Effects of Turbulence Induced Viscosity and Plasma Flow on Resistive Wall Mode Stability

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Abstract. In this paper, we investigate the effects of a new dissipation mechanism, turbulence induced viscosity, on the resistive wall mode (RWM) stability. The eigenmode equation for RWM is derived, including the turbulence induced viscosity and the plasma flow. The test computations are carried out to study the dependence of the mode growth rate on the wall conductivity, for a case without the viscosities and the plasma flow. With the turbulence induced viscosity but without flow, the numerical results show that the growth rate of the RWM decreases quickly with enhancement of the turbulence induced viscosity. In the presence of the plasma flow, the results show that the RWM is completely suppressed when the plasma flow velocity exceeds a critical value. Especially, the numerical results show that the turbulence induced viscosity significantly reduces the threshold of flow velocity required for the RWM stabilization. The effect of the turbulence induced viscosity on the stability window, in terms of the wall minor radius, has also been investigated.

1. Introduction

The stabilization of large-scale magneto-hydrodynamic (MHD) modes is necessary for the magnetic confinement of toroidal plasma such as the International Thermonuclear Experimental Reactor (ITER). In tokamaks, the maximum achievable value of the parameter β ($\beta = 2\mu_0 < P > / B^2$, the ratio of the plasma pressure to the magnetic field pressure) is often limited by the external kink modes, which can be stabilized by placing a perfectly conducting wall sufficiently close to the edge of the plasma. However, the wall of the actual tokamaks has finite conductivity. This converts the external kink mode into a slowly growing MHD mode which is called as the resistive wall mode (RWM). The RWM instability can be driven by the pressure gradient and the current gradient of the plasma. In this study, we investigate the behaviors of the RWM driven by the current gradient of the plasma.

As for the stabilization of the RWM in tokamak plasma, two approaches are investigated extensively during recent years, namely rotational stabilization [1-5] and feedback control [6-10]. It has been shown, both in theories and experiments, that the RWM can be completely suppressed by the toroidal plasma rotation, provided that the rotation velocity exceed a certain threshold value, which is typically a few percent of the Alfven wave speed at the plasma centre. And the threshold rotation speed is rather sensitive to the damping model.[10] . The physics mechanisms of the rotational stabilization of the RWM have not been fully understood. For example, the present MHD theory can not explain the recent experimental results [11, 12] clearly, which show that RWM can be stabilized with very slow toroidal rotation speed. Understanding the damping mechanism of the RWM is crucial not only for studying the critical rotation speed required to stabilize RWM in tokamak plasmas but also for understanding other related physics such as the plasma momentum damping.

In this study, we developed a cylindrical model including turbulent viscosity, which is related to the gradients of the magnetic field fluctuation and the plasma flow fluctuation in the plasma, and applied this model to study the RWM stability in tokamak plasma. The effects of the turbulent viscosity are incorporated into the model via a viscosity term χ in the momentum equation. Authors of Ref.[13] have demonstrated that the role χ plays in the derivation process of MHD equations, actually is the turbulent counterpart of the kinematic viscosity ν (as described in Ref.[13]). Therefore, for simplicity, in the paper we consider χ as a control parameter (the same as χ in Ref.[13]), the value of which is estimated on the basis of the experimental measurements[14].

2. Eigenmode equation and boundary condition

We consider the incompressible single fluid MHD equations in the cylindrical plasma. The linearized momentum equation, where the damping terms and the plasma flow are involved, can be written[3]

$$-\rho\omega^2\boldsymbol{\xi} = \frac{1}{\mu_0}[\mathbf{B}\cdot\nabla\tilde{\mathbf{b}} + \tilde{\mathbf{b}}\cdot\nabla\mathbf{B}] - \nabla(-\boldsymbol{\xi}\cdot\nabla P + \frac{\mathbf{B}\cdot\tilde{\mathbf{b}}}{\mu_0}) - \nabla\cdot\vec{\Pi}_1 - \nabla\times\chi\nabla\times\mathbf{v}, \quad (1)$$

where $\boldsymbol{\xi}$, $\mathbf{v}$, $\tilde{\mathbf{b}}$ represent the plasma perturbed displacement, the perturbed velocity and the perturbed magnetic field, respectively; P, $\mathbf{B}$ and ρ denote the equlibrium plasma pressure, the magnetic field and the plasma density, respectively; ω is the Doppler shifted frequency as that given in [15] with the assumption of uniform equilibrium flow velocity $\mathbf{V}$. $\vec{\Pi}_1$ denotes the parallel viscosity term induced by the ions collision. The turbulence induced viscosity term enters into the momentum equation via the last term shown in Eq. (1), where χ is considered to be the coefficient of the turbulent viscosity which is related to the gradient of the magnetic field fluctuation and the flow velocity fluctuation [13]. The value of χ in the paper is estimated based on the experimental measurements [14].

The perturbed displacement has the form $\boldsymbol{\xi} = \boldsymbol{\xi}_1 \exp(-i\hat{\omega}t + ikz + im\theta)$, where m is the poloidal wave number, k is the wave number in the longitudinal direction. After tedious straightforward manipulations, Eq. (1) can be coordinated to be the 4th order differential equation with unknown $\psi = r\xi_r$

$$C_4\frac{\partial^4\psi}{\partial r^4} + C_3\frac{\partial^3\psi}{\partial r^3} + C_2\frac{\partial^2\psi}{\partial r^2} + C_1\frac{\partial\psi}{\partial r} + C_0\psi = 0, \quad (2)$$

Due to the length limit of the paper, the detailed expression of coefficients in Eq. (2) are not shown in the paper. Here, C_3 and C_4 depend on the value of $\omega\chi$. We can simplify Eq. (2) according to the condition $\omega\chi << 1$, which is reasonable for such a low frequency mode as the RWM. That is, the 4th and 3rd terms in the eigenmode equation Eq. (2) can be deleted. Then Eq.(2) is reduced to be the following 2nd order differential equation

$$C_2\frac{\partial^2\psi}{\partial r^2} + C_1\frac{\partial\psi}{\partial r} + C_0\psi = 0, \quad (3)$$

where C_0, C_1, and C_2 are the functions of r, $\hat{\omega}$, χ, Ω_0, η_0, k, m, and ρ. Here, $\hat{\omega} = \omega_r + i\gamma$ is the eigenfrequency, $\Omega_0 \equiv \mathbf{k}\cdot\mathbf{V}$ is defined as the plasma toroidal rotation frequency, η_0 presents the coefficient of the parallel viscosity. In addition to the parallel viscosity and the plasma flow, a new dissipation term, turbulent viscosity has been taken into account in the eigenmode equation. The influences of the turbulent viscosity, the parallel viscosity and the toroidal rotation on the RWM stability will be investigated in detail in the numerical part.

Boundary conditions are required to resolve the eigenmode equation numerically. We consider a cylindrical plasma with the minor radius $r = a$ and the major radius $r = R$, surrounded by a resistive wall at $r = b$, with the wall thickness d and conductivity σ. The wall diffusion time is defined as $\tau_w = bd\mu_0\sigma$, where μ_0 is the magnetic constant. At the wall position ($r = b$), the perturbed magnetic field B_{1r}^{v} in the vacuum is continuous across the wall surfaces and meanwhile satisfies the thin wall jump condition. At the plasma-vacuum interface ($r = a$), both the perturbed radial magnetic field and the perturbed pressure are also continuous. The perturbed pressure condition can be obtained by integrating the radial component of Eq. (1) across the plasma-vacuum surface, where the turbulent viscosity correction has been taken into account.

3. Numerical results

In this section, we present the computational results obtained by solving the eigenmode equation numerically. We investigate the effects of the turbulent viscosity χ, the toroidal rotation frequency Ω_0, the parallel viscosity η_0, the wall position b and the wall conductivity σ on the RWM in detail. Here the frequencies ω, Ω_0, and $1/\tau_w$, the coefficients χ, η_0, the wall conductivity σ, the length scales r, b, d, and $1/k$ are normalized to ω_A, $\rho a V_A$, $1/\omega_A\mu_0 a^2$, and a, respectively, where $\omega_A \equiv V_A/R$ and $V_A = B\big/\sqrt{\mu_0\rho}$, V_A is the Alfven wave speed. In the following numerical researches, we consider the large aspect ratio equilibrium configuration, with $B_z = constant$, $B_\theta = B_{\theta a}[1-(1-r^2)^2]/r$, $J = (0, 0, J_0(1-r^2))$, $\nabla P = \mathbf{J}\times\mathbf{B}$, and $R/a = 10$. In addition, we assume that the turbulent viscosity, the plasma equlibrium density and the toroidal rotation are constants along the minor radius.

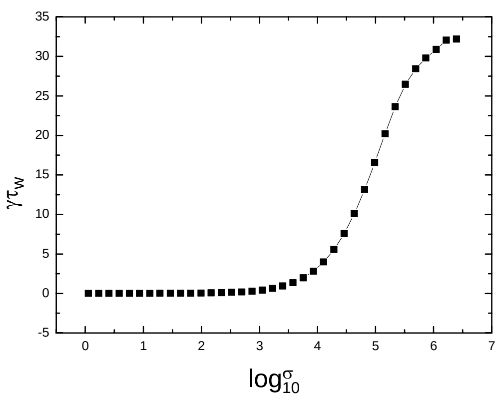

FIG. 1. The product of the growth rate and the wall time vs the denary logarithm of wall conductivity. The parameters are given as $q_a = 1.6$, $m = 2$, $k = -0.1$, $b = 1.1$, *and* $d = 0.01$, *where* q_a *is the safety factor at the plasma edge.*

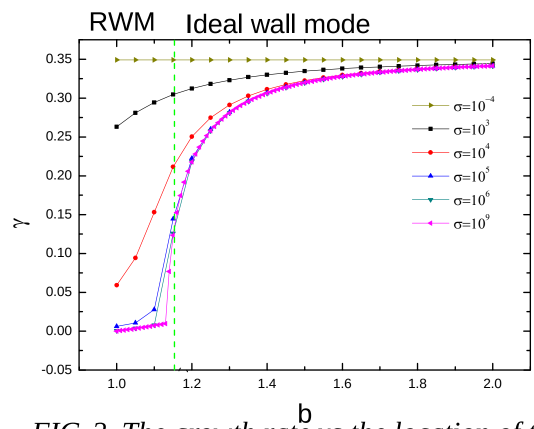

FIG. 2. The growth rate vs the location of the wall for the different wall conductivities. The parameters are taken as $d = 0.01$, $q_a = 1.6$, $m = 2$, *and* $k = -0.1$. *No plasma rotation and without damping are assumed.*

Without the plasma flow and any damping, we calculate the product ($\gamma\tau_w$) of the growth rate and the wall time as the function of the denary logarithm of the wall conductivity, which is shown in Fig.1. The computations show that, at high enough wall conductivity, the value of $\gamma\tau_w$ tends to be a constant, in other words, the mode growth rate scales is inversely proportional to the wall time, then the growth rate of the instability is determined by the wall conductivity. The result is consistent with that shown in [16]. Figure 2 shows the dependence of the RWM growth rate on the wall position for the different wall conductivities. The computations identify a critical wall radius $b_c \approx 1.15$. In the region $b < b_c$, the growth rate of the instability decreases significantly as the increase of the wall conductivity. However, it is shown that the perfect conducting wall ($\sigma = 10^9$) can not make the marginal stability ($\gamma \approx 0.0$) become a completely stability ($\gamma < 0.0$). Thus, the damping terms or other stable effects are required for the full stabilization of the RWM. On the other hand, in the region $b > b_c$, the RWM growth rate increases significantly as the increase of the wall position for the given wall conductivity. If the wall position is more further from the plasma surface, the wall does not have any effect on the growth rate, even the wall is perfectly conducting.

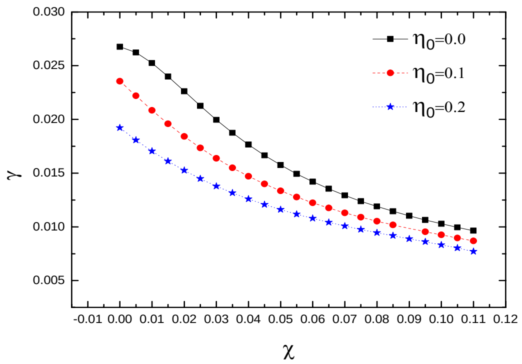

FIG. 3. The RWM growth rate vs the turbulent viscosity coefficient for the different values of the parallel viscosity coefficient. Here, the plasma toroidal rotation is not taken into account. The parameters are assumed as $q_a = 1.6$, $\sigma = 10^5$, $b = 1.1$, $d = 0.01$, $\Omega_0 = 0.0$, $m = 2$, $k = -0.1$.

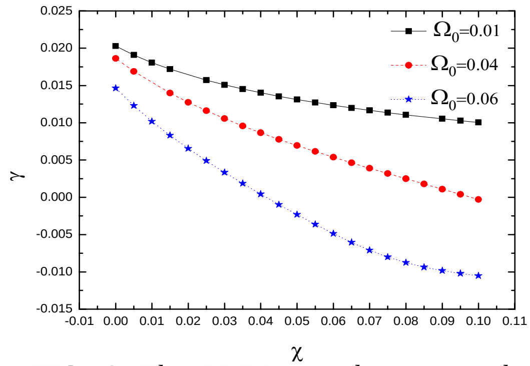

FIG. 4. The RWM growth rate vs the turbulent viscosity coefficient for the different values of the plasma toroidal rotation. It is shown that the RWM can be fully stabilized when the turbulent viscosity is larger than a certain value for the cases $\Omega_0 = 0.04$ *and* $\Omega_0 = 0.06$*.The parameters are the same as these used in Fig. 3, but* $\eta_0 = 0.1$

Figure 3 plots the RWM growth rate versus the coefficient of the turbulent viscosity with the different parallel viscosities but without plasma rotation. It can be seen that the growth rate γ decreases largely with the increase of χ. That is, the turbulent viscosity has a strongly stable effect on the RWM instability. For the present case without the plasma flow, the RWM instability can not be completely stabilized (i.e. $\gamma < 0.0$), even the value of χ is significantly large. Furthermore, it is found that the parallel viscosity has the stability influence on the RWM . The mode frequency for these cases nearly vanishes, that is, $\omega_r \approx 0.0$.

Shown in Fig. 4 is the RWM growth rate as the function of the turbulent viscosity for the different toroidal rotation frequencies Ω_0. The results show that the RWM can be completely suppressed when the turbulent viscosity is larger than a critical value for a certain value of Ω_0. The critical values of turbulent viscosity are, respectively, $\chi_c = 0.085$ and $\chi_c = 0.04$ for $\Omega_0 = 0.04$ (red-dotted line) and $\Omega_0 = 0.06$ (blue-dotted line). Thus, the larger Ω_0 needs the smaller χ_c for the completely stabilization of RWM. Based on Eq.(15) in [13], the value of χ is estimated by the formula $\chi \approx 0.07(\delta B^4 / (\partial \delta B / \partial r)^2)(1/\rho a V_A)\sigma_p$, where σ_p denotes the plasma conductivity and δB is the perturbed magnetic field. According to [14], the fluctuation of the magnetic field is represented by the analytical expression: $(\delta B(r)/B)^2 = 0.25\times10^{-9} + 7.7\times10^{-9}(r/a)^5$. The value of χ is in the order of 0.1, which is normalized by $\rho a V_A$, where the experimental parameters are taken as $B = 3.7T$, $a = 0.75m$, $\sigma_p = 10^8\Omega^{-1}m^{-1}$, the density $n = 4\times10^{19}m^{-3}$, respectively.

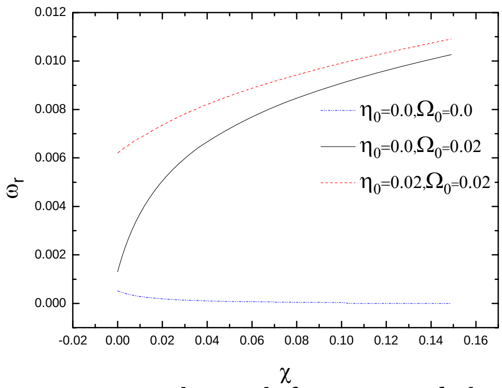

FIG .5 .The real frequency of the RWM vs the turbulent viscosity coefficient for the different values of η_0 and Ω_0. The turbulent viscosity coefficient varies from 0.0 to 0.15.

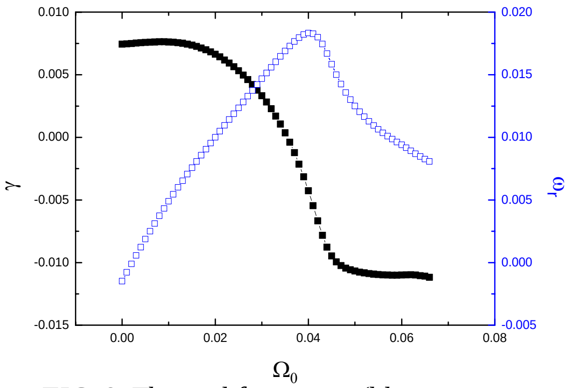

FIG. 6. The real frequency (blue-square line) and the growth rate of the RWM vs the plasma toroidal rotation. The viscosity coefficients are assumed as $\eta_0 = 0.2$, and $\chi = 0.1$, respectively.

In the presence of the plasma toroidal rotation, the mode frequency is not equal to zero anymore, but has a finite value. Figure 5 shows the influences of the turbulent viscosity, plasma flow speed and parallel viscosity on the mode frequency. The computations show that the mode frequency increases with the enhancement of the turbulent viscosity when the plasma flow is taken into account. Furthermore, for the given Ω_0 and χ, the mode frequency corresponding to the $\eta_0 = 0.2$, is larger than that corresponding to $\eta_0 = 0.0$. In Fig. 6, both the real frequency and the growth rate of the RWM are plotted as the function of the toroidal plasma rotation frequency Ω_0. The RWM growth rate monotonically decreases with the increase of the value of Ω_0. It is noted that, when the plasma rotation frequency is larger than a critical value, $\Omega_c = 0.03$, the RWM instability can be fully stabilized. In addition, it can be seen that the mode frequency is roughly proportional to the plasma rotation frequency in a small slope at the beginning when $\Omega_0 < \Omega_c$; As the further increase of the Ω_0, when $\Omega_0 \approx \Omega_c$, the mode frequency reaches a maximum value; When $\Omega_0 > \Omega_c$, the mode frequency decreases gradually with the further increase of Ω_0.

In order to investigate the dependence of the critical rotation frequency required for the RWM stabilization on the turbulent viscosity χ in detail. Figure 7 plots the critical rotation frequency as the function of turbulent viscosity for the different parallel viscosities. The numerical results show that the critical rotation frequency Ω_C decreases rapidly with the enhancement of the χ for a given η_0. Furthermore, Figure 7 also indicates that the presence of the parallel viscosity reduces the critical toroidal rotation frequency required for the RWM stabilization. However, the influence of the parallel viscosity on the critical rotation frequency become smaller and smaller as the increase of the turbulent viscosity. Finally, the effects of the η_0 would vanish when the value of χ is larger enough. The critical toroidal rotation frequency obtained in the paper is larger than the experimental result[11, 12], due to that the other stable effects, such as Alfven wave damping, sound wave damping and kinetic damping, have not been taken into account in the present model.

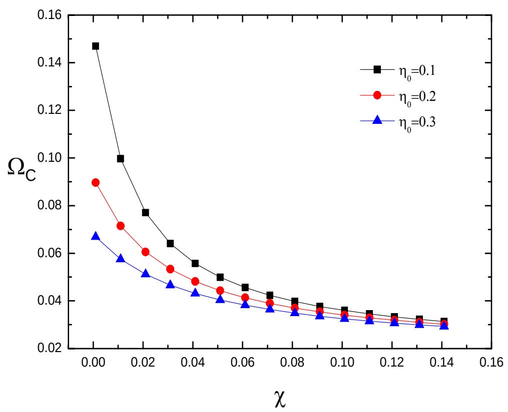

FIG. 7. Plotted is the dependence of the critical toroidal rotation frequency on the turbulent viscosity coefficient χ under various η_0 ($\eta_0 = 0.1$, $\eta_0 = 0.2$, and $\eta_0 = 0.3$). Different colors for the curves correspond to the different choices of the parallel viscosity coefficient.

The behavior of the stability window in terms of the wall minor radius has also been studied. Figure 8 plots the mode growth rate versus the wall position with the different plasma toroidal rotation frequencies for the given χ and η_0. It is identified that, when the plasma rotation frequency reaches a certain value, Ω_0=0.05, a stability window appears in the wall position. The effects of the Ω_0 on the stability window are similar to that shown in Ref. [3]. However，the presence of χ reduces the critical value of the Ω_0 required for the appearance of the stability window. We also study the effect of the turbulent viscosity on the stability window. For the case $\eta_0 = 0.2$ and $\Omega_0 = 0.05$, we calculate the growth rate by varying the wall distance b for different values of χ. It is presented that, when the turbulent viscosity χ reaches a certain value, the stability window also appears in the wall position as that shown in Fig.9. A further increase in χ widens the stability window toward the plasma boundary. The right side of the stability window is very close to b_c and change little with the χ, while the left side significantly depends on it. We can notice that, the role of the damping terms, such as the turbulence viscosity, on the stability window is similar to that of the plasma rotation frequency on the stability window.

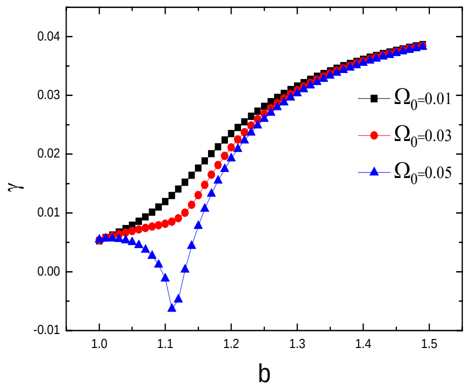

FIG.8. The growth rate vs the wall position for the different values of toroidal rotation frequency. Here, the values of the χ and η_0 are fixed, $\chi = 0.05$ and $\eta_0 = 0.2$. The parameters are assumed as $q_a = 1.6$, $\sigma = 10^5$, $b = 1.1$, $d = 0.01$, $m = 2$, and $k = -0.1$.

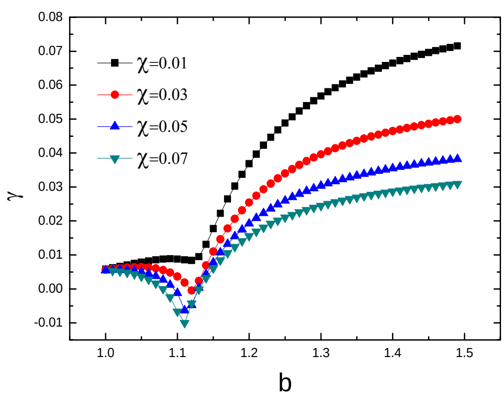

FIG.9. The growth rate vs the wall position for various turbulent viscosity with the fixed plasma toroidal rotation and the parallel viscosity. The parameters are assumed as $q_a = 1.6$, $\sigma = 10^5$, $b = 1.1$, $d = 0.01$, $m = 2$, $\eta_0 = 0.2$, $k = -0.1$ and $\Omega_0 = 0.05$.

4. Conclusions

In this paper, an eigenmode equation in the tokamak plasma is derived, which considers the turbulent viscosity, the parallel viscosity and the plasma flow. We have numerically solved the eigenmode equation to obtain the normalized growth rate and the real frequency of the RWM with the appropriate boundary conditions.

The computations show that the turbulent viscosity has the stable influence on the RWM instability. In the presence of the plasma flow, the numerical results show that the RWM is completely suppressed when the plasma rotation frequency exceeds a critical value Ω_C for the given χ and η_0. The critical rotation frequency Ω_C significantly decreases with the enhancement of the turbulent viscosity. The results indicate that the presence of the parallel viscosity also reduces the critical toroidal rotation frequency required for the RWM stabilization. It is also observed that, when the turbulent viscosity reaches a certain value, the stability window first appears in the terms of the wall minor radius. The width of the stability window is proportional to the value of the turbulent viscosity coefficient. In addition, it is presented that, when the mode starts to be stabilized, the real frequency of the mode reaches a maximum value, and then decays gradually. The calculations show that the mode frequency is proportional to the damping terms, such as turbulence viscosity, when the plasma rotation is taken into account.

In the paper, we investigated the effects the turbulent viscosity and the plasma flow on the RWM which is driven by the plasma current gradient. However, the conclusions obtained, such as that the turbulent viscosity has the stable effect on the RWM, are expected to be applicable qualitatively for the pressure driven RWM.

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