates proportional to the mode number [49]: thus, short wavelengths tend to be favored. On the other hand, due to the fundamental role of wave-particle interactions in the transport processes [49], the typical lower bound for $\lambda_\perp$ is set by the characteristic E.-P. orbit width, ρ_{EP}, which, in toroidal devices such as tokamaks, is determined by magnetic drifts and is generally larger than the E.-P. Larmor radius. For this reason, modes with $\lambda_\perp \gtrsim \rho_{EP}$ are expected to play a dominant role for both resonant excitations of collective Alfvén instabilities as well as for producing fluctuation enhanced fast ion transport. This condition corresponds to $n_{max} q \lesssim (r/\rho_{EP})$. Generally, AE in the same gap have nearly degenerate frequency for the various toroidal mode numbers, as in the case of TAE [5]. Moreover, each nth mode has $\sim \mathrm{O}(nq)$ different possible realizations (radial eigenstates) of AE localized at different radial locations. Thus, *e.g.*, within the TAE gap we may expect $\sim \mathrm{O}(n^2 q)$ AE, forming a dense population of eigenmodes (*lighthouses*) with *unique* (equilibrium-dependent) frequencies and locations. In Section 3 we discuss the significant implications of this fact on the non-linear AE physics.

Numerical investigations of AE and EPM stability in B.-P., such as ITER, have shown that the most unstable (least stable) mode numbers are in the medium to high-n range, i.e. $n_{max} \simeq \mathrm{O}(10)$ [50, 51]. In realistic geometry, stability analyses of the ITER “positive shear” reference scenario with perturbative E.-P. dynamics demonstrate that AE are marginally stable in the presence of fusion-αs only, while instability is to be expected when E.-P. tails due to $1MeV$ Negative Neutral Beam Injection (NNBI) are accounted for [50]. Meanwhile, AE and EPM stability studies with self-consistent E.-P. physics in simplified circular geometry show that, in the presence of fusion-αs only, ITER is marginally unstable for AE in all the three reference scenarios; i.e., “positive shear” (SC2), “reversed shear” (SC4) and “hybrid scenario” (SCH) [51]. The slight discrepancy between [50] and [51] is likely due to the different equilibrium representation (shaped vs. circular), the treatment of fast ion dynamics (perturbative vs. self-consistent) and the profile differences in the reference scenarios. Simulations results also show that AE close to the plasma center are more easily excited by precession and precession-bounce resonances with trapped fusion-αs [51], due to the fact that the maximum drive due to NNBI is more externally located (at about mid radius of the plasma cross section). Similarly, AE structures mostly weighted in the outer plasma column are preferentially excited by NNBI and transit resonances [50, 52]. This fact explains why, due to orbit averaging effects, mode numbers of AE localized in the plasma core tend to be smaller. Generally, AE stability in ITER is most critical for the “reversed shear” (SC4) scenario [51, 52]. Assuming fusion-αs only, the threshold for $n \simeq \mathrm{O}(10)$ EPM excitation in the ITER SCH scenario is reached by artificially raising the on axis β_α by ≈ 1.6 at fixed profiles [51].

3. Non-linear Shear Alfvén Waves and Energetic Particle physics

The fundamental problem to be addressed in studies of collective mode excitation by E.-P. in B.-P. is to assess whether or not significant degradation in the plasma performance can be expected in the presence of Alfvénic fluctuations and, if yes, what level of wall loading and damaging of plasma facing materials can be caused by energy and momentum fluxes due to collective fast ion losses. The possible detrimental effects of collective mode excitation by E.-P. on the B.-P. performance can also manifest themselves in more restrictive requirements on the ignition condition even in the absence of global E.-P. losses. This is, *e.g.*, the case of fluctuation induced profile relaxations. Examples of these various effects are given below.

Energetic particle losses up to 70% of the entire fast particle population have both been predicted theoretically and found experimentally [53]. The non-linear SAW dynamics and their consequence on E.-P. transports is largely influenced by the characteristics of the AE and EPM spectra, described in Section 2. These issues are analyzed in the following.

3.1. Non-linear Alfvén Eigenmode physics

For weakly unstable AE, $|\gamma/\omega| \sim \mathrm{O}(10^{-2})$, a possible non-linear saturation mechanism is via phase-space non-linearities (wave-particle trapping) [54, 55]. This fact has been confirmed by many numerical simulations [56, 57, 58, 59]. The analogy with the single-wave bump-on-tail paradigm has been proposed for the case of a single linear TAE and non-linear resonant E.-P. response [54, 55]. Assuming wave-particle precession resonance only, both μ and J are conserved, P_ϕ being the only broken invariant of the particle motion. In this way, the correspondence $\omega_{TAE} \Rightarrow \omega_{pe}$ (the plasma frequency), $P_\phi \Rightarrow v$ and $F_{EP}(P_\phi|_{\mu,J}) \Rightarrow F_b(v)$ (the beam distribution function), demonstrates the analogy with the single-wave bump-on-tail paradigm conjectured in [54]. With the inclusion of background dissipation and $F_b(v)$ restoring via collisional processes (or $F_{EP}(P_\phi|_{\mu,J})$ restoring via source inputs), the trapping of E.-P. in the potential well of the wave causes hole-clump pair production in phase space and sideband generation [10]. This paradigm has been adopted for the theoretical explanation of the pitchfork splitting of TAE spectral lines observed on JET [9].

Another possible saturation mechanism for weakly unstable AE is via non-linear frequency shifts due to mode-mode couplings. For a single TAE, the dominant mode structures produced non-linearly are ($n = 0, m = 0$) zonal flows/fields [60] and ($n = 0, m = \pm 1$) $\delta \mathbf{B}_\perp$ [61] and δn [62] fluctuations. These radially localized non-linear structures generate local neighboring non-linear equilibria, narrowing the TAE frequency gap and/or lowering ω_{TAE}, thereby enhancing continuum or radiative damping. Recently, numerical simulations confirmed that $n = 0$ non-linear structures produced by mode-mode couplings give an estimate for AE saturation amplitudes on TFTR that are closer to (lower) experimentally measured levels than if they were not included in the simulation model [63].

When the non-linear dynamics of AE refer to single wave-particle resonances or to local saturation mechanisms, as in the examples discussed above, it is not surprising that AE yield negligible E.-P. losses, unless phase-space stochasticity is reached, possibly via phase-space explosion (*domino effect* [64]). This fact has been confirmed also by numerical simulations of alpha particle driven Alfvén gap modes in ITER [59]. For this reason, the dominant loss mechanism below stochastic threshold is expected to be that of scattering of barely counter-passing particles into unconfined (trapped particle) “fat” banana orbits [65, 66]; however, this loss mechanism is expected to be weak in ITER, due to the small ratio of banana orbit width to system size. Diffusive losses above the stochastic threshold are expected to scale as $\approx (\delta B_r/B)^2$, due to energetic particle stochastic diffusion in phase space and eventually across the orbit-loss boundaries. The stochastic threshold for a single AE is $(\delta B_r/B) \approx 10^{-3}$, although that may be greatly reduced ($(\delta B_r/B) \lesssim 10^{-4}$) in the multiple mode case [65, 66].

In the presence of multiple AE, as it is typical for TAE in a B.-P. (see Section 2.3), non-linear interactions can induce a downward frequency cascading via non-linear wave-particle resonances, *e.g.* non-linear ion Landau damping or Compton scattering off the thermal ions [67]. This process enhances continuum or radiative damping, analogously to the non-linear wave-wave interaction case described above, because of the cascading to lower frequency and more stable TAE [67]. As anticipated in Section 2.3, the fact that the most unstable (least stable) mode numbers in B.-P. are in the medium to high-n range, i.e. $n_{max} \simeq \mathrm{O}(10)$ [50, 51], implies a dense AE *lighthouse* spectrum with *unique* frequencies and locations in the (ω, r) space. Thus, we expect significant multiple-TAE non-linear interactions, yielding a diffusive redistribution of $F_{EP}(\mathcal{E}, P_\phi(r)|_\mu)$, where the notation in $P_\phi(r)|_\mu$ reminds that $P_\phi \simeq -M\omega_c\psi/B$ in strongly magnetized plasmas. Diffusive redistributions, meanwhile, are expected to take place via AE-avalanches; i.e., via gradient steepening and relaxation processes, closely resembling those typical of turbulence spreading [68, 69, 70].

3.2. Non-linear Energetic Particle Mode physics

Energetic Particle Modes are generally characterized by stronger drive than AE, $|\gamma/\omega| \sim$

$\mathrm{O}(10^{-2} \div 10^{-1})$, due to the threshold condition for their excitation, which is set by continuum damping [1]. In contrast with AE, EPM have frequencies, ω_{EPM}, which are set by the characteristic E.-P. dynamic frequencies, ω_{EP}. Furthermore, EPM are excited *in situ*, where $\alpha_{EP} = -R_0 q^2 (d\beta_{EP}/dr)$ is maximum [71]; i.e., the EPM spatial localization is set by the strength of the E.-P. resonant drive. For the intrinsic EPM resonant character and their natural localization at the radial position where the drive is strongest, EPM rapidly redistribute $F_{EP}(\mathcal{E}, P_\phi(r)|_\mu)$. Simulation results indicate that, above the linear stability threshold, strong EPM induced fast ion transport occurs in *avalanches* [11]. Such strong transport events occur on time scales of a few inverse linear growth rates (generally $100 \div 200$ Alfvén times, $\tau_A = R_0/v_A$) and have a ballistic character [12] that basically differentiates them from the diffusive and local nature of weak transport. In this respect, EPM induced avalanches are similar but also qualitatively different from the AE-avalanches (see Section 3.1), closely resembling those typical of turbulence spreading [68, 69, 70]. Energetic Particle Mode induced avalanches consist of a radially propagating unstable front producing a convective E.-P. radial redistribution [11]. This peculiarity is due to the fact that EPM growth rate has a stronger n dependence, producing a narrow toroidal mode number unstable spectrum, in contrast with the AE case. As a consequence, single-n non-linear dynamics dominates the initial rapid convective phase. The radial propagation of a single-n non-linear EPM localized mode structure takes place via couplings between poloidal harmonics and their interplay with non-linear distortion of the E.-P. source. An analytical description has been proposed for elucidating the EPM avalanche paradigm

$$D^{\ell}_{EPM} \left(-i\omega + \partial_t, \partial_r, r\right) A(r,t) = \delta \hat{W}^{n\ell}_k \left(\partial_t, \partial_r, r, |A|^2\right) A(r,t) \ , \tag{6}$$

where $A(r,t)$ is the radial envelope of the single-n EPM poloidal harmonics, $D^{\ell}_{EPM}(-i\omega + \partial_t, \partial_r, r)$ is the linear EPM dispersion function, including radial dispersiveness, and $\delta \hat{W}^{n\ell}_k(\partial_t, \partial_r, r, |A|^2)$ is the E.-P. $\delta \hat{W}_k$ due to non-linear wave particle interactions. This model predicts radial convective amplification of the EPM envelope and E.-P. source propagation, consistent with numerical simulations [11].

Experimental observations on the JT-60U tokamak have confirmed macroscopic and rapid energetic particle radial redistributions in connection with the so called Abrupt Large amplitude Events (ALE) [13]. Recent numerical simulation of $n = 1$ EPM modes on JT-60U, show that $F_{EP}(\mathcal{E}, P_\phi(r)|_\mu)$ is rapidly redistributed in the $(\mathcal{E}, r)$ space by the non-linear wave-particle interactions, consistently with ALE observations and in quantitative agreement with experimental results [72, 73]. Similar considerations can be made for the non-linear E.-P. dynamics in the presence of strong excitations of low-frequency MHD modes. Experimental observations of the fishbone mode [74] were originally interpreted as the resonant excitation of the $n = 1$ internal kink mode at $\omega = \bar{\omega}_{dE}$ [2], causing E.-P. convective losses with ballistic character [12], similar to those of EPM avalanches. Recent numerical simulations of both kink and fishbone instability confirm the fact that rapid E.-P. transport are expected when the system is significantly above marginal stability [75]. Fast radial redistributions involving $F_{EP}(\mathcal{E}, P_\phi(r)|_\mu)$ lead to fishbone mode saturation and downward frequency chirping. In addition to the complex interplay between mode structure and E.-P. source, simulation results also elucidate the role of fluid non-linearities, which are important for the correct evaluation of the saturated amplitude and mode structures [75].

Recent numerical simulations of B.-P. operations proposed for ITER indicate that significant fusion-α losses ($\approx 5\%$) may occur due to a rapid broadening of α-particle profiles in the hollow-q "advanced"-tokamak scenario [51]. Meanwhile, only moderate internal E.-P. relaxation are expected for "conventional" q-profiles, whereas strong EPM excitation and significant convective fusion-α losses are predicted in the "hybrid" centrally flat-q-profile case only if the volume averaged fast ion density is increased by a factor $\simeq 1.6$ [51]. These results are obtained assuming an initial given fusion-α profile without NNBI and Ion Cyclotron Resonance Heating (ICRH). Recalling the results on linear stability, discussed at the end of Section 2.3, the presence of NNBI (and ICRH) induced E.-P. tails will make the system more unstable and prone to losses, especially considering the more external radial location of the mode structures, which tend to easily interact with the radial profiles of the NNBI deposited tangential fast ions [50].

4. Summary and Discussions

The linear physics of collective SAW excitation by E.-P. in B.-P. is well at hand. Still, it is necessary to develop a comprehensive linear numerical code to accurately evaluate the B.-P. stability properties in realistic conditions, which include non-perturbative treatment of wave-particle interactions of SAW with E.-P., (gyro) kinetic analyses of the thermal (core) plasma particles and actual equilibrium geometries in the presence of X-points and divertor regions. This task is well within reach of the various integrated tokamak modeling initiatives and will involve a thorough benchmarking of different numerical codes among themselves and with existing and future experimental data on mode structures and damping/growth rates.

Medium to high mode number AE and EPM will be excited by the synergistic effect of fusion-αs and E.-P. due to external heating and current drive systems in ITER. The consequences of E.-P. transports remain unclear, although the excitation of EPM is expected to be characterized by convective transports/relaxations, in contrast with the typical diffusive nature of the transport processes due to AE. However, EPM are characterized by a higher excitation threshold.

The key non-linear physics mechanisms for collective SAW excitation by E.-P. in B.-P. are identified: some of these non-linear dynamics are "verified" either by customized numerical simulations or by experimental observations, whose comparisons have often produced in depth understanding of specific physical issues. Multi-mode simulations up to $n \sim \mathrm{O}(10 \div 20)$, with non-perturbative E.-P. dynamics and accurate background damping (realistic geometries and boundaries), are needed to push forward this area.

On the longer time scales, interactions between SAW–E.-P. dynamics and Drift/Alfvén-thermal particles dynamics will emerge and pose challenging multi-scale physics issues. For example, AE and EPM non-linear evolutions can be predominantly affected by either spontaneous generation of zonal flows and fields [76, 77] or by radial modulations in the fast ion profiles [11], depending on the proximity to marginal stability. Generally, non-linear evolution equations for the zonal structures can be derived, whose time-asymptotic behavior corresponds to non-linear equilibria, consistent with zonal flows and fields as well as non-linear modulations in the E.-P. radial profiles. Within this framework, it is possible to explore the possible non-linear interplay between zonal structures, drift wave turbulence and collective modes excited by energetic ions.

As a concluding remark, it is worthwhile emphasizing that SAW-E.-P. research is intellectually challenging (complexities in geometries and non-linearities) and programmatically important. Strong and healthy positive interplays among experiments, theory and simulations are necessary to push further the science in this field.

Acknowledgments

This work was supported by the U.S. DOE Contract No. DE-AC02-CHO-3073. This work was also supported by the Euratom Communities under the contract of Association between EURATOM/ENEA.

[1] CHEN, L., Phys. Plasmas **1** (1994) 1519
[2] CHEN, L., WHITE, R.B., and ROSENBLUTH, M.N., Phys. Rev. Lett. **52** (1984) 1122
[3] BIGLARI, H., and CHEN, L., Phys. Rev. Lett. **67** (1991) 3681
[4] HEIDBRINK, W.W., et al., Phys. Rev. Lett. **71** (1993) 855
[5] CHENG, C.Z., CHEN, L., and CHANCE, M.S., Ann. Phys. **161** (1985) 21
[6] SHARAPOV, S.E., et al., Phys. Lett. A **289** (2001) 127
[7] ZONCA, F., et al., Phys. Plasmas **6** (1999) 1917
[8] NAZIKIAN, R., et al., Phys. Rev. Lett. **96** (2006) 105006
[9] FASOLI, A., et al., Phys. Rev. Lett. **81** (1998) 5564
[10] BERK, H.L., et al., Phys. Lett. A **234** (1997) 213
[11] ZONCA, F., et al., Nucl. Fusion **45** (2005) 477
[12] WHITE, R.B., et al., Phys. Fluids **26** (1983) 2958
[13] SHINOHARA, K., et al., Nucl. Fusion **41** (2001) 603
[14] ZONCA, F., et al., "Electron fishbones: theory and experimental evidence", Fusion Energy 2006 (Proc. 21st Int. Conf. Chengdu, 2006), CD-ROM file TH/3-2
[15] CHEN, L., and HASEGAWA, A., J. Geophys. Res. **74** (1974) 1024
[16] PRIEST, E.R., "Solar Magnetohydrodynamics", D. Reidel Publsihing Co. (Boston, 1982)
[17] HASEGAWA, A., J. Geophys. Res. **81** (1976) 5083
[18] ROSENBLUTH, M.N., and RUTHERFORD, P.H., Phys. Rev. Lett. **34** (1975) 1428
[19] MIKHAILOVSKII, A.B., Sov. Phys. JETP **41** (1975) 890
[20] CHEN, L., and HASEGAWA, A., Phys. Fluids **17** (1974) 1399
[21] GRAD, H., Phys. Today **22** (1969) 34
[22] ZONCA, F., and CHEN, L., Plasma Phys. Control. Fusion **48** (2006) 537
[23] KIERAS, C.E., and TATARONIS, J.A., J. Plasma Phys. **28** (1982) 395
[24] BETTI, R., and FREIDBERG, J.P., Phys. Fluids B **3** (1991) 1865
[25] VAN DAM, J.W., and ROSENBLUTH, M.N., Bull. Am. Phys. Soc. **43** (1998) 1753
[26] ZHENG, L.-J., and CHEN, L., Phys. Plasmas **5** (1998) 444 and 1056
[27] TURNBULL, A.D., et al., Phys. Fluids B **5** (1993) 2546
[28] ZONCA, F., et al., Plasma Phys. Control. Fusion **38** (1996) 2011
[29] METT, R.R., and MAHAJAN, S.M., Phys. Fluids B **4** (1992) 2885
[30] FU, G.Y., Phys. Plasmas **2** (1995) 1029
[31] APPERT, K., et al., J., Plasma Phys. **24** (1982) 1147
[32] BERK, H.L., et al., Phys. Rev. Lett. **87** (2001) 185002
[33] TAKECHI, M., et al., "Property of Alfvén Eigenmode in JT-60U Reversed Shear and Weak Shear Discharges", Fusion Energy 2002 (Proc. 19th Int. Conf. Lyon, 2002), C&S Papers Series No. 19/C, IAEA, Vienna (2003), CD-ROM file EX/W-6 and http://www.iaea.org/programmes/ripc/physics/fec2002/html/fec2002.htm
[34] HASEGAWA, A., and CHEN, L., Phys. Rev. Lett. **32** (1974) 454
[35] ZONCA, F., and CHEN, L., Phys. Rev. Lett. **68** (1992) 592
[36] ROSENBLUTH, M.N., et al., Phys. Rev. Lett. **68** (1992) 596
[37] COPPI, B., and PORCELLI, F., Phys. Rev. Lett. **57** (1986) 2272

[38] GLASSER, A.H., GREEN, J.M., and JOHNSON, J.L., Phys. Fluids **18** (1975) 875
[39] GRAVES, J.P., et al., Plasma Phys. Control. Fusion **42** (2000) 1049
[40] HEIDBRINK, W.W., Phys. Plasmas **9** (2002) 2113
[41] FU, G.Y., and BERK, H.L., Phys. Plasmas **13** (2006) 052502
[42] BREIZMAN, B.N., et al., Phys. Plasmas **12** (2005) 112506
[43] HEIDBRINK, W.W., and SADLER, G.J., Nucl. Fusion **34** (1994) 535
[44] JAUN, A., FASOLI, A., and HEIDBRINK, W.W., Phys. Plasmas **5** (1998) 2952
[45] BORBA, D., et al., Nucl. Fusion **42** (2002) 1029
[46] FU, G.Y., BERK, H.L., and PLETZER, A., Phys. Plasmas **12** (2005) 082505
[47] LAUBER, PH., et al., Phys. Plasmas **12** (2005) 122501
[48] HASEGAWA, H., and CHEN, L., Phys. Fluids **19** (1976) 1924
[49] CHEN, L., J. Geophys. Res. **104** (1999) 2421
[50] GORELENKOV, N.N., et al., Nucl. Fusion **43** (2003) 594
[51] VLAD, G., et al., Nucl. Fusion **46** (2006) 1
[52] GORELENKOV, N.N., et al., Nucl. Fusion **45** (2005) 226
[53] STRAIT, E.J., et al., Nucl. Fusion **33**(1993) 1849
[54] BERK, H.L., and BREIZMAN, B.N., Phys. Fluids B **2** (1990) 2235
[55] BERK, H.L., BREIZMAN, B.N., and YE, H., Phys. Rev. Lett. **68** (1992) 3563
[56] WU, Y., and WHITE, R.B., Phys. Plasmas **1** (1994) 2733
[57] TODO, Y., et al., Phys. Plasmas **2** (1995) 2711
[58] FU, G.Y., and PARK, W., Phys. Rev. Lett. **74** (1995) 1594
[59] CANDY, J., et al., Phys. Plasmas **4** (1997) 2597
[60] SPONG, D.A., et al., Phys. Plasmas **1** (1994) 1503
[61] ZONCA, F., et al., Phys. Rev. Lett. **74** (1995) 698
[62] CHEN, L., et al., Plasma Phys. Control. Fusion **40** (1998) 1823
[63] TODO, Y., et al., (Proc. 9th IAEA TCM-EP Takayama, 2005) paper OT09 and http://htpp.lhd.nifs.ac.jp/IAEATM-EP2005/index.html
[64] BERK, H.L., et al., Phys. Plasmas **3** (1996) 1827
[65] SIGMAR, D.J., et al., Phys. Fluids B **4** (1992) 1506
[66] HSU, C.T., and SIGMAR, D.J., Phys. Fluids B **4** (1992) 1492
[67] HAHM, T.S., and CHEN, L., Phys. Rev. Lett. **74** (1995) 266
[68] GARBET, X., et al., Nucl. Fusion **34** (1994) 963
[69] DIAMOND, P.H., et al., Phys. Plasmas **2** (1995) 3685
[70] DIAMOND, P.H., and HAHM, T.S., Phys. Plasmas **2** (1995) 3640
[71] ZONCA, F., and CHEN, L., Phys. Plasmas **7** (2000) 4600
[72] FOGACCIA, G., et al., "Particle simulations of energetic particle driven Alfvén modes in JT-60U", (Proc. 33rd EPS Conf. on Plasma Phys. Rome, 2006) P5.073 to be published
[73] VLAD, G., et al., "Particle Simulation Analysis of Energetic-Particle and Alfvén-Mode Dynamics in JT-60U Discharges", Fusion Energy 2006 (Proc. 21st Int. Conf. Chengdu, 2006), CD-ROM file TH/P6-4
[74] McGUIRE, K., et al., Phys. Rev. Lett. **50** (1983) 891
[75] FU, G.Y., et al., Phys. Plasmas **13** (2006) 052517
[76] CHEN, L., et al., Nucl. Fusion **41** (2001) 747
[77] GUZDAR, P.N., et al., Phys. Rev. Lett. **87** (2001) 015001