Nonlocal dynamics of Turbulence, Transport and Zonal Flows in Tokamak Plasmas

G. Dif-Pradalier 1-2), S. Ku 3-4), P. H. Diamond 1-2-3), C. S. Chang 4-5), Y. Sarazin 6), V. Grandgirard 6), J. Abiteboul 6), S. Allfrey 6), X. Garbet 6), Ph. Ghendrih 6), G. Latu 6-7), A. Strugarek 6)

1) Center for Momentum Transport & Flow Organisation, UCSD, La Jolla, CA 92093, USA
2) Center for Astrophysics and Space Sciences, UCSD, La Jolla, California 92093, USA
3) National Fusion Research Institute, Daejon, Korea 305-806
4) Courant Institute of Mathematical Sciences, New York Univ., New York 10012, USA
5) Department of Physics, KAIST, Daejeon, Korea 305-701
6) CEA, IRFM, F-13108 Saint Paul-lez-Durance cedex, France
7) Université de Strasbourg, LSIIT, Bd. Sébastien Brant, 67400 Illkirch, France

E-mail contact of main author: gdifpradalier@ucsd.edu

Abstract. The understanding of plasma turbulent transport in tokamaks used to rely on a local process, in the sense that locally excited fluctuations exhibit short radial correlation lengths only, ultimately leading to diffusive transport. We find here that the intrinsic nature of turbulent heat transport in tokamaks is nonlocal. This nonlocality is thoroughly defined and quantified. In the same vein, it is also found that the global structure of turbulence and transport results from a synergy between edge-driven inward propagation of turbulence intensity with outward heat transport. This synergy results in inward-outward pulse scattering leading to spontaneous production of strong internal shear layers in which the turbulent transport is almost suppressed over several radial correlation lengths. These two examples represent different sides of the same coin: the turbulence-generated self-organised processes which occur at *mesoscales* are central to our understanding of transport processes as they govern shear generation and flow pattern formation.

1. Introduction

Transport and turbulence in tokamaks have traditionally been treated as local processes, in which localised instabilities drive local mixing and diffusive transport, so that well separated regions of the plasma interact with one another only by diffusive pulse propagation. Recast differently, this paradigm states that any particle or heat flux can accurately be described using a set of local transport coefficients –diffusivities or conductivities– which should be *locally* related through a generalised Fick's law to the thermodynamic forces which induce them. Models with such assumptions will be referred to as *local* or *quasi-local*: they assume successive transport events to be either mutually independent or with both a short correlation and short memory; in other words transport events are random and accurately described by a classical Gauss-Markov process.

Theoreticians have long alerted to the dubiety of local, diffusive approaches to nonlinear transport, using e.g. continuous time random walks (CTRWs) [1] in as different areas as chaotic dynamics [2,3], geophysics, financial mathematics [4], hydrodynamics [5] or even hydrology [6]. Early works in fusion research have also worked beyond local, diffusive models: either invoking *(i)* linear toroidal mode coupling to account for fast pulse propagation time scales [7], *(ii)* self-organised criticality [8-10] and the concept of marginal stability to connect nonlocal transport events to scale-free avalanches [11,12] and explain ion profile stiffness, *(iii)* turbulence spreading where rapid pulse propagation is considered as a more general consequence of the nonlinear dynamics [13] or *(iv)* based on the fractional kinetic equation [14,15], following pioneering works by Mandelbrot or Zaslavsky [16].

And recently, while studying the turbulent heat conduction generated by the Ion Temperature Gradient (ITG) instability, evidence was shown from a different standpoint [17] that the ``standard", local model described above is surely incomplete, in that turbulence and transport dynamics are intrinsically nonlocal. We first present evidence of this strong nonlocal, nondiffusive, avalanche-mediated character of heat transport found in state-of-the-art flux-driven gyrokinetics (section 2) and show how these nonlocal dynamics lead to the discovery of a novel self-organised flow structure which we called the ``**ExB** staircase" [17] (section 3). We then discuss how this nonlocal behaviour can crucially affect our understanding of the edge-core interaction, especially through the large-scale fast propagation of turbulent fronts which connect core and edge regions (section 4).

2. Quantifying nonlocal, nondiffusive behaviour: avalanching and the heat conduction

It is possible to provide a simple, systematic, constructive and *self-consistent* procedure to quantify nonlocal and nondiffusive behaviour in complex geometry and realistic plasma parameters. Self-consistency is key: the analysis presented in Ref.[17] and detailed below is based on a large database from the state-of-the-art full-f, flux-diven gyrokinetic GYSELA [18] and XGC1 [19] codes: *velocity fields, flows and heat fluxes* are fully self-consistently evaluated, differing from either CTRW or particle-following methods. Avalanching, namely intermittent bursts due to overturning of neighbouring modal convection cells, has been observed in digital and physical experiments on plasma turbulence. The process of thermal avalanching suggests a nonlocal description of the heat flux which translates mathematically as follows. We wish to move from a *local* or *quasi-local* formalism: $Q(r)=-n(r)\chi(r)\nabla T(r)$, Q being the turbulent heat flux, n the density, χ the turbulent diffusivity and T the temperature – each of these three latter quantities are expressed *locally* at radius r – to a generalised heat transfer integral: $Q(r)=-\int\kappa(r,r')\nabla T(r')dr'$ since the heat flux at radius r must necessarily have a memory of where the avalanche was triggered. Here, the kernel κ – physically, a generalised diffusivity, which includes the density dependence– is the crucial quantity. At this point, insofar as κ is not specified, let us emphasise that no assumption of dominance of nonlocality over locality is made: the above heat transfer integral embeds both nonlocal and local (κ can be a Dirac distribution) formalisms.

One of our goals here is to provide a straightforward and systematic way to infer the form of κ, no assumptions on the nature of the dynamics (local or not) being made *a priori*. The basic idea is to interpret the above heat transfer integral as a convolution product and hence recast, using the convolution theorem, the integral as a mere product in Fourier space: $F(\kappa)=-F(Q)/F(\nabla T)$, F being the radial Fourier transform. Thus, given any set of data, insofar as the turbulent heat flux Q and the external thermodynamic forces are known, one can straightforwardly compute the (Fourier transform of) kernel $F(\kappa)$. To illustrate this idea, we performed this procedure on a vast sample of simulation data (see Fig.1), encompassing significantly different plasma parameters. For an up-front comparison with local transport models, the quantities below are both flux-surface averaged and radially averaged over the central half of the simulation domain, as is the case for the normalised temperature gradient R/L_T, known to quantify the strength of the turbulence drive:

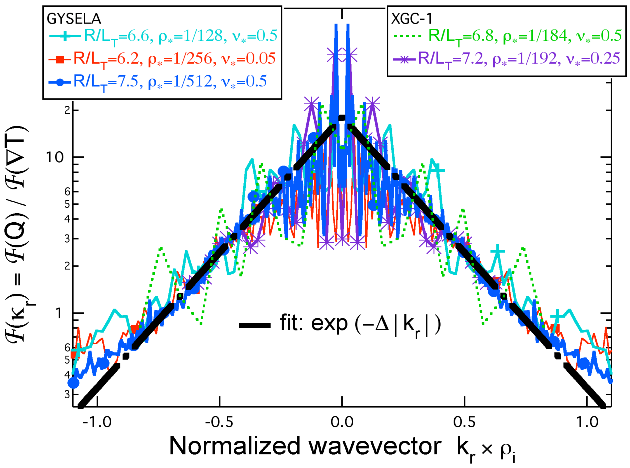

Figure 1 A Cauchy-Lorentz distribution robustly fits the kernel $F(\kappa)$ in Fourier space.

$Q \propto (R/L_T-5.9)^{1/2}$ [20]. We scan *(i)* the importance of SOC features while scanning this key control parameter from weak and close to marginal $(R/L_T \sim 6.2)$ to moderate/strong $(R/L_T \sim 7.5)$ turbulence regimes, *(ii)* the collisionality ν^* over one order of magnitude in the tokamak-relevant low-collisional so-called "banana regime", from 0.05 up to 0.5. At last, the system size, parametrised *(iii)* by the dimensionless $\rho_*=\rho_i/a$ number spans from today's smallest tokamaks (ρ_*=1/128) down to tomorrow's largest ones (with the (ρ_*=1/512) Iter-like value). Here, a denotes the minor radius and ρ_i is the ion gyroradius.

As the first remarkable feature from Fig.1, *(i)* a universal pattern arises, despite the widely different underlying plasma conditions. A generic exponential form $L_\gamma \sim \exp(-\Delta|k_r|^\gamma)$ can indeed be tailored in each case to best fit the data, k_r denoting the radial wavevector, normalised to ρ_i at mid radius. The Fourier transforms are performed between $0.35<\rho<0.65$ after temporal averaging over a collision time. At this point, γ and Δ are free parameters: γ controlling the shape of the fit, Δ its width. Note that the absolute value of k_r is reminiscent of the non-diffusive behaviour found in radiation hydrodynamics, later rediscovered in closure theory [21,22]. A set of optimal (γ,Δ) pairs [*e.g.* in the case ρ_*=1/512, pairs between (γ,Δ)=(0.9,18) and (1.1,24)] may be found to equally well fit the data through a systematic minimisation in the (γ,Δ) space of quantity $F(\kappa)-F_{\mathrm{fit},(\gamma,\Delta)}(\kappa)$. Interestingly, in any case, $0.8<\gamma<1.2$, readily implying that *(ii)* the kernel κ is a Lévy distribution with index γ. This result is especially attractive since Lévy distributions are characterised by a divergent second moment (infinite variance), making them choice candidates for modeling nonlocality.

At this point, let us strongly emphasise on the fact that finding the kernel κ to be of Lévy type appeared *self-consistently*, as an outcome of our procedure and that *no pre-conceived hypothesis* has led us to this conclusion. The current study, based on the self-consistent fluxes from GYSELA or XGC1, is strongly different in essence to procedures based on test particle following in a pre-chosen Lévy-like nondiffusive formalism. For the sake of simplicity, we now *(iii)* choose to discriminate amongst all optimal *(γ, Δ)* pairs by setting $\gamma=1$. This choice has several advantages: with limited loss of generality, it allows for physical intuition while remaining fully analytic. As a special case of Lévy distributions, $\gamma=1$ is trivially Fourier invertible, its inverse being the well-known Lorentz distribution. Thus, κ in real space now has the attractively simple analytic expression: $\kappa(r,r') \propto (\Delta/2)\{(\Delta/2)^2+|r-r'|^2\}^{-1}$. As the Lorentzian width, parameter Δ may now further be interpreted as a radial *influence length*: a transport event happening at location r can drive a flux up to a distance Δ from this event. One can now readily see that Δ does indeed take over a special relevance to assess the question of locality *v.s.* nonlocality: the larger Δ, the stronger the nonlocality.

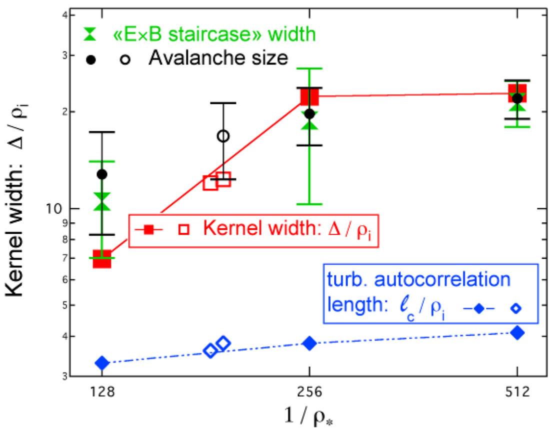

Figure 2 The `influence length' Δ is compared to the turbulence autocorrelation length, the avalanche size and the `ExB staircase' width (solid symbols GYSELA; open symbols XGC1).

Interestingly also, this radial influence length Δ fills in the mesoscale range –*i.e.* $\ell_c \ll \Delta \ll L_{Ti}$, where ℓ_c is the turbulence autocorrelation length and L_{Ti} is the profile scale– and is systematically 3-4 times larger than the turbulence autocorrelation length, as shown in Fig.2. If we further recall that κ may equivalently be described by a second moment divergent Lévy distribution, **reconciliation of this data with local or quasi-local models can only appear as fortuitous.**

On the grounds of this difficult reconciliation, any transport model should endeavour to encompass the possibility for a nonlocal, nondiffusive action at a distance. A prominent manifestation of action at a distance in plasma turbulence are the commonly observed, intermittent, large-scale heat avalanches; in that spirit, the typical avalanche size –comparable to the tail in the autocorrelation function– is compared in Fig.2 to the kernel width Δ for each simulation. Close agreement is found, which interestingly grounds this intuitive nonlocal *influence length* Δ –self-consistently obtained from actual heat fluxes– to the oft-invoked physics of self-organised near-critical transport [5,8–15,20].

Organisation of the transport dynamics based on realistic, unconstrained and self-consistent full-f, flux-driven (statistical) state-of-the art modeling shows that a crucial piece of physics occurs at intermediate scales, in-between the *local-like* turbulence autocorrelation length ℓ_c and the system size (or the profile scale) L_{Ti}. A crystallisation happens at these intermediate *mesoscales* where shear and turbulent stresses are generated, flows are formed and dissipation happens.

3. Puzzling staircases

Avalanches are often thought of as scale-invariant; having an intrinsic "nonlocal length" Δ emerging as in section 2 and saturating at mesoscales $\ell_c << \Delta << L_{Ti}$ for large enough systems (ρ_*<1/256) (see Fig.2) is a remarkable feature. So is the fact that it corresponds to a typical avalanche size in the system. Remarkably also, this influence length leaves a salient footprint on the flow structure. Strongly organised and persistent **ExB** sheared flows emerge and organise the turbulence into radial domains (see Fig.3, inset). The typical size of these domains [the spacing between the "jets"] is shown as the "**ExB** staircase width" in Fig.2; it also is in remarkable agreement with the avalanche size and the kernel influence length Δ, reflecting the surprising tendency of the stochastic avalanche ensemble to self-organise in a jet-like pattern: few avalanches do indeed exist on scales larger than Δ. Though the causality question "is it the avalanching process that sets the jet spacing or is it the creation of this pattern that limits the radial extension of the avalanches" is yet unresolved [23], *any proposed mechanism will have to account for arresting the "upscale of nonlocality" to the system size and the emergence at this* mesoscale Δ *of a jet-like pattern of coherent structures of alternating sign: the "***ExB** *staircase"*, see Fig.6 and Ref.[17], which we have named after its planetary analogy [24]. Thus, in large systems like *e.g.* Iter, Δ saturates within the mesoscale range. Note that the location of these jets is not tied to rational values of the safety factor profile q, as commonly emphasised. An interesting perspective, under investigation, is whether at the location of these jets, in monotonic q-profile plasmas, a transport barrier may nucleate.

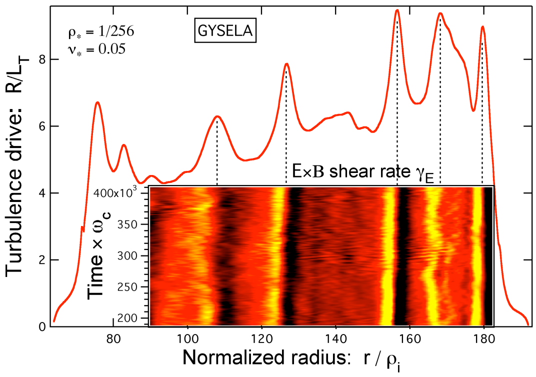

Figure 3 Steady-state corrugations of the mean temperature profile correlate well with dynamically-driven perennial ExB sheared flows which self-organise non-linearly in a jet-like pattern: the "ExB staircase" [17].

A bigger picture starts to emerge, in which self-organisation is present at all scales yet manifests itself differently at different scales; its most prominent feature being arguably the dynamical emergence of the *mesoscale* Δ: at scales smaller than Δ, transport is scale-invariant, avalanche-mediated, nonlocal and non-diffusive; at scales larger than Δ, few genuinely scale-free avalanches exist; rather, strongly coherent and persistent flows organise the turbulence into a jet-like pattern, the "**ExB** staircase". This structure represents the footprint of the radial influence length on the flow structure, and reflects the surprising tendency of the stochastic avalanche ensemble to self-organise in a jet-like pattern. The system may thus either be seen as scale-invariant, nonlocal, non-diffusive [fractal or self-similar] on some scale or, equivalently, strongly organised on some other. A thorough characterisation of the interplay between scales may thus only be achieved through a self-consistent and simultaneous treatment of all scales, especially allowing for the back-reaction of fast, small turbulent scales on the slower and larger equilibrium (neoclassical) scales.

Interestingly, in a global, full-f and flux-driven (statistical) system, the dynamics of the mean background profiles provides the turbulence with another efficient channel for regulation, especially through alteration of the mean thermodynamic forces acting upon the system. This alteration is dominantly located at the steps of the latter staircase and is responsible for the generation of a "mean flow shear" (or "equilibrium shear"), dominant as compared to the usual *fluctuating* "zonal flow" shear [20], which is the only turbulence-regulating mechanism possibly modeled in *local or quasi-local* models. The dominance near marginality of this "mean flow shear" certainly commends a shift to our current understanding of the *hierarchy of shears* and begs for a detailed and self-consistent accounting of turbulence-induced mean profile dynamics in "predictive" modeling, as detailed in Refs.[20,25]. This observation also emphasises on the central importance of the self-organising processes happening at *mesoscales*.

The remaining question of the physical mechanism that sets the staircase width is beyond the scope of the present paper. Its connection to a Rhines-like scale or Rossby-like wave breaking mechanism which could account for the emergence of the coherent structures in Fig.3 most notably the zonal/mean jets of alternating sign that organise the turbulence into radial domains will be reported elsewhere. Ongoing work is also concerned with exploring the dependencies of Δ and elucidating the mechanisms whereby the staircase emerges.

Such nonlocal phenomena appear to be very generic features in tokamaks, not merely confined to the core region. As we now discuss in the case of the core-edge interaction, a similar nonlocal, front (avalanche)-like transport dynamics is found.

4. Edge-Centre Coupling in Turbulence Dynamics, or how the Tail wags the Dog

H-mode density pedestals are usually observed to be narrower than T_i-pedestals, so that a region of large $\eta_i=L_n/L_{Ti}$ can drive strong turbulence in the near-edge region of the T_i profile. As a result, a strong ITG turbulence is excited in the "near-edge" region of the T_i pedestal knee, just inside the density pedestal and originates in this region. Remarkably, gyrokinetic simulations using the full-f code XGC1 [26], which self-consistently evolves flows and profiles, reveal the robustness of this phenomenon, as shown in Fig.4: rapid *inward* propagations of turbulence intensity [19], starting from the near edge, occur at a ballistic speed –a fraction of the diamagnetic velocity: $v \sim 0.1 v_{dia}$, see Fig.5. These inward and outward avalanches are consistently observed in any flux-driven model and are well understood in the context of Self-Oraganised Criticality as

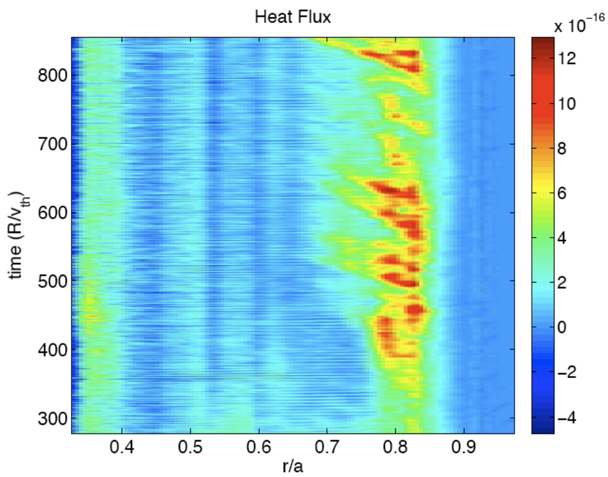

Figure 4 Ingoing intensity pulses keep originating from the near edge.

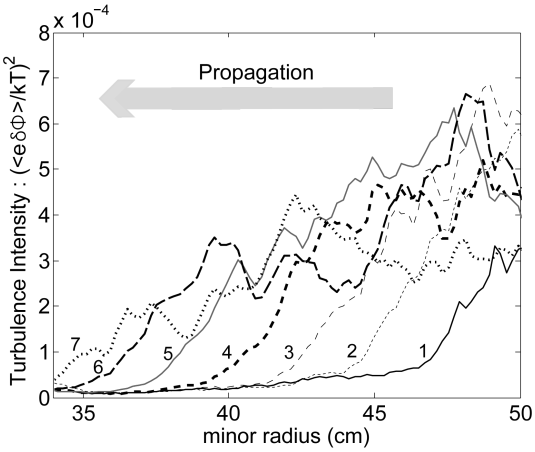

Figure 5 Inward (times series) propagation of turbulence intensity.

"avalanches of holes" which can excite turbulence in both directions as the gradient fronts propagate either up-hill or down-hill. This inward propagation of turbulence intensity is accompanied by an increase in the outward ion heat flux in the region of active turbulence. Simultaneously, core fluctuations grow and drive outward propagating intensity bursts which collide with the incoming pulses and thus further stir the core turbulence. The strong scattering between inward and outward propagating fronts triggers the formation of an **E**x**B** flow shear layer in the core, see Fig.6. In this layer, the turbulent diffusivity χ_i drops to near-neoclassical levels, so that the global profile consists of a near marginal (stiff) core, a strongly turbulent near edge and a pedestal region where turbulence is suppressed. As a consequence, χ_i is low in the core and rises as one approaches the pedestal, since fluctuation intensity peaks in the near edge.

These results have several interesting implications for our understanding of tokamak confinement and suggest a paradigm shift in how we conceive the basic nature of tokamak microturbulence. Previously, the turbulence was thought to be local and locally driven by the local temperature gradient. However: *(i)* as a basic consequence of H-mode pedestal structure, a strongly turbulent `near edge' region forms and connects the stiff core and the pedestal zones; much of the core plasma is actually activated by an inward flux of intensity from the ion temperature pedestal. Thus, the pedestal is *not* merely a ``boundary condition'' for the core transport process. Also, *(ii)* this incoming front scatters off of ambient fluctuations, producing a bursty outwards heat flux with Q(f) ~ 1/f and also triggering a shear layer formation. Models of core turbulence dynamics should therefore address the interaction and the collision of inward and outward propagating turbulence intensity fronts, and inward turbulence spreading for this latter is a key player in core turbulence and transport. Also *(iii)* the universally observed increase in $\chi_i(r)$ with radius may be explained by the dynamics of ITG turbulence spreading alone, without the need of *additional* edge turbulence mechanisms: our results indicate that both fluctuation intensity profiles remain peaked at the pedestal as time progresses and that η_i remains peaked at the edge in the long time, quasi-stationary state. Taken together, *(iv)* these results indicate that the edge pedestal temperature is a key parameter for core turbulence and transport dynamics and that both the temperature and density profile structure may exert a strong nonlocal influence on core transport and turbulence.

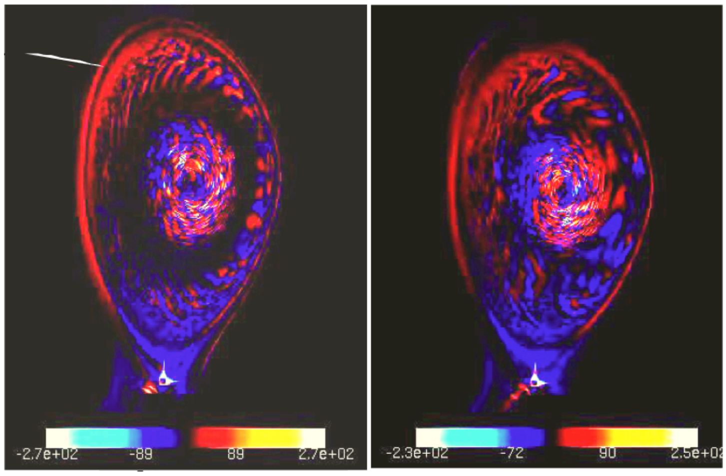

Figure 6 The building up of a shear flow layer resembling an internal transport barrier where the *inward* propagating intensity meets the *outgoing* heat flux. On the left, the turbulent diffusivity; on the right, the turbulence intensity.

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