A Method for Error Field Detection in ITER

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Abstract. The methods of the error field diagnostics are discussed and compared from the viewpoint of possible application in ITER. The analyzed methods rely on measuring the plasma dynamic response to the finite-amplitude external magnetic perturbations, the error fields and probing pulses. In ITER, such pulses can be created by the coils designed for the static error field correction and for stabilization of the resistive wall modes (RWM), the technique used now in several tokamaks, including DIII-D and JET. Here the estimates for ITER diagnostics are based on the theory predictions for the resonant field amplification (RFA). To achieve the desired level of the error field correction in ITER, the diagnostics must be sensitive to signals of several Gauss. Therefore the measurements should be performed near the plasma stability boundary, where the RFA effect is stronger. While the proximity to the marginal stability is important, the absolute values of plasma parameters are not. This can be used by lowering the stability threshold in diagnostic discharges below the nominal level. The discussed diagnostics is an extension of the ‘active MHD spectroscopy’ used recently in the DIII-D tokamak and the EXTRAP T2R reversed-field pinch.

1. Introduction

Error fields, or magnetic field anomalies, are the magnetic perturbations breaking the axial symmetry of tokamaks. Such fields even with amplitude 10^{-4} of the main toroidal field can strongly impact the tokamak plasma [1]. They ultimately induce severe instabilities resulting in degradation of energy/particle confinement and the disruptions, see [1-4] and references therein. Importance of the problem for ITER was also emphasized by theoretical forecasts that next-step tokamaks may be hypersensitive to error-fields [5] and by some experimental scalings for the critical error field level [1, 2, 6]. Recent experimental studies [4] provided improved constraints on predictions for locked mode thresholds on ITER. However, rather large discrepancies in the existing theories and scalings and, sometimes, even inconsistencies in experimental results [3, 4, 7] do not allow precise extrapolation to ITER. With all implications and uncertainties, the error field experimental detection and correction is a prerequisite for the desired operation of ITER.

The error fields can be directly measured when the magnetic system is energized without plasma. This technique was applied in the tokamaks DIII-D [3] and Alcator C-Mod [4]. However, plasma based error field correction assessment means or procedure were considered a better choice for ITER [1, 6]. Such methods have also been developed. In experiments with plasma, the error field can be evaluated using the resonant magnetic perturbations (RMPs) with varying amplitudes and phases to induce and suppress the locked modes [1, 2, 4, 6, 7], to minimize the plasma rotation decay or the field asymmetry measured by magnetic sensors [7], to avoid the anharmonicity of the oscillations [8]. A variety of methods brings, however, another problem: the error field magnitudes inferred from different magnetic diagnostics differ by a factor of 2-3 [3, 4].

All the mentioned plasma based methods are indirect, they are aimed to detect the error-field related effects and find when these effects are best compensated. This involves a search in operation space, with variations of either RMP or plasma parameters, or both. Here we discuss another possibility: the error field is calculated from a formula with coefficients determined from measurements of the plasma response to external magnetic perturbations, the unknown error field plus the pre-programmed RMPs. For the analysis we use the model described in [9, 10].

2. The model

In a cylindrical geometry the radial component b_r of the magnetic perturbation is expressed as a sum of harmonics $b_{mn}(r,t)\exp(im\theta - in\zeta)$ with $\zeta = z/R$ staying for the 'toroidal angle' ($2\pi R$ is the length of equivalent torus), r, θ, and z being the cylindrical coordinates related to the axis of symmetry. The Ohm's law for the thin wall and the Maxwell equations yield finally

$$\tau_w \frac{\partial B_m}{\partial t} = \Gamma_m B_m - \Gamma_m^0 B_m^{ext}, \tag{1}$$

where $\tau_w = \mu_0 \sigma r_w d$ is the 'wall time' with σ the wall conductivity, r_w the radius, and d the thickness of the wall, B_m is the amplitude of the (m,n) harmonic of b_r at the wall,

$$B_m \equiv b_{mn}(r_w) = B_m^{pl} + B_m^{w} + B_m^{ext}, \tag{2}$$

B_m^{pl} is the contribution to $B_m(t)$ from the plasma, B_m^{w} is the field created by the currents in the wall, and B_m^{ext} is the part of B_m created by all the sources outside the wall (in the region $r > r_w$):

$$B_m^{ext} = B_m^{er} + B_m^{CC}. \tag{3}$$

Here B_m^{CC} describes the pre-programmed field produced by correction coils, and B_m^{er} the (m,n) error field harmonic. All the amplitudes in (2) and (3) are the time-dependent complex quantities. Other symbols are: $\Gamma_m = \Gamma_m^0(1 - B_m^{pl}/B_m)$, and $\Gamma_m^0 \approx 2|m|$. Note that Γ_m can be expressed as

$$\Gamma_m = \tau_w(\gamma_0 + in\Omega_0), \tag{4}$$

where, according to (1), γ_0 and Ω_0 are the natural instantaneous decay/growth rate of the mode and the frequency of the mode toroidal rotation (under fixed B_m^{ext} or $B_m^{ext} = 0$).

3. General requirements to measurement scenarios

Equation (1) combined with (3) gives us an explicit expression for the error field amplitude:

$$B_m^{er} = \frac{\tau_w}{\Gamma_m^0}\frac{\partial B_m}{\partial t} - \frac{\Gamma_m}{\Gamma_m^0} B_m - B_m^{CC}. \tag{5}$$

The control field B_m^{CC} is known, $\partial B_m / \partial t$ can be directly measured by the magnetic probes, and, accordingly, the variation of the perturbation amplitude can be found:

$$\delta B_m = \int_0^t \frac{\partial B_m}{\partial t} dt. \tag{6}$$

Equation (5) contains, however, $B_m(t) = B_m^0 + \delta B_m$ with $B_m^0 \equiv B_m(0)$ unknown since it contains a part proportional to B_m^{er}. Another unknowns are Γ_m / Γ_m^0, which may be complex, and τ_w / Γ_m^0. We have to prescribe the algorithm of measurements providing the data on these three values.

For an equilibrium plasma stable with respect to particular (m,n) mode ($\mathrm{Re}\,\Gamma_m < 0$), with the error field as the only external perturbation at $t < 0$, equation (1) allows a stationary state with

$$\Gamma_m B_m^0 - \Gamma_m^0 B_m^{er} = 0\,. \tag{7}$$

If we start from this state and keep Γ_m fixed, variation of B_m will be described by the equation

$$\tau_w \frac{\partial}{\partial t}\delta B_m = \Gamma_m \delta B_m - \Gamma_m^0 \delta B_m^{ext} \tag{8}$$

with $\delta B_m(t=0)=0$. Here $\delta B_m^{ext} = B_m^{ext}(t>0) - B_m^{ext}(0)$ does not include the static error field.

Equation (8) implies that τ_w / Γ_m^0 and Γ_m / Γ_m^0 can be determined by measuring the stable plasma response to the applied pulses of the external field. With equation (8), the perturbation evolution can be calculated for any given $\delta B_m^{ext}(t)$. Here for discussion and estimates we consider the step-like pulses $\delta B_m^{ext} = B_m^{CC}, \quad 0<t<T$ with constant B_m^{CC}. In this case, the solution to equation (8) with $\Gamma_m = \text{const}$ and $\delta B_m(t=0)=0$ is

$$\delta B_m = -\frac{\Gamma_m^0}{\Gamma_m}\delta B_m^{ext}\left[\exp(\Gamma_m t/\tau_w) - 1\right], \tag{9}$$

and both Γ_m^0/τ_w and Γ_m/τ_w can be found from $\partial \delta B_m / \partial t$ known in two points of time. This also suggests two kinds of measurements, 'fast' and 'slow'. For the solution (9) we have

$$\frac{\partial}{\partial t}\delta B_m = -\frac{\Gamma_m^0}{\tau_w}\delta B_m^{ext}\exp(\Gamma_m t/\tau_w)\,. \tag{10}$$

Here the rate of the δB_m growth is maximal when the pulse is switched on, and exponentially goes to zero in a time of several $|\gamma_0|^{-1}$ (for $\operatorname{Re}\Gamma_m < 0$). The maximum at $t=0$ is always $\delta B_m^{ext}\Gamma_m^0/\tau_w$. Short probing pulses can be used then to find Γ_m^0/τ_w. The other option is using the pulses long enough for δB_m to reach the saturation level ($|\gamma_0| T >> 1$):

$$\delta B_m^{\max} = \frac{\Gamma_m^0}{\Gamma_m}\delta B_m^{ext}\,. \tag{11}$$

This gives the 'amplification coefficient' Γ_m^0/Γ_m through the measurable $\delta B_m^{\max}$. Note that the latter formula does not necessarily requires the abrupt change of B_m^{ext} at $t=0$: we just need $B_m^{ext} = \text{const}$ for the final state and, as before, $\Gamma_m = \text{const}$.

The diagnostics based on Eq. (1) and assumption $\Gamma_m = \text{const}$ was discussed in [10], and the model was validated in the DIII-D [11] and EXTRAP T2R [12] experiments. These experiments confirmed that $\Gamma_m = \text{const}$ was a reliable approximation for a given equilibrium state within some range of δB_m^{ext}, which can be determined experimentally. For example, in the DIII-D experiments [11] the linearity of the plasma response was a good assumption for the externally applied resonant magnetic fields $10^{-4} \div 10^{-3}$ of the main toroidal field.

Finally, there are three unknowns on the right hand side of (5): τ_w/Γ_m^0, complex Γ_m/Γ_m^0, and B_m^0. The first two can be determined by measuring the reaction δB_m to the pre-programmed perturbations δB_m^{ext} and using equation (8). This equation comes from (1) under two conditions: $\Gamma_m = \text{const}$ and (7). The first means fixed discharge parameters, and the second the existence of a stationary plasma equilibrium with nonzero error field (therefore, $\Gamma_m \neq 0$). To find or eliminate the unknown B_m^0, we have to exploit other regimes. We consider two options: the measurements covering the transition between two stable states with different $\Gamma_m = \text{const}$, and the measurements in the nonstationary state with $\Gamma_m = \text{const}$ and $B_m^{ext} = \text{const}$. The latter case is only possible when $\Gamma_m = 0$, which is the stability boundary of the locked modes.

4. Calculation and measurements of Γ_m / Γ_m^0

The ratio $\Gamma_m / \Gamma_m^0 = 1 - B_m^{pl} / B_m$ implicitly depends on the mode eigenfunction in the plasma through unknown plasma contribution to the magnetic perturbation $\mathbf{b}$. Without the plasma, $\Gamma_m / \Gamma_m^0 = 1$. With plasma, Γ_m / Γ_m^0 could be found if the solution for b_r in the plasma would be known to yield the boundary conditions for $\mathbf{b}$ at the plasma surface, as described in [10]. For the practical purposes here, an important outcome from the theoretical analysis is that Γ_m can be considered, at some conditions, a constant independent on B_m^{ext}. The experiments [11, 12] prove that Eq. (1) with $\Gamma_m = \text{const}$ is indeed a good model for describing the interaction between the RWM and externally applied magnetic fields. With $\Gamma_m = \text{const}$ we can use simple expressions (9)-(11) for determining Γ_m from the magnetic measurements.

The mode rotation makes smaller the δB_m in (9), (11). The amplitude of the measured signals with information on Γ_m can be increased by using the rotating RMPs. For

$$\delta B_m^{ext} = b_m^{st} + b_m^{os} \exp(P_m \tau), \tag{12}$$

switched on at $t = 0$, Eq. (8) gives us for $t > 0$:

$$\delta B_m = B_m^0 \exp(\Gamma_m \tau) - \Gamma_m^0 b_m^{st} \frac{\exp(\Gamma_m \tau) - 1}{\Gamma_m} - \Gamma_m^0 b_m^{os} \frac{\exp(\Gamma_m \tau) - \exp(P_m \tau)}{\Gamma_m - P_m}. \tag{13}$$

Here $\tau \equiv t / \tau_w$, $P_m = in\omega\tau_w$, and ω is the toroidal rotation frequency of the applied field. For a stable plasma with $\gamma_0 < 0$ this gives for $|\gamma_0| t >> 1$

$$\delta B_m = A_{st} b_m^{st} + A_{os} b_m^{os} \exp(in\omega t), \tag{14}$$

where $A_{st} = \Gamma_m^0 / \Gamma_m$ describes the amplification of the static part of δB_m^{ext}, and

$$A_{os} = \frac{\Gamma_m^0}{\Gamma_m - in\omega\tau_w} \tag{15}$$

is the amplification factor for the oscillating resonant field, both are the complex quantities. The amplitude of the measured field (14) must vary with ω according to

$$|A_{os}| = \frac{2\mu}{\tau_w \sqrt{\gamma_0^2 + n^2 (\Omega_0 - \omega)^2}}. \tag{16}$$

A resonant dependence of this value on the controlled frequency ω can be used to find both γ_0 and Ω_0. A resonant behaviour of the measured perturbation, consistent with (16), has been observed in tokamaks DIII-D [11] and T-10 [13].

If the desired information is obtained from the magnetic measurements outside the plasma, there is one phenomenon that deserves special attention: mode locking. The locked modes near the stability boundary is described by $\Gamma_m = 0$. If this state is experimentally identified, the Γ_m is known without using the probing pulses.

5. Special regime: locked modes near marginal stability

Operation near the stability boundary of the locked modes would greatly simplify the problem since, by definition, this marginal state corresponds to $\Gamma_m = 0$. For $\Gamma_m = 0$ and $B_m^{ext} = \text{const}$, equation (1) has a solution with a linear growth, without saturation:

$$\delta B_m = B_m - B_m(t_0) = -\Gamma_m^0 B_m^{ext}(t - t_0)/\tau_w . \tag{17}$$

This gives us $B_m^{ext} = B_m^{er} + B_m^{CC}$ in terms of the measurable δB_m, if τ_w / Γ_m^0 known (calculated or measured). If, however, τ_w / Γ_m^0 is not known in advance, this can be found with one additional measurement with constant $B_m^{CC} \neq 0$. Then B_m^{er} is determined from

$$\frac{\delta \dot{B}_m(w)}{\delta \dot{B}_m(wo)} = \frac{B_m^{er} + B_m^{CC}}{B_m^{er}} . \tag{18}$$

through $\delta \dot{B}_m = \partial B_m / \partial t$ measured with and without B_m^{CC}.

Solution (17) for the locked mode at the stability boundary was discussed in more detail in [9, 10]. Here we emphasize the fact that this solution shows an easy way of measuring both Γ_m^0 / τ_w and B_m^{er}. Also, it shows that the decreasing rate of δB_m growth can be a good indicator of the efficiency of the error field suppression, when attempted.

Theoretically, this method is the most simple and direct, and, therefore, deserves attention. Whether or not it can be recommended may depend on several experimental factors. First, it is hardly possible that the regime (17) with a linear growth of the locked mode can exist longer than several τ_w. A natural termination of this state, which allows a substantial growth of the perturbation with $\Omega_0 = 0$ in a short time, must be much faster instability or disruption. Nonlinearity of the plasma response may also appear because of the plasma rotation. When the RFA changes the plasma rotation and hence, the growth rate of the mode, the evolution of the perturbed field is expected to be faster than linear [14].

Smaller $|\Gamma_m|$ means larger plasma response to the external perturbation and easier detection of the error field. On the other hand, without a proper control, a discharge can readily slide down from $\Gamma_m = 0$ to instability. Because of this danger we have to consider a possible diagnostics below the stability boundary.

6. Measurements with a transition between two stable states

In the discussion above a single state with $\Gamma_m = \mathrm{const}$ was considered. Equation (5) shows that the magnetic measurements in such a state could give the error field in case only if $B_m^0 = B_m(0)$ would be known in advance. However, B_m^0 is determined by the unknown B_m^{er} and history of the discharge through the unknown $\Gamma_m(t)$. With B_m^0 unknown and $\Gamma_m \neq 0$, the next scheme of the error field diagnostics can be proposed. Consider a transition between two stable states characterized by $\Gamma_m(I)$ and $\Gamma_m(F)$ (*I*, *F*: initial and final). Assume that $B_m^{ext} = \mathrm{const}$ is the same in these states (either B_m^{er} or $B_m^{er} + B_m^{CC}$). According to (1), the stationary solution in each state is

$$B_m^{\max} = \frac{\Gamma_m^0}{\Gamma_m} B_m^{ext}, \tag{19}$$

which may be unknown at the initial state. The difference

$$B_m^{\max}(F) - B_m^{\max}(I) = B_m^{ext} \left. \frac{\Gamma_m^0}{\Gamma_m} \right|_I^F \tag{20}$$

is, however, a well defined measurable quantity. In each state, the ratio Γ_m^0 / Γ_m can be determined by measuring the variation of the perturbation δB_m after the step-like change in B_m^{CC}, as described in Sections 3 and 4. Then, the measured signal (20) gives us the desired B_m^{ext}. To facilitate the detection, we need larger $(\Gamma_m^0 / \Gamma_m)\Big|_I^F$. This requires one of the states be close to the stability boundary, while another deeply stable. Rotation of the mode always makes the signal smaller, so the best case can be the nonrotating modes, at least in the state with smaller $|\mathrm{Re}\,\Gamma_m|$.

The measurements of RFA with square pulses of $n=1$ fields in the JET discharge with β increasing towards the ideal stability limit have been reported in [15]. The pulses were long enough for reaching the stationary RFA (19), and β increase between the pulses was large enough to see the difference in the measured plasma response B_r.

Let us summarize. First, we have to produce a discharge with a transition from one equilibrium state (*I*) to another (*F*). Second, the measurement of δB_m must be done on the time interval covering the transition (*I*) $\rightarrow$ (*F*) and the perturbation relaxation to the stationary level (19). For this operation, the control field is not needed, though it can be used, if necessary, for increasing the measured δB_m. Finally, separate measurements with active pulses must be done to determine Γ_m^0 / Γ_m in each state. With that, the error field is determined from (20).

7. Application to ITER

With uncertainties in the theory and experimental scalings, it could be a prudent approach to consider 10^{-5} of the main toroidal field B_t as a lower scale for the successful error field compensation in ITER. With $B_t = 5.3$ T this means that $0.5 \div 1$ G may be a target for the measurements at the final stage of this work. In a more optimistic scenario, the ultimate goal will be detection of the error field harmonics with $n = 1 \div 2$, $m = 1 \div 3$ at the level of several Gauss.

The upper limit for the amplitude of the probing pulses is determined by the critical level of B_m when the model is not longer valid. With insufficient theoretical knowledge, this can be roughly evaluated on empirical basis. Taking $B_m / B_t = 10^{-3}$ as acceptable value, according to DIII-D experiments [11], we obtain 50 G as a restriction for δB_m^{ext} maximum in ITER. This also determines a scale of the expected diagnostic signal δB_m.

The 'wall time' estimated as $\tau_w = \mu_0 \sigma d r_w$ is 0.34 s for the ITER first wall [16]. This is one to two orders of magnitude larger than τ_w in existing tokamaks [14], which will result in slower rate $\partial \mathbf{B}/\partial t$ to be measured in ITER at similar absolute values of the error field. For the discussed measurements, this rate is $\leq 2|m|B_m^{ext}/\tau_w$, see (10) and (17). This gives at maximum approximately $20 B_m^{ext}$ s^{-1} for $m = 3$ and, assuming $\max \delta B_m = 50$ G, only 0.25 s for the measurements in the state described by (17) with $B_m^{ext} = 10$ G, or 2.5 s with $B_m^{ext} = 1$ G. If these intervals will be too short, the regimes below the locked mode stability boundary should be used.

To find Ω_0 and γ_0 by using the regime described by (14) we need $\omega \tau_w = O(1)$. The stationary solutions (11) and (14) are reached on the time interval T that depends on the growth rate, $|\gamma_0| T >> 1$. The RWMs are the modes with $\gamma_0 \tau_w = O(1)$. For larger diagnostic signal, the measurements should be performed closer to the stability boundary, then T may be order of magnitude larger than τ_w.

8. Discussion

The proposed algorithm allows error field detection by means of dynamic magnetic measurements outside the plasma and equation (1). Dynamic means that only $\partial \mathbf{B}/\partial t$ is a measurable quantity, while the static error field is unseen. Either pre-programmed probing pulses of the magnetic field or transition of the plasma from one equilibrium state to another is needed to get a diagnostic signal. Both possibilities are considered with necessary sequence of measurements and the expressions relating the measurable values to the unknown error field. No prior knowledge of the plasma parameters is needed for this diagnostics.

The discussed method employs the plasma property to react stronger on the same magnetic perturbation when plasma is closer to the marginal stability, with the largest response for the locked modes. The analysis is performed within the RFA model described in [9, 10]. The model is based on Maxwell equations and Ohm's law for the magnetic perturbation outside the plasma. The plasma comes to the problem through boundary conditions only, which are incorporated in the model without assumptions on the equilibrium plasma pressure and current profiles. The plasma contribution to the measured signal is then characterized by the natural growth/damping rate γ_0 and toroidal rotation frequency Ω_0 of the mode, the unknowns to be determined experimentally. Recent experiments on DIII-D proved that this model provides good description of time evolution and frequency dependence of the plasma response and can be used to obtain a measurement of both γ_0 and Ω_0 for a marginally stable RWM [11]. The model was also validated in the EXTRAP T2R reversed field pinch experiments [12].

Being a logical extension of the technique already developed and tested [7, 11–15], the discussed diagnostics requires standard experimental set-up. The accuracy and resolution of magnetic measurements demonstrated in the experiments [2–4, 6–8, 11–15] would be sufficient for ITER diagnostics. The main difference will be the time scale determined by τ_w which is above 0.3 s for the ITER first wall, order of magnitude larger than that in DIII-D and JET. The diagnostic equipment and the described methods can also be used for the 'active MHD spectroscopy'.

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