Experimental Studies and Analysis of Alfvén Eigenmodes in Alcator C-Mod

M. Porkolab 1), E. Edlund 1), L. Lin 1), R. Parker 1), C. Rost 1), J. Sears 1), J. Snipes 1), S.J. Wukitch 1), B. Breizman 2), N. Gorelenkov 3), G.J. Kramer 3), A. Fasoli 4), H. Smith 5)

PSFC, MIT, Cambridge, MA 02139 USA; 2) IFS, U Texas, Austin, TX 78712 USA
 PPPL, Princeton, NJ 08540 USA; 4) CRPP-EPFL, CH-1015, Lausanne, Switzerland
 Chalmers U., SE-41296, Goteborg, Sweden

Porkolab@psfc.mit.edu

Abstract.

Alfvén cascades (ACs) or equivalently, reverse shear Alfvén eigenmodes (RSAE) have been observed in Alcator C-Mod during current ramp experiments in the presence of intense ICRF driven energetic minority tails. These experiments were carried out at ITER relevant densities and magnetic field. Phase contrast imaging (PCI) and magnetic pick-up coils have been used for detecting the Alfvén eigenmodes. The chirping evolution (increasing in time) of the frequency spectrum has allowed us to deduce the temporal evolution of the reversed shear q-profile. Numerical results from the latest version of the NOVA-K MHD code, including finite pressure reproduces the experimental measurements, including the minimum frequency. The harmonic of the cascade frequency was also observed by the diagnostics. In a different set of C-Mod experiments a pair of active MHD antennas were used to excite ITER relevant intermediate toroidal mode number (3 < n < 14) toroidal Alfvén eigenmodes (TAE), and their measured weak damping rates indicate that such modes may be excited by alpha particles in ITER. We have also studied the feasibility of adding PCI detection optics outside the ITER vessel that shares the CO2 laser interferometer port hardware. Implementing PCI on ITER would greatly extend our capabilities to assess the importance of Alfvén mode excitation on alpha particle confinement.

1. Experimental Results.

Plasma experiments with ICRF heating during the current-ramp and diffusion phase have produced frequency chirping modes known as Alfvén cascades (AC), or reverse shear Alfvén eigenmodes (RSAE). These modes exist only in the presence of a very flat or hollow current distribution, residing in the vicinity of the minimum of the q profile. An analytic expression describing the frequency of the modes is taken from Ref. [1] as

$$\omega_{AC}^{2} = \frac{V_{A}^{2}}{R_{0}^{2}} \left(\frac{m}{q_{\min}(t)} - n\right)^{2} + \frac{2T_{e}}{M_{i}R_{0}^{2}} \left(1 + \frac{7}{4}\frac{T_{i}}{T_{e}}\right) + \omega_{V}^{2} \quad (1)$$

The frequencies of these modes are generally seen to chirp up for modes with $m>n\cdot q_{min}$, as is described by the modes' frequency being proportional to the quantity $m/q_{min} - n$ (when the first term in Eq. (1) dominates). As q_{min} decreases as the current diffuses, this quantity becomes more positive resulting in a frequency up-chirp. The cascade modes are recognized by a single dominant poloidal harmonic with $m = n \cdot q_{min}$ when q_{min} is integral. As the mode moves toward the TAE gap the m+1 mode becomes co-dominant and the cascade evolves into a toroidicity-induced Alfvén eigenmode (TAE) [2]. Shown below in Fig.1 is a spectrogram of phase contrast imaging (PCI) data [3] and the overlaid results from modeling with NOVA-K [4].

This fit to the data was performed using experimentally derived profiles for density and temperature. The pressure profiles used in the NOVA equilibrium q-solver was scaled slightly from the experimental values to match the initial frequency only. In some cases, especially with the n=2 and n=3 modes, the numerical results fit the experimental data so well that the traces are obscured. As the magnetic pick-up coils on C-Mod do not have enough spatial coverage to accurately measure the low-n modes, the n numbers here have been inferred from the slopes of the cascades as $d\omega/dt$ is proportional to m, and m is coupled to a specific n.

A recent study of the cascade modes [1] employing an earlier observation made by Chu *et al.* [5] has shown that there exists a significant geodesic acoustic deformation of the Alfvén continuum, closely related to geodesic acoustic coupling, giving rise to an initial frequency that scales as $T_e^{1/2}$. Possible sources of uncertainty in this calculation come from the assumption, supported by modeling, that the radial position of the peak of the mode is near r/a=0.33. The electron temperature for this scaling was taken at this position. Allowing for a radial uncertainty of $\Delta r/a = +/-0.10$ gives rise to typical variations in electron temperature of about - 20% to + 30%, which translates to a $T_e^{1/2}$ error of up to 15%. Results from ramp-up experiments on C-Mod support the $T_e^{1/2}$ scaling with a correlation value of 0.92, calculated using a weighted fit inversely proportional to the frequency width (see Fig. 2).



Figure 1: PCI data is shown in red scale, overlaid are numerical results from NOVA in white. The initial frequency calculated from NOVA matches the experimental data within the uncertainty of the experiment. The peak frequencies of the n=2 and n=3 modes calculated by NOVA disagree with the measurements by about 20 kHz. This effect is as yet unaccounted for.

240 220 frequency (kHz) 200 180 160 140 correlation = 0.92120 100 2.0 1.0 1.2 1.4 1.6 1.8 sart(Te)

Figure 2: Initial frequency data compiled from four run days shows a correlation of 0.92 with a linear fit to a $T_e^{1/2}$ model, where T_e was taken at r/a = 0.33 with an uncertainty derived from the line width of the mode as measured by PCI.

More recent work [2] shows that the mode frequency should be somewhat above the Alfvén continuum due to additional terms arising from the gradients of the temperature of the bulk plasma [6,7], as well as from the gradient of the density of the energetic particles [8]. The deviation from the continuum is described by the term ω_{∇}^2 in Eq. 1 where

$$\omega_{\nabla}^{2} = -\frac{2}{MR^{2}} r \frac{d}{dr} \left(T_{e} + T_{i} \right) - \frac{\omega}{m} \left| \frac{eB}{Mc} \right| \frac{r}{n_{plasma}} \frac{d}{dr} \left\langle n_{fast} \right\rangle$$
(2)

Using approximate values for the terms in Eq. 2, it appears that the first term has a small effect on the frequency, of the order of 1%. The second term, involving ω , must be solved selfconsistently within the eigenvalue equation. Estimates of this term using the measured ω show that it may make a correction to the frequency on the order of 10% or less. A more detailed analysis of the resonant fast particle distribution is necessary before any conclusive statements can be made. We are also examining the possibility of modeling the temperature variation with the NOVA-K code. The uncertainties in the input parameters, in regard to the value of gamma (ratio of specific heats), the ion to electron temperature ratio and the values of the gradients of the bulk plasma and the energetic particles need to be resolved before further comparisons can be made. Work is continuing in this direction, as well as assessing the importance of Doppler shift due to internal dc electric fields.

Typical PCI signals from the 32 element detector array (Fig. 3 left) show multiply peaked structures (Fig. 3 right). Before the development of the "synthetic PCI" tool used with the NOVA-K results, it was unknown whether this was a measurement of a mode with multiple peaks in its radial structure, or an effect arising from the line integration of density fluctuations. First results from NOVA-K using typical eigenmode solutions with a single peak in the radial structure showed a triplet of peaks in the simulated PCI data. The results shown below put the minimum of the q profile at r/a of 0.30 and used an average central shear of 5%.



Figure 3: Schematic of the PCI vertical viewing chords (in pink) in Alcator C-Mod (left). On the right, PCI data shown in black, synthetic PCI results from NOVA shown in red. The q profile in this calculation had a minimum at r/a = 0.30 with an average value of the central shear of 5%. The location of the minimum value of q determines the peak spacing and it aids to "dial-in" the q profile.

Variation in the radial position of the minimum of the q profile results in a spreading of the peaks. Thus, the radial position can be used to "dial in" the "synthetic" PCI predictions to the experimental measurements. It has been found that the modes are most sensitive to this parameter. Notice that there is poor agreement between measurement and model with respect to the right-most peak. This is under investigation and may provide further information about the q-profile. Other model variable parameters such as the difference between q on axis and q at minimum, can give rise to variation of the individual peak heights. This data, though potentially useful, is currently questionable because of systematic uncertainties in the PCI sensitivity. This analysis provides an upper limit on the q-profile's central shear and together with the localization of q_{min} begins to provide a good constraint on the global shape of the q-profile. Without the information provided by the MSE diagnostic, the q profiles were generated using the edge EFIT solutions and a smooth fourth order extension to the center with

continuity in the profile and its first derivative at r/a of 0.9. This is done under the assumption that the current in the last 10% of the minor radius has achieved equilibrium and the diffusion is occurring interior to that.

As shown in new data in Fig. 4, in some cases the second harmonic of the cascade frequency was also observed by the PCI diagnostic [9], and recent theoretical analysis has offered an explanation for this phenomena [10]. This second harmonic perturbation can be interpreted as a nonlinear sideband produced by the Alfvén cascade eigenmode via quadratic terms in the MHD equations. The signal at 2ω is nearly resonant with the (2m;2n) branch of the Alfvén continuum. The resulting second enhancement of the harmonic density perturbation $\rho_{2\omega}$ is counteracted by the relatively weak non-linearity of the shear Alfvén wave. As



Figure 4: Second harmonics of cascades and TAE activity observed with the PCI diagnostic on C-Mod.. The steady horizontal lines are pick-up noise from the ICRF system and should be ignored.

shown in Ref. [10], the ratio of $\rho_{2\omega}$ to the first harmonic density perturbation ρ_{ω} is roughly $\rho_{2\omega} / \rho_{\omega} \approx mq_0 (|\delta \mathbf{B}_{\omega}| / B_0) (R / r_0)$, where $|\delta \mathbf{B}_{\omega}|$ is the perturbed magnetic field at the fundamental frequency. Using the PCI data, in principle the expressions derived for $\rho_{2\omega} / \rho_{\omega}$ can be used to determine the perturbed fields inside the plasma.

2. Active MHD Spectroscopy.

Stable Toroidal Alfvén Eigenmodes (TAEs) with a broad spectrum of moderate toroidal mode numbers ($3 \le |n| \le 14$) are excited with very low amplitude ($\tilde{B}_{\theta}/B_{\theta} \sim 10^{-7}$ at the wall) on Alcator C-Mod with a pair of antennas at one toroidal location (Fig. 5) [9]. This is in the range of mode numbers that are expected to be unstable in ITER [11,12], at the same toroidal field and density and in reactor relevant regimes where $T_i \approx T_e$. By sweeping the frequency through the expected center of the TAE gap frequency ($\omega_{TAE} = v_A/2qR$) for a given q value, the width of the mode resonance can be measured with magnetic pick-up coils to find the TAE damping rate as a function of plasma parameters, as has been done previously on JET for low $n \le 2$ [13]. Understanding the physics of moderate *n* TAEs may prove to be important for fusion burn control or controlling the loss of fast ions in ITER. TAE damping rates between $0.5\% < \gamma/\omega <$ 6% have been observed in diverted and limited Ohmic plasmas and up to 8% in ICRF heated limited plasmas.



Fig 5. (a) Active MHD antenna drawing showing the proximity to the plasma; (b) photo of the active MHD antennas; (c) geometric toroidal mode spectrum of each antenna.

Recent experiments have measured the damping rate of stable TAE resonances as a function of triangularity, ion ∇B drift direction, density, toroidal field, and ICRF power. The damping rate does not increase for n = 6 modes with increasing triangularity (Fig. 6), in contrast to the JET result for n = 1 [14] where increased triangularity increased the damping rate. Measured moderate n TAE damping rates appear to be nearly independent of the ion ∇B drift direction in diverted plasmas. This is again in sharp contrast with the JET result for n = 1 modes where the damping rate was 3 times higher when the ion ∇B drift was directed away from the divertor [15]. These results indicate that moderate n TAE damping is not strongly affected by edge conditions in contrast to low n TAE damping measured on JET. The moderate n damping rate was also found to be independent of ICRF power, possibly because the fast ion toroidal precession was opposite to the TAE direction or that the fast ion drive was at a different radial location than these modes, as suggested by NOVA-K modeling.

The measured damping also appears to increase at very low v* and increase with ρ_* and β . TAE damping rates depend sensitively on a number of plasma parameters that are difficult to measure to the required precision, such as the detailed shape of the q and density profiles. It is possible that changes in these or other parameters, such as the toroidal mode number, could confuse the parametric dependences presented here. Modeling of a number of the stable TAE resonances was performed with the NOVA-K code [16, 17]. One example is shown in Fig. 7, where a clear resonance at about 0.85 s was found at a frequency of $f_{meas} = 487$ kHz with n=9. The measured damping rate was found to be $\gamma/\omega \approx -1.5\%$. NOVA-K was run using the calculated q profiles from TRANSP [18] and EFIT [19], and in addition, the sensitivity to the



Fig. 6. (a) Low triangularity EFIT equilibrium; (b) high triangularity EFIT equilibrium; (c) measured stable TAE damping rate versus average triangularity for n=6 modes indicating a slight decrease in damping with increasing triangularity.

q profile was checked by artificially scaling the entire EFIT q profile by 0.9, 1.0, and 1.1. While this does not preserve the correct edge q value, it does provide a simple means to check the sensitivity of the calculated mode structure and damping rate to relatively small changes in the overall q profile without changing the shape of the profile or other inputs to the code. Note that the TRANSP central q value is 0.54, which is much lower than the EFIT q₀ value, which is constrained to be near 1. The measured core ion plasma toroidal rotation was ~ 5 kHz in the electron direction, so the mode frequency in the plasma frame should be approximately $f_{plasma} = f_{meas} - 45$ kHz ~ 442 kHz. Note that given the relatively large toroidal mode number, a small change in the plasma rotation will make a much larger change in the Doppler shifted frequency, so there is a large uncertainty in the mode frequency in the plasma frame. Eigenmodes with calculated frequencies in the range from 420 kHz to 450 kHz were found for each of these q profiles, but the calculated damping rate was found to be quite sensitive to q, varying by nearly an order of magnitude with these changes in q. The variation in the damping rates is from the different radial localization of these modes with respect to the TAE gap.

For the TRANSP q profile, the low central q value leads to a less radially aligned gap structure and thus to modes with fairly strong continuum interaction in both the inner half of the plasma and out near the plasma edge. Figure 7c shows the radial profile of the eigenfunction for a mode at 449.5 kHz. There is strong continuum interaction inside $\Psi_{pol}^{1/2} \le 0.6$ and outside $\Psi_{pol}^{1/2} \ge 0.9$ as can be seen in Figure 7b showing the n = 9 TAE gap structure. This mode had a calculated damping rate of $\gamma/\omega = -1.48\%$, in agreement with the measured value.

The NOVA-K calculations found a more open TAE gap structure for 0.9, 1.0, and 1.1 times the EFIT q profile. For the EFIT q profile multiplied by 0.9, a mode was found at 440.0 kHz with a damping rate of -1.2%, very close to the measured damping rate. For the EFIT q profile, the mode found with the frequency closest to the Doppler shifted measured mode frequency had a frequency of 446.3 kHz. This mode had a calculated damping rate of -0.33%, which is considerably lower than the measured value of -1.5%. However, when the EFIT q profile was multiplied by 1.1, the closest mode to match the experiment was found at 435.8 kHz which had a damping rate of only -0.17%, nearly an order of magnitude smaller than the measured



Fig. 7. (a) Spectrogram of a magnetic pick-up coil signal showing the frequency ramp imposed by the active MHD antennas together with the average poloidal field fluctuation amplitude and the calculated TAE center of the gap frequency for q=1.5; (b) NOVA-K calculated Alfvén eigenmode radial gap structure for n=9; (c) NOVA-K calculated eigenfunction displacement radial profile for an n=9 mode at 449.5 kHz together with the TRANSP calculated q profile.

damping rate. For 0.9 times the EFIT q profile, the mode amplitude near the edge is particularly large where there is strong interaction with the continuum that leads to a larger total damping rate. For the EFIT q profile, the edge interaction is not as large, but there is interaction with the continuum for $\Psi_{pol}^{1/2} < 0.5$, which leads to an intermediate damping rate. At 1.1 times the EFIT q profile, there is some continuum interaction at mid radius as well as near the edge, but the resulting damping rate is much smaller. This shows how the continuum damping, in particular, is quite sensitive to details of the q profile. External excitation of TAEs with moderately high n numbers up to 15 potentially allows investigation of not only the continuum damping but also the radiative damping, which is expected to dominate in ITER at high n numbers with n ~ 20. The present C-Mod experiments indicate that radiative damping is small since good agreement could be obtained without including it in the NOVA-K analysis.

3. Conclusions.

We have presented results on excitation of RSAE due to energetic ion tails during ICRF minority heating, as well as a study of damping rates of actively launched TAE modes with intermediate mode numbers in Alcator C-Mod. A comparison of these results with the NOVA-K code has also been presented and further analysis is continuing. It was shown that the initial frequency of RSAE scales as $T_e^{1/2}$ as predicted by theory when accounting for geodesic acoustic deformation of the Alfvén continuum. For the first time, ITER relevant moderate toroidal mode number ($3 \le n \le 12$) stable TAE damping rates have been measured in the range of $0.5\% < |\gamma/\omega| < 5\%$ using dedicated antennas to excite these modes on C-Mod. Calculated damping rates using the NOVA-K code compare very well with the experimental measurements for reasonable estimates of the q profile, but the results are very sensitive to even small changes in the q profile. An intriguing possibility is to implement PCI diagnostic on ITER. We propose to implement a PCI design that shares the already approved CO₂ laser interferometer port hardware and adds PCI detection optics outside the vessel. This would greatly extend our capabilities on ITER to search for RSAE and TAE in the presence of alpha particles.

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