Improved Stability and Confinement in a Self-Organized High-β Spherical-Torus-Like Field-Reversed Configuration

H. Y. Guo 1), A. L. Hoffman 1), K. E. Miller 1), R. D. Milroy 1, 2), L. C. Steinhauer 1)

1) Redmond Plasma Physics Laboratory, University of Washington, Seattle, Washington, USA
2) Plasma Science and Innovation Center, Seattle, Washington, USA

e-mail contact of main author: hguo@aa.washington.edu

Abstract. A novel spherical-torus- (ST-) like field-reversed configuration (FRC), with an extremely high-β (over 85%), has been produced in the translation, confinement, and sustainment (TCS) experiment by highly super Alfvénic translation of a spheromak-like plasmoid. Such a self-organized compact FRC-ST state carries predominantly a diamagnetic current with the toroidal field magnitude much smaller than that of the poloidal field. However, when combined with the high elongation and small aspect ratio, it results in a safety factor exceeding unity over much of the configuration with a significant magnetic shear near the edge. Relaxation has been demonstrated, for the first time, in such a high-β plasma state. Modeling using the newly developed nearby-fluids theory shows that a broad core of the FRC-ST resembles a two-fluid minimum energy state. This FRC-ST state exhibits significantly reduced transport with up to four times improvement in confinement over the scaling of conventional θ-pinch formed FRCs. It also exhibits remarkable stability to global low-*n* modes such as the potentially disrupting *n*=2 centrifugally-driven interchange mode. This is explained, for the first time, by a simple stability model, accounting for the magnetic shear of the unique FRC-ST configuration.

1. Introduction

Experimental "relaxed" plasma states were first identified thirty years ago [1]. Taylor conjectured that a plasma may self-organize into a state of minimum magnetic energy $W_m = \int d\tau B^2 / 2\mu_o$ via turbulent relaxation bounded by a magnetic surface, subject to fixed magnetic helicity, $K_m = \int d\tau \mathbf{A} \cdot \nabla \times \mathbf{A}$ where $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field; the integrals are on the system volume. Some experiments confirmed the approximate preservation of K_m, and the appearance of relaxed states with current density $\mathbf{j} = const\mathbf{B}$ in agreement with the Taylor principle. The Taylor theory predicts force-free ($\mathbf{J} \times \mathbf{B} = 0$) plasma states with $\nabla \times \mathbf{B} = \lambda \mathbf{B}$, where λ is a global constant. However, these theoretical states have intrinsically $\beta = 0$. Recognizing that realistic plasmas have finite β, more general relaxation principles [2,3] have been advanced. One of the more promising is the *modern relaxation principle* [2] based on a two-fluid plasma model where the magnetofluid energy, $W_{mf} = \int \left(\sum m_\alpha n_\alpha u_\alpha^2 / 2 + B^2 / 2\mu_0 \right) d\tau$ is minimized subject to *two* preserved helicities $K_\alpha = \int (\mathbf{P}_\alpha \cdot \nabla \mathbf{P}_\alpha) d\tau$, where $\mathbf{P}_\alpha = m_\alpha \mathbf{u}_\alpha + q_\alpha \mathbf{A}$ is the canonical momentum, and m_α, n_α, $\mathbf{u}_\alpha$, q_α are the mass, density, fluid velocity and charge, respectively, of each species $\alpha = i$ (ions), e (electrons); the sum in W_{mf} is over both species. In a two-fluid plasma object the two global helicities are ideal integrals of motion for suitable boundary conditions. These finite-β minimum energy states (MES) are identical to the "natural" two-fluid states found elsewhere by a more intuitive approach [3].

The state of minimum energy represents an important subset of flowing two-fluid equilibria [4]. The possibility of relaxation into a state of minimum energy is very intriguing because a MES plasma object would be stable to all two-fluid instabilities, ideal and nonideal, and would thus have very low transport. Such states have intrinsically strongly sheared flows. Strong flow shear itself has stabilizing effects for global magnetohydrodynamic (MHD) modes, and is also effective at turbulence suppression, thus improving transport. In axisymmetric two-fluid equilibria there are two distinct sets of characteristic surfaces, the magnetic (and massless electron) surfaces $\psi(r,z) = const$, and the ion surfaces $Y(r,z) = \psi + \varepsilon r u_\theta = const$. This differs from standard single-fluid MHD in which there is only one set of surfaces $\psi = const$. The parameter charactering two-fluid effects is $\varepsilon = \ell_i / L$, the ratio of the ion skin depth $\ell_i = c/\omega_{pi}$ to the characteristic length in the system L; here $\omega_{pi} = \sqrt{ne^2/m_i\,\varepsilon_0}$ is the ion plasma frequency. In equilibrium the electron and ion stream functions, which characterize the poloidal (r,z) flows are functions of the surface variables, $\psi_e = \psi_e(\psi)$ and $\psi_i = \psi_i(Y)$. Note that the magnetic and massless-electron flow surfaces coincide, while observe that the ion flow surfaces differ from the electron (magnetic) surfaces due to the toroidal rotation (u_θ) and the two-fluid effects (ε). In a MES the ion and electron stream functions are required to have a linear dependence on the surface functions, i.e., $\psi_i \propto Y$ and $\psi_e \propto \psi$. This is analogous to constant λ in a Taylor state.

Relaxation in low-β plasma states, such as solar and space plasmas, as well as laboratory plasmas including spheromaks and reversed field pinches, has long been recognized and is usually described in terms of the Taylor relaxation principle. Recently, strong evidence for relaxation in high-β plasmas has appeared in the translation, confinement and sustainment (TCS) experiment by highly super-Alfvénic translation of a spheromak-like plasmoid produced by conventional θ-pinch technology [5]. The plasmoid self-organizes into a spherical-torus- (ST-) like FRC state [6], with a broad core conforming to a two-fluid minimum energy state [7]. Substantial flux conversion from toroidal to poloidal occurs during the highly dynamic formation process with the magnetic helicity approximately preserved. The final relaxed state carries predominantly a diamagnetic current with the toroidal field magnitude much smaller than that of the poloidal field, thus retaining the high-β nature of an FRC. However, although the toroidal field is small, when combined with the high elongation and small aspect ratio, it results in a safety factor q exceeding unity over much of the configuration, along with a significant magnetic shear at the edge. This *relaxed* plasma state exhibits remarkable stability with significantly reduced transport [6].

The present paper is organized as follows. Section 2 introduces the TCS-translation experiment and shows the first evidence of relaxation in an extreme high-β (over 85%) plasma state. Section 3 presents the modeling results using a newly-developed nearby-fluids model, showing that a broad core of the final relaxed ST-like FRC state is very close to a MES. Section 4 demonstrates improved confinement and stability in such an FRC-ST state and the significance of the small toroidal field in the FRC-ST. A simple stability analysis, accounting for the magnetic shear of the unique FRC-ST, is also presented to explain, for the first time, the lack of development of the potentially disrupting n=2 rotating interchange mode in the FRC-translation experiment. Section 5 concludes the paper with a summary.

2. Evidence of Relaxation in TCS-Translation Experiment

Evidence for relaxation in an extremely high-β plasma has appeared, for the first time, in the TCS-translation experiment. Figure 1 shows the TCS facility with the LSX/mod (half scale Large *s* Experiment) field-reversed pinch attached [8]. High density compact toroids were formed in LSX/mod, and ejected at either 0.45 T (high energy) or 0.3 T (low energy). The initial compact toroids had typical conditions of $n_e = 6\times10^{20}$ m^{-3}, T_t = 900 eV, and ϕ_p^{RR} ~ 2 mWb (high energy) or of $n_e = 6\times10^{20}$ m^{-3}, T_t = 400 eV, and ϕ_p^{RR} ~ 1.5 mWb (low energy). $T_t = T_i + T_e$ is the sum of electron plus ion temperatures. ϕ_p^{RR} is the poloidal flux, which is estimated from the measured excluded flux, $\Delta\phi$, by the Rigid Rotor (RR) approximation $\phi_p^{RR} = 0.31 x_s \Delta\phi$. $x_s \equiv r_s / r_c$ is the ratio of separatrix radius to flux conserver radius, and r_s is generally taken to be the measured excluded flux radius $r_{\Delta\phi} = \sqrt{\Delta\phi / \pi B_e}$ where B_e is the external axial field at the flux conserver boundary.

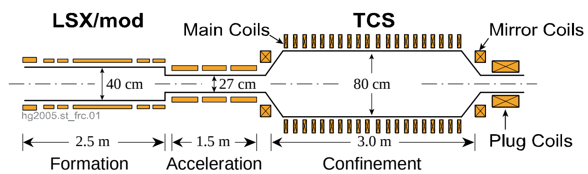

FIG. 1. Schematic of TCS device with LSX/mod attached.

Figure 2 shows time traces of ϕ_p^{RR}, B_e, and the line integrated density $\int ndl$, obtained from a two-pass CO_2 interferometer, for the typical translated FRC conditions (low energy) at the center of TCS. The plasmoids were ejected from the LSX/mod source at a highly super-Alfvénic speed, ~300 km/s, and expanded into the large-diameter TCS confinement chamber with a lower bias field (~50 mT). After the extremely violent reflections at the end magnetic mirrors of the confinement chamber, the plasmoids settled down into a quiescent equilibrium with a substantial increase in the poloidal flux. The final confined FRC-like object has a separatrix radius r_s ~ 0.22 m, n_e ~ 4×10^{19} m^{-3}, T_t ~ 200 eV. The detailed dynamics of translation were described in Ref. [5]. To illustrate the reproducibility, three discharges are shown in Fig. 2, including one with an internal probe inserted radially at the center of TCS, 2 cm past the axis, for the measurement of internal magnetic fields. The probe had 31 magnetic loops inside a 5 mm diameter boron nitride tube, and could be oriented to measure either $B_z(r)$ or $B_\theta(r)$ with a high time resolution. The data points shown in

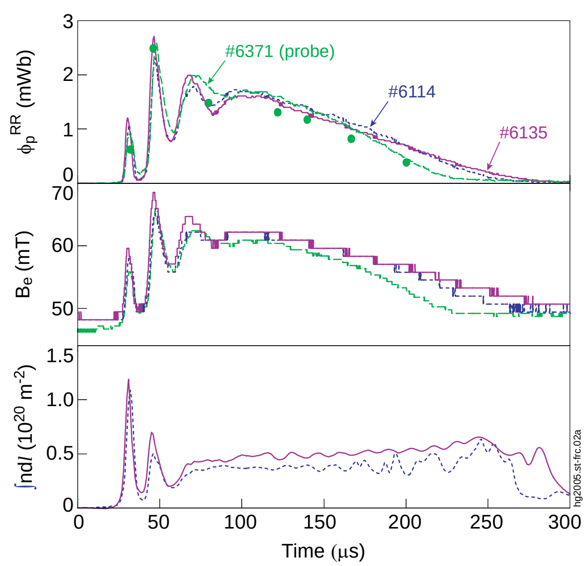

FIG. 2. Time histories of basic plasma properties at the TCS midplane for several translated FRCs, including one with the insertion of an internal probe 2 cm past the axis. The data points are obtained from the internal field measurements from the probe. Note that $\int ndl$ is not available for the discharge with the probe, which blocked the CO_2 interferometer.

Fig. 2 were obtained from the detailed internal probe measurements. The measured poloidal flux is in agreement with the RR value obtained from diamagnetism, and the equilibrium value is substantially higher than that present during the first pass. There is essentially no difference between the discharges except for some small variations in densities, partially due to small changes in initial formation conditions, and partially due to variations in centering of the FRC. The insertion of the internal probe only slightly reduces the lifetime.

The rise in poloidal flux has major implications. In an axisymmetric plasma ϕ_p decays at a rate that depends on the resistivity. *It can never increase*. The only way to "amplify" ϕ_p is by a topology-breaking, three-dimensional process that converts toroidal flux into poloidal. *Flux conversion* is a marker of a turbulent relaxation. Detailed internal probing shows that the initial plasmoid in translation had little poloidal field, but strong, oppositely-directed toroidal fields at its ends. After extremely violent reflections from the magnetic mirrors at the ends of the confinement chamber, the plasmoid quickly *relaxes* into a near-FRC state with a modest, unidirectional toroidal field with magnitude ~ 1/3 the peak poloidal field at the separatrix. Substantial flux conversion from toroidal to poloidal occurs during the highly dynamic formation process with the magnetic helicity approximately preserved. The $B_z(r)$ and $B_\theta(r)$ profiles obtained from the internal probe for the final equilibrium state (~140 μs) are shown in Fig. 3(a). A striking feature is that the toroidal field is highly localized inside the field null, which is consistent with a "nearby-fluids" model for a high-β self-organized plasma with strong flows [7]. Due to low aspect ratio and large elongation, the highly localized B_θ, although small, leads to a significant magnetic shear with a large q at the edge, as shown in Fig. 3(b), producing an ST-like FRC configuration, but without a center column.

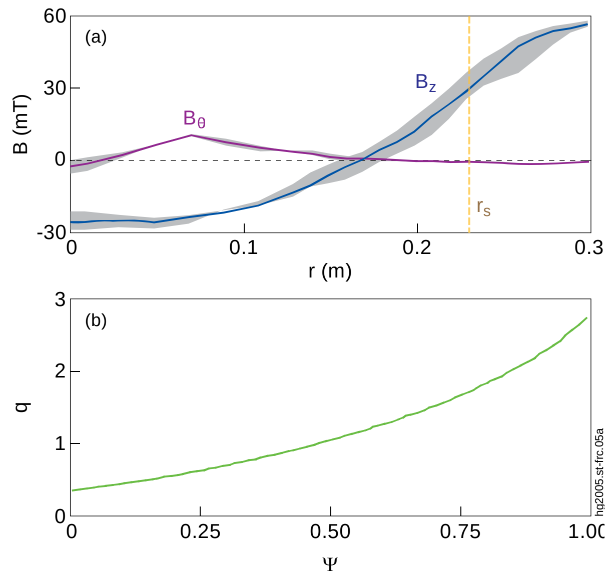

FIG. 3. (a) Radial profiles of axial and toroidal fields, $B_z(r)$ and $B_\theta(r)$, for the equilibrium FRC-ST state. Residual dynamical motions and shot-to-shot deviations are indicated by the shades. (b) q profile derived from the measured internal fields plotted against the normalized flux $\Psi = 1 - \psi/\psi_{\max}$, $\Psi = 0$ at the magnetic axis, and $\Psi = 1$ at the edge.

3. Newly Discovered Two-Fluid MES

A recently developed nearby-fluids equilibrium model for flowing two-fluid plasma equilibria [9] has been used to interpret the field and flow structure observed in the FRC-ST [7]. The most remarkable outcome is the discovery that a broad core of this *relaxed* high-β plasma object is very close to a MES. As described in Section 1, in a two-fluid MES state, the electron stream function ψ_e and the ion stream function ψ_i are *linear* functions of their respective surface variables, ψ and $Y = \psi + \varepsilon r u_\theta$. Figure 4 shows the inferred electron and ion stream functions from the TCS data. Both the electron and ion stream functions appear to be linear in a large core region, indicating that a broad core of the plasma conforms to a minimum energy state. This likely results in the suppression of instabilities in the core, including all microinstabilities that can be described by the two fluid model. This may promote good stability and confinement, as observed in TCS. However, the edge region differs significantly from the linear structure required for a MES: thus confinement would be dominated by higher transport rates in the edge region.

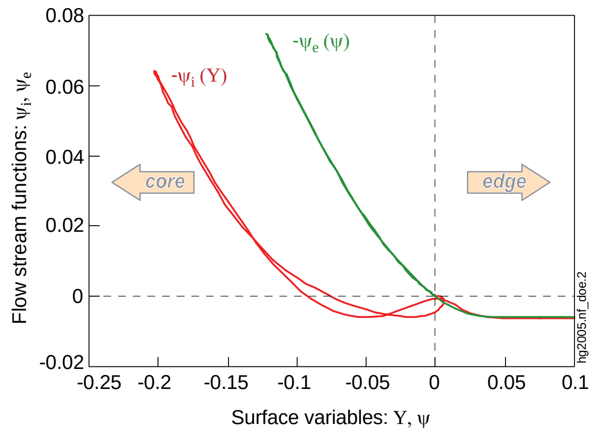

FIG. 4. Surface functions for electron and ion stream functions $\psi_e(\psi)$ and $\psi_i(Y)$*, calculated using a dimensionless variable scheme (see Ref. 7).*

4. Improved Stability and Confinement

The most prominent physics issue with an FRC has been global stability. The FRC-ST exhibits remarkable stabilizing properties for global low-*n* MHD modes, in particular, the normally lifetime terminating *n*=2 centrifugally driven interchange mode present in the conventional θ-pinch formed FRCs. This instability has usually been stabilized in the θ-pinch formed FRCs by externally applied static multipole fields [10]. However, from technological and confinement considerations, multipole fields are highly undesirable; the steady magnetic fields should be axisymmetric. A new stabilization technique has been demonstrated recently in TCS by rotating magnetic fields (RMF) for the RMF formed FRCs [11].

To investigate the significance of the unique magnetic topology of FRC-ST for the observed stability, we have developed a simple model for an *elongated* compact toroid (CT) by extending the energy principle approach used by Bellan [12]. The added feature is the effect of rotation represented by an extra term in the expression for δW, which is, in effect, a *gravitational* potential energy arising from the centrifugal effect of rotation. Although the principal driving forces are different, both pressure gradient and centrifugally driven interchange instabilities are essentially Rayleigh-Tayler fluid instabilities driven by an unfavorable *gravitational* gradient [13]. The centrifugal effect may be taken into account by simply replacing the normal interchange drive with the effective centrifugal drive, *i.e.*, $2\kappa P \sim \rho\Omega^2 r$, thus $2\kappa\nabla P \sim \Omega^2 r\nabla\rho$, where $\kappa = 1/R_c$ and R_c is the radius of the field line curvature. κ is very small over the central portion of an elongated FRC, and is not responsible for the rotational *n*=2 instability. With a number of pessimistic approximations as in Bellan's treatment, we obtain

$$\delta W \approx \frac{1}{4}\int dV\ \xi_\perp^2\left(B_{1\perp}^2/\mu_0 + \Omega^2 r\nabla\rho\right) \qquad (1)$$

for a radial displacement $\xi_\perp$. Since the translated FRCs carry predominately a diamagnetic current, with a small toroidal field highly localized inside the field null, i.e., in the good curvature region, both the parallel current drive term and the curvature related pressure gradient drive term are neglected. Following Bellan's analysis in expanding the safety factor

$q(\psi) = d\Phi/d\psi$ (where Φ is the toroidal flux and ψ the poloidal flux, $B_p = \nabla\psi/2\pi$) about a rational surface $\psi^{(m,n)}$ with $q = m/n$, the perpendicular component of the poloidal magnetic field perturbation becomes

$$B_{1\perp} = n\left(\psi - \psi^{(m,n)}\right)\frac{dq}{d\psi}\left(B_p \cdot \nabla\theta\right) \tag{2}$$

with *m* the poloidal and *n* the toroidal mode numbers.

For an elongated FRC equilibrium,

$$B_p \cdot \nabla\theta = \left|B_z(u)\right| \sin^2\theta / y \tag{3}$$

where $u = \left(\frac{r}{R}\right)^2 - 1$, $y = r - R$ is the minor radius and $R = r_s/\sqrt{2}$ is the major radius at the magnetic axis. With the usual RR assumption for an FRC,

$$B_z = B_e \tanh(K_{RR}u), \ \rho = \rho_0 \operatorname{sech}^2(K_{RR}u). \tag{4}$$

We focus on the edge region, $r \sim 0.9r_s$, where the centrifugal drive term, $\Omega^2 r\nabla\rho$, is most negative. $q \sim 2$ at this position, and the *m*=4 mode would be the most unstable of the *n*=2 distortions. The *n*=2, *m*=1 mode, seen in most untranslated FRCs without significant toroidal field, would be restricted to a small region near the magnetic axis. With the most pessimistic choice of $\psi^{(4,2)} = \psi_{\max}/2$, and only counting the unfavorable $\nabla\rho$ outer portion, again a pessimistic situation, the stability criterion $\delta W > 0$ becomes

$$\left(\frac{dq}{d\Psi}\right)^2 > \frac{3R^2 Z_0 \Omega^2}{n^2 V_A^2}\left|\frac{\nabla\rho}{\rho_0}\right| \tag{5}$$

where $\Psi = 1 - \psi/\psi_{\max}$, Z_0 is the axial distance to the midplane, $V_A = B_z/\sqrt{\mu_0\rho_0}$ is the Alfvén speed based on the edge poloidal field, and $\left|\frac{\nabla\rho}{\rho_0}\right| = \frac{4r}{R^2}K_{RR}\operatorname{sech}^2\left(K_{RR}u\right)\tanh\left(K_{RR}u\right)$.

For a typical translated FRC in TCS, $B_z \sim 30$ mT at the edge, $\langle n\rangle \sim 4\times10^{19}$ m^{-3}, $\Omega \sim 1.5\times10^5$ rad/s, $R \sim 0.16$ m, $Z_0 \sim 1.0$ m, and $K_{RR} \sim 0.7$, we obtain the following stability criterion for the centrifugally driven *n*=2 mode: $\left(\frac{dq}{d\Psi}\right)^2 > 1$. For the *q* profile shown in Fig. 3(b), $\left(\frac{dq}{d\Psi}\right)^2 \sim 5$. Thus, it appears that the magnetic shear due to the appearance of the small and highly localized toroidal field is *sufficient* to stabilize the *n*=2 rotational mode. Note that these translated FRCs are highly kinetic with *s* (the number of ion Larmor radii between the field null and the separatrix) about unity, but low *s* has not been noted to prevent the *n*=2 rotational instability from developing in previous static FRC experiments.

For a given ratio of B_θ/B_z, the magnetic shear, $\frac{dq}{d\Psi}$, is proportional to the FRC elongation *E*. Thus, the stabilizing term $\left(\frac{dq}{d\Psi}\right)^2$ in the energy analysis increases as E^2, while the destabilizing rotational term only scales with *E*. Improvement with elongation is consistent with previous MHD simulations which predicted a reducing growth rate, especially for high *n* modes, with increasing elongation [14, 15]. On the other hand, the translated FRC is like an ST in that essentially all of the toroidal wrapping occurs on the inside leg of the poloidal surface. The stabilizing effect of the inside leg is communicated to the outside leg along field lines at the Alfvén speed with a communication time $\Delta t = Z_0/V_A$. The above analysis is valid if the growth rate of the n=2 mode $\gamma_{n=2} < V_A/Z_0$. This is usually satisfied for FRCs with low *s*, in which the growth rates of the $n \geq 2$ modes are strongly reduced due to finite Larmor radius (FLR) effect [14, 15].

It is interesting that the magnetic flux lifetimes of the translated FRCs are much greater than the previous τ_ϕ scaling measured at high densities (LSX), $\tau_\phi(\mu s) = 9 \times 10^3 x_s^{0.5} \left\{ r_s(m) \big/ \sqrt{r_L(cm)} \right\}^{2.14}$ in conventional θ-pinch FRCs [16], where r_L is the Larmor radius. This is shown in Fig. 5 (a), assuming $T_i = T_e$. The very best lifetimes, obtained with the FRCs well centered on the centerline of the confinement chamber during the reflection process, have lifetimes about four times the conventional high density FRC scaling. The magnetic flux decays on a resistive diffusion time scale. For a rigid rotor profile and a uniform plasma resistivity, the flux lifetime $\tau_\phi \approx (\mu_o/\eta) r_s^2/16$ [16]. Figure 5(b) shows the corresponding values of $D_\perp = \eta/\mu_o$ plotted against the predictions of the high density LSX scaling. The resistivity and, thus, the radial transport appear to be significantly improved in these plasmas.

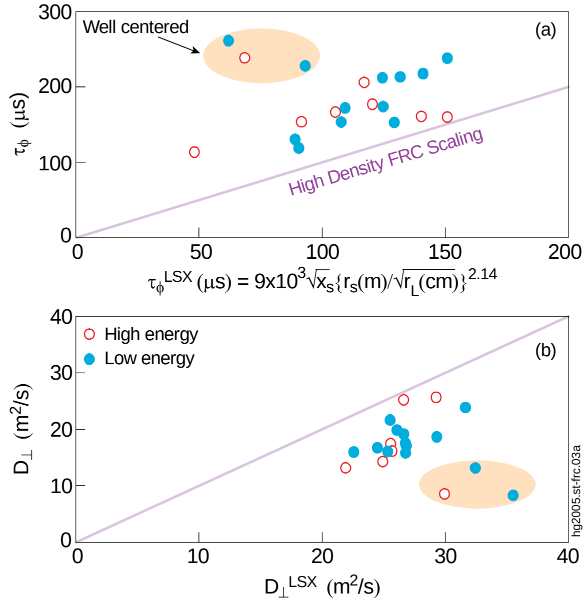

FIG. 5. (a) Measured magnetic flux lifetimes for the FRC-STs versus the LSX scaling for conventional θ-pinch formed FRCs. Those with low bias fields (B_o < 45 mT) are not included due to strong impurity contamination from the surrounding quartz walls. (b) $D_\perp = \eta/\mu_o = r_s^2/16\tau_\phi$ derived from the flux lifetimes assuming a RR profile.

5. Summary

Relaxation in high-β plasmas has appeared in the TCS-translation experiment, producing a novel ST-like FRC plasma state, i.e., clearly not a Taylor state, strongly suggesting that a more general principle regulates the relaxation. Although the toroidal field in the final relaxed state is small, it leads to q ~ 1 over much of the configuration, along with a significant shear at the edge, as a result of the high elongation and small aspect ratio. Modeling using the newly-developed nearby-fluids theory shows that a broad core of FRC-ST is very close to a two-fluid minimum energy state. This relaxed plasma state exhibits remarkable stability to the potentially disrupting n=2 rotating interchange mode, and significantly improved confinement.

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