Global Hybrid Simulations of Energetic Particle-driven Modes in Toroidal Plasmas

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Abstract

Global hybrid simulations of energetic particle-driven MHD modes have been carried out for tokamaks and spherical tokamaks using the hybrid code M3D[1]. The Numerical Results for the National Spherical Tokamak Experiments (NSTX) show that Toroidal Alfvén Eigenmodes are excited by beam ions with their frequencies consistent with the experimental observations. Nonlinear simulations indicate that the n=2 mode frequency chirps down as the mode moves out radially. For ITER, it is shown that the alpha particle effects are strongly stabilizing for internal kink mode when central safety factor $q(0)$ is sufficiently close to unity. However, the elongation of ITER plasma shape reduces the stabilization significantly.

1 Introduction

The physics of energetic particles is an area of importance in magnetic fusion devices such as tokamaks. A key issue in burning plasmas is whether alpha particle-driven Alfvén instabilities can induce large alpha particle transport and can damage the first wall in a fusion reactor. To answer this question, we need self-consistent numerical simulations which can predict alpha particle-driven instabilities in the new parameter regime of burning plasmas. Towards this end, we have developed a particle/MHD hybrid code M3D to study the physics of energetic particle-driven modes for present and future fusion devices. In this paper, we report recent results of global hybrid simulations of energetic particle-driven MHD modes in tokamaks and spherical tokamaks.

M3D is a multi-level 3D extended MHD code[1]. This code contains multiple levels of physics model including resistive MHD, two fluids[2], and particle/MHD hybrid. In this work, the particle/MHD hybrid model is used. Recently, we have extended the hybrid model of M3D code to general 3D geometry by using linear finite element in poloidal planes and finite difference in toroidal direction[3]. The code has been parallelized using MPI so now it runs routinely on massively parallel computers. These new capabilities have enabled us to study energetic particle-driven modes in many types of fusion devices including shaped tokamaks, spherical tokamaks and stellarators. In this work, we will present results of beam-driven Alfvén modes in NSTX and alpha particle stabilization of internal kink mode in ITER. The results of fast ion-driven TAE in stellarators have been reported elsewhere[4].

2 Particle/MHD Hybrid Model

In this work, we use a particle/MHD hybrid model to describe the interaction of energetic particles and MHD waves. In the model[1], the plasma is divided into two parts: the thermal component and the energetic particle component. The thermal ions and electrons are either treated as a single resistive fluid (this work) or a two-fluid model. The energetic species is treated as drift-kinetic particles or gyrokinetic particles. The energetic particle effects enter through the particle stress tensor $\mathbf{P}_h$ in the momentum equation:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P - \nabla \cdot \mathbf{P}_h + \mathbf{J} \times \mathbf{B} \ , \tag{1}$$

Equation (1) is closed by the Maxwell equations

$$\mathbf{J} = \nabla \times \mathbf{B}, \qquad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \ , \tag{2}$$

Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} \ , \tag{3}$$

and the pressure equation for thermal species

$$\partial P/\partial t + \mathbf{v} \cdot P = -\gamma P \nabla \cdot \mathbf{v}. \tag{4}$$

The particle stress tensor is written in the CGL form

$$\mathbf{P}_h = P_\perp \mathbf{I} + (P_\parallel - P_\perp)\mathbf{bb} \ , \tag{5}$$

and the particle distribution function is represented by an ensemble of markers as

$$f = f(\mathbf{R}, v_\parallel, \mu) = \sum_i \delta(\mathbf{R} - \mathbf{R}_i)\delta(v_\parallel - v_{\parallel,i})\delta(\mu - \mu_i) \tag{6}$$

with marker's orbits following the gyrokinetic equations:

$$\frac{d\mathbf{R}}{dt} = \frac{1}{B^{\star\star}} \left[v_\parallel (\mathbf{B}^\star - \mathbf{b_0} \times (< \mathbf{E} > - \frac{1}{q} \mu \nabla (B_0 + < \delta B >)) \right] \tag{7}$$

$$m \frac{dv_\parallel}{dt} = \frac{q}{B^{\star\star}} \mathbf{B}^\star \cdot (< \mathbf{E} > - \frac{1}{q} \mu \nabla (B_0 + < \delta B >)) \tag{8}$$

Here $\mathbf{R}$ is the gyro-center coordinates, $\mathbf{E}$ the total electric field, $\mathbf{B}$ the total magnetic field, μ the magnetic moment, b_0 is the unit vector in direction of equilibrium magnetic field, B_0 and δB is the equilibrium and perturbed magnetic field respectively. Note that the bracket $<>$ represents gyro-average. The variable $\mathbf{B}^\star$ is given by

$$\mathbf{B}^\star = \mathbf{B_0} + < \delta \mathbf{B} > + \frac{mv_\parallel}{q} \nabla \times \mathbf{b_0}, \qquad B^{\star\star} = \mathbf{B}^\star \cdot \mathbf{b_0} \tag{9}$$

The system of the coupled equations (1-9) is a complete set of particle/MHD hybrid model which is self-consistent, three dimensional, and nonlinear. The coupled equations are solved as an initial value problem. A δf method is used for the particle part of the model. Details of the model and numerical methods will be presented elsewhere.

3 Beam-driven Toroidal Alfvén Eigenmodes in NSTX

In the NSTX neutral beam-heated plasmas, fast beam ion-driven Alfvén modes were routinely observed in the experiments. Figure 1 shows frequency spectrum of beam-driven modes in such a plasma (shot #108530) where bursting modes of multiple toroidal mode number ($n \sim 1-5$) in the range of TAE frequency were observed[5]. Here we apply our hybrid code M3D to simulate the beam-driven modes near the time slice t=0.267sec. Experimental parameters and profiles are used in the simulations. They are as follows: magnetic field $B = 0.43T$, major radius $R = 87cm$, minor radius $a = 63cm$, electron density $n_e(0) = 2.5e13$, ion and electron temperature $T_i(0) = 1.7kev$, $T_e(0) = 1.4kev$, central total beta $\beta(0) = 21\%$, central beam ion beta $\beta(0) = 13\%$. The safety factor profile is weakly reversed near the center with $q(0) = 1.9$ and $q(a) = 12.9$. The beam ion distribution is specified as a function of constants of motion: $P_\phi = e\Psi + Mv_\parallel R$ is the toroidal angular momentum, $\Lambda = \mu B_0/E$ the pitch angle and E the energy. The distribution is a slowing-down in energy and anisotropic in pitch angle and is given by

$$f = \frac{1}{v^3 + v_c^3} \exp\left(-\frac{\bar{P}_\phi}{\Delta\Psi}\right) \exp\left(-\frac{(\Lambda - \Lambda_0)^2}{\Delta\Lambda^2}\right) \tag{10}$$

$$\bar{P}_\phi = \frac{P_\phi - P_{\phi,min}}{P_{\phi,max} - P_{\phi,min}} \tag{11}$$

where v_c is the critical velocity and $P_{\phi,min}$ and $P_{\phi,max}$ is the minimum and maximum value of P_ϕ respectively for co-injected beam ions. We note that the distribution is specified by four parameters: v_c, $\Delta\Psi$,Λ_0 and $\Delta\Lambda$. The values of these parameters are chosen by fitting the numerical distribution obtained from a Monte Carlo simulation using the

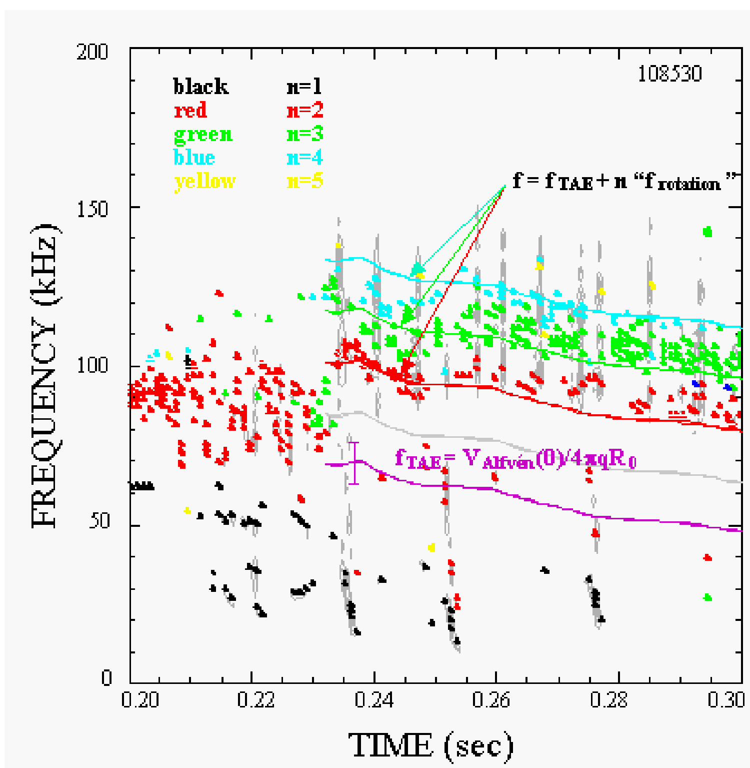

Figure 1: The measured frequency spectrum of the Mirnov signal as function of time. The color of solid symbol denotes toroidal mode number. The solid line shows the analytic TAE frequency in the lab frame.

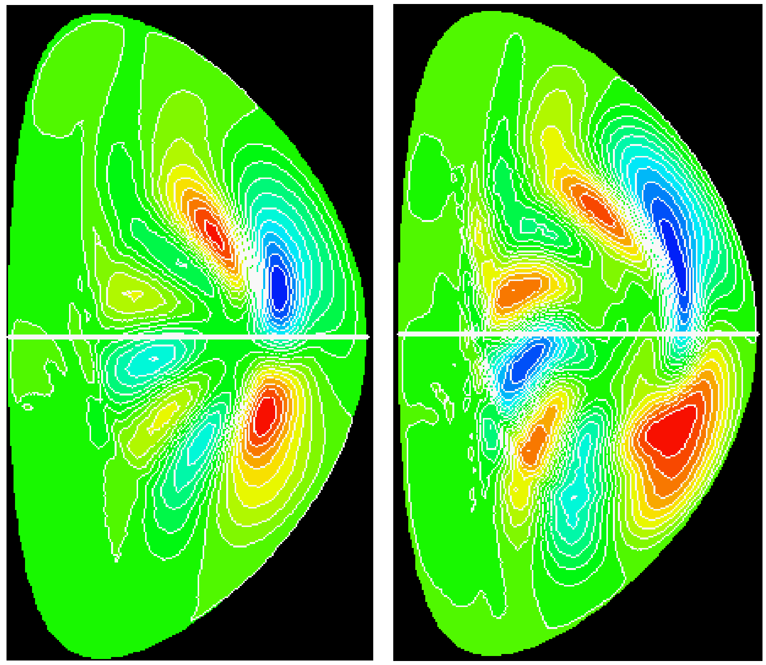

Figure 2: The n=2 mode structure of stream function U for the incompressible component of the fluid velocity in linear regime (left, at $t = 0$) and in nonlinear regime (right, at $t = 335$).

TRANSP code. Using these parameters and profiles, M3D simulations show that TAEs of $n = 2 - 4$ are excited by the beam ions with frequency consistent with the measurement. In particular, the n=2 linear TAE mode structure is shown in the plot on the left in Fig. 2. The calculated mode frequency is $68kHz$, which agrees well with the experimental frequency in the plasma frame ($f_{plasma} = f_{lab} - nf_{rotation} = 67kHz$ with $f_{rotation} \sim 15kHz$). Nonlinear simulations of the same n=2 TAE shows bursting behavior with evolving mode structure. Figure 3 shows the mode amplitude as function of time starting from the linear eigenmode at $t = 0$. We observe that the mode bursts after the initial saturation at $t \sim 200$. Correspondingly, the mode frequency chirps down by about 20% on the time scale of saturation. After the initial saturation, the mode structure moves out radially as shown in Fig. 2. This illustrates the importance of a self-consistent simulation where the driven modes can change as the particle distribution function evolves nonlinearly. We believe that the changing in the mode structure during the nonlinear evolution plays a major role in the mode bursting behavior. Finally, we have also carried out nonlinear simulations with multiple modes ($n = 1 - 4$) and similar nonlinear features are observed with mode bursting and mode shifting out radially.

4 Alpha Particle stabilization of internal kink mode in ITER

We now consider the effects of fusion alpha particles on the stability of internal kink mode in burning plasmas. Specifically, we consider parameters and profiles of ITER-FEAT. It is well know that energetic particles such as alpha particles have a significant stabilization effect on internal kink mode due to fast precession of trapped ions. There were many previous studies related to this physics. However, most of previous work assumes large aspect ratio and circular flux surfaces. Here, we apply M3D hybrid code which can treat finite aspect ratio and strongly shaped plasma cross sections such as ITER configuration. We will show that effects of elongation are important for alpha particle stabilization of internal kink.

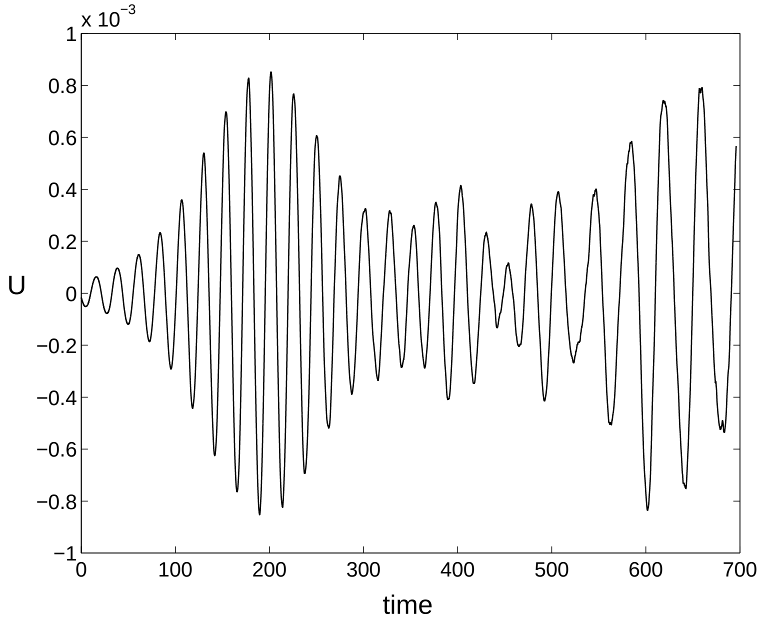

Figure 3: The mode amplitude U of the n=2 mode as a function of time for obtained with distribution paramaters of $v_c = 0.37v_0$, $\Delta\Psi = 0.23$, $\Lambda_0 = 0.5$ and $\Delta\Lambda >> 1$.

The alpha particle stabilization of internal kink can be described below in the limit of small alpha particle beta:

$$\frac{\gamma}{\omega_A} = \frac{\gamma_{MHD}}{\omega_A} - \beta_\alpha(0)\delta W_\alpha \tag{12}$$

where ω_A is the shear Alfvén frequency, γ and γ_{MHD} is the growth rate with and without alpha particle effects respectively. The second term on the right in Eq. 12 represents the stabilization effects of alpha particles (due to non-adiabatic response of alpha particles) with $\beta_\alpha(0)$ being the central alpha particle beta and δW_α is an order of unity numerical factor which depends on q profile and alpha pressure profile. In limit of large aspect ratio, circular geometry and $1 - q(0) << 1$, δW_α can be written as[6]:

$$\delta W_\alpha = -\frac{\sqrt{3}\pi}{8s_1}\sqrt{\frac{R}{r}}\int_0^{r_1} rdr[(0.6 + 3.2(1 - q - 0.5s))(\frac{r}{r_1})^{1.5}\frac{d\hat{p}_\alpha}{dr}] \tag{13}$$

where $r = r_1$ is the minor radius at which $q(r_1) = 1.0$, s_1 is the magnetic shear at $r = r_1$, and $\hat{p}_\alpha$ is the alpha pressure normalized to unity at the center. This analytic results agree quite well with our numerical results for a model tokamak equilibrium with circular flux surfaces.

We now consider the case of ITER-FEAT. The main parameters and profiles, obtained by a TRANSP simulation[7], are as follows: $B = 5.05T$, $R = 620cm$, $a = 200cm$, electron density $n_e(0) = 1.0e15$, $T_i(0) = 19kev$, $T_e(0) = 23kev$, central total beta $\beta(0) = 6.5\%$, $\beta_\alpha(0) = 1\%$, $q(0) \sim 0.9$, $q(a) = 3.83$. The alpha particle distribution is a slowing-down function with alpha pressure profile given by $p_\alpha(\Psi) = p_\alpha(0)\exp(-\Psi/0.25)$ where Ψ is the poloidal flux normalized to zero at the center and unity at the edge. Using these parameters and profiles, linear simulations have been carried out to determine the alpha particle stabilization effects. Figure 4 shows n=1 internal kink eigenmode structure with (righ) and without alpha particles (left). It is calculated that the MHD growth rate

$\gamma_{MHD}/\omega_A = 0.0070$ is reduced to $\gamma/\omega_A = 0.0039$ with alpha particles, which corresponds to $\delta W_\alpha = 0.31$. However, the simple analytic estimate in Eq.13 gives $\delta W_\alpha = 0.82$ which is much larger. We have shown analytically and numerically that this discrepancy is mainly due to elongation of the ITER plasma shape. Figure 5 shows δW_α as a function of elongation at zero triangularity while all other parameters and profiles are fixed. We observe that the stabilization effects of alpha particles decreases as elongation increases. At the full ITER shape (elongation=1.8), alpha particle stabilization is reduced by a factor of 2.5 as compared the circular shape case. This result shows that the elongation of plasma shape is an important factor for alpha particle stabilization of internal kink mode and it must be taken into account for realistic modeling of internal kink stability and sawteeth in ITER.

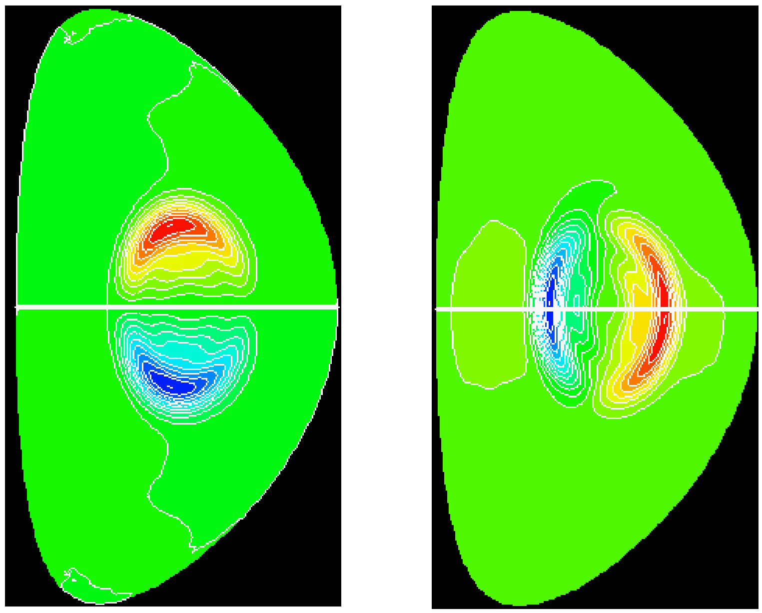

Figure 4: The n=1 internal kink mode structure with (right) and without(left) alpha particles.

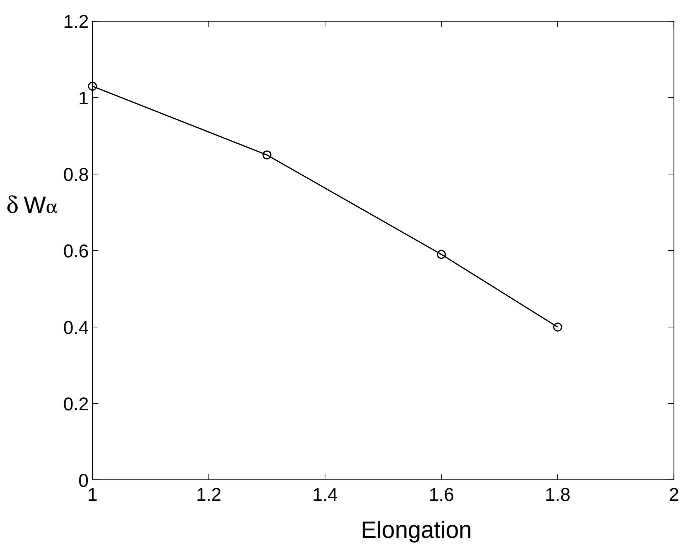

Figure 5: The alpha particle stabilizing contribution δW_α as a function of elongation.

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References

[1] W. Park, E.V. Belova, G.Y. Fu et al., Phys. Plasmas **6** 1796 (1999).

[2] L.E. Sugiyama and W Park, **7**, 4644 (2000).

[3] G.Y. Fu et al., the 2002 APS division of plasma physics meeting.

[4] H.R. Strauss, L.E. Sugiyama, G.Y. Fu et al., Nucl. Fusion **44**, 1008 (2004).

[5] E. Fredrickson, C.Z. Cheng, D. Darrow et al., Physics of Plasmas **10**, 2852 (2003).

[6] K.G. McClements, R.O. Dendy, C.G. Gimblett et al., Nucl. Fusion **35**, 1761 (1995).

[7] R. Budny, private communication, 2002.