The Impact of 3-D Fields on Tearing Mode Stability of H-modes

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Abstract. New processes have been discovered in the interaction of 3-D fields with tearing mode stability at low torque and modest β on DIII-D and NSTX. These are thought to arise from the plasma response at the tearing resonant surface, which is theoretically expected to depend strongly on plasma rotation and underlying intrinsic tearing stability. This leads to sensitivities additional to those previously identified at low density where the plasma rotation is more readily stopped, or at high β_N where ideal MHD responses amplify the fields (where β_N is the plasma β divided by the ratio of plasma current to minor radius multiplied by toroidal field). In particular, the field threshold to induce modes tends to zero as the natural tearing β_N limit is approached. 3-D field sensitivity is further enhanced at low rotation, with magnetic probing detecting an increased response to applied fields in such regimes. Typical field thresholds to induce modes in torque-free β_N~1.5 H-modes are well below those in Ohmic plasmas or plasmas near the ideal β_N limit. This strong interaction with the tearing mode β_N limit is identified through rotation shear, which is decreased by the 3-D field, leading to decreased tearing stability. Thus both locked and rotating mode field thresholds can be considered in terms of a torque balance, with sufficient braking leading to destabilization of a mode. On this basis new measurements of the principal parameter scalings for error field threshold have been obtained in torque free H-modes leading to new predictions for error field sensitivity in ITER. The scalings have similar exponents to Ohmic plasmas, but with seven times lower threshold at the ITER baseline β_N value of 1.8, and a linear dependence on proximity to the tearing mode β_N limit (~2.2 at zero torque). This reinforces the need to optimize error field correction strategies in ITER, and consider the need for and optimization of plasma rotation.

1. Introduction

The effects of 3-D fields have long been a cause of concern for tokamaks. Such fields, commonly termed “error fields”, naturally arise from asymmetries in device design and construction. Harmonics of the fields can resonate with rational surfaces in the plasma, potentially driving confinement-degrading magnetic islands. However, when the plasma is rotating, the fields are generally shielded out by image currents at the rational surface. But with finite resistivity this interaction generates a torque [1] that reduces the rotation and changes the phase of the imaging response from perfect shielding to enable slight tearing. If the field grows, this torque and consequent phase shift increase, leading to less perfect shielding and further increased torque until a bifurcation point is reached, termed “penetration” – a transition to large scale tearing and braking, usually leading to a stationary “locked” mode and plasma termination [2].

The most performance-limiting of such modes is generally the m/n=2/1 tearing mode (denoting *poloidal/toroidal* mode number). The process was initially thought to pose the greatest risk to low density Ohmic operation, where the rational q=2 surface can be readily stopped due to its low viscosity and weak coupling to the bulk plasma; error field correction systems were designed for ITER on this basis [3]. More recently the ideal shielding response has been generalized to the whole plasma [4], and is then found to drive a kink-like response. The resulting perturbed currents amplify the fields across the plasma, leading to increased sensitivity as the ideal kink β limit is approached [5] and to changes in the structure of the perturbed field at the resonant surface at all β [6]. The consequences of this on tolerable error field levels and structures in high β H-modes were set out in [7], where a reduction in error field tolerance with β [8] was linked to increases in plasma response, particularly as β rose above the no-wall ideal MHD limit. This highlighted the potential for error fields to trigger tearing modes in H modes, particularly at high β or as torque is lowered.

However, from the underlying theory [1] it is clear that further effects associated with the plasma response directly at the resonant surface can play a role. Firstly, if plasma rotation is lowered, then the shielding response will decrease, leading to increased residual tearing and a stronger coupling to the external field. Secondly, if underlying tearing stability is weaker, the partially shielding-suppressed island will be driven to larger amplitude by the error field, again increasing coupling to the external field and likelihood of penetration. These effects will only occur when the shielding response is susceptible to weakening – either at low rotation or with weak tearing stability. However, this turns out to be precisely the regime where we expect future burning devices to operate. These will have low rotation due to their size and low injected torque. Recent studies [9,10] show that lower rotation regimes have weaker intrinsic tearing stability, with 2/1 modes growing out of the noise at lower β_N values. This linkage also raises the prospect of a third mechanism of mode triggering – if electromagnetic braking slows rotation sufficiently to make the plasma intrinsically tearing unstable.

Experiments were executed on DIII-D and NSTX to explore these potential resonant surface effects. In particular, the expected increasing sensitivity to error fields in plasmas with decreased tearing stability or decreased rotation is confirmed on DIII-D (Sec. 2) using n=1 fields applied with the 12 segment “I-coil” set. The way in which the fields act on plasma rotation to change tearing stability is elucidated further in parallel experiments on NSTX (Sec. 3), where an action through rotation braking is determined. Finally, scalings for error field thresholds to trigger 2/1 modes have been obtained for H modes with no injected torque on DIII-D (Sec. 4), indicating thresholds seven times lower than previous Ohmic scalings. Conclusion and implications are discussed in Sec. 5. We start, however, with a discussion of how to calculate the applied fields & measure correction requirements.

1.1 Calculating the Applied Field & Accounting for Harmonic Mix in DIII-D

To predict requirements for future devices, 3-D field thresholds must be couched in terms of tolerable external fields. However, from the above arguments, the total resonant field at a given surface of interest depends strongly on the plasma ideal response. Calculating this with the IPEC code [7,11] for DIII-D plasmas similar to those used here indicate a total resonant field at q=2 of 3.26 G (Gauss)/kA of I-coil current, compared to 1.1 G/kA of vacuum field. The vacuum field is not an appropriate measure of external field, as other harmonics couple through the plasma to make a 2/1 field at q=2. However, other measures of total field amplitude are similarly problematic, as the plasma response will depend on the harmonic content of the external fields. A solution emerges from IPEC, which identifies a single dominant eigenstructure of field at the plasma boundary that generates resonant fields at q=2 [11] – it is the degree to which this component is applied which governs the amount of resonant q=2 field. This can be computed by an overlap integral of the applied boundary field with this dominant eigenstructure, providing a consistent basis to quantify applied fields of different structure and mode trigger thresholds. We therefore adopt this measure here. For the DIII-D I-coils and this particular plasma configuration, this yields a “resonant boundary field strength” of 1.57 G/kA.

Further, when measuring applied fields, such as from the DIII-D I coils discussed in Sec. 2, it is also important to account for any intrinsic error field sources. This can be addressed by representing the applied (I-coil) field and intrinsic error field by equivalent distributions of normal magnetic field on an external reference surface such as the plasma boundary. The intrinsic error field distribution, B_E, can then be divided into two parts: B_E=B_{EN}+B_{EA}. The “non-aligned” part, B_{EN}, is defined to have zero overlap integral with both the I-coil field, B_I, and the aligned part, B_{EA}. The electromagnetic torque, T, from these fields is proportional to $\langle B^2 \rangle$, where $\langle\ \rangle$ denotes the surface integration, the same operation as the overlap integral, and B is the total field. Thus, $T \propto \langle (B_{EN}+B_{EA}+B_I)^2 \rangle$. As by definition $\langle B_{EN}B_I \rangle=0$ and $\langle B_{EN}B_{EA} \rangle=0$, this gives $T \propto \langle B_{EN}^2 \rangle + \langle (B_{EA}+B_I)^2 \rangle$. However, B_{EA} is equal and opposite to the empirically measured I coil field that gives optimal error field correction on DIII-D, $B_{Ioptimal}$. B_{EN} can be determined using density ramp-down studies: as the minimum density to avoid triggering modes is found to be proportional to B [1], a non-zero B_{EN} implies a non-zero density limit even with optimal I-coil correction. Thus the ratio of $B_{EN}/B_{Ioptimal}$ can be deduced from the fractional fall in density limit (–38%) between zero I-coil and optimal error correction. This gives a total torque and equivalent field magnitude that captures the empirical trends with density and I coil current amplitude of the form:

$$T \propto B^2 = (B_I - B_{Ioptimal})^2 + 0.61 B_{Ioptmal}^{\ 2}\ .$$

2. Increased 3-D Field Sensitivity in Torque Free H Modes on DIII-D

Experiments were undertaken on the DIII-D tokamak to explore the sensitivity of low torque H-modes to n=1 3-D fields, utilizing DIII-D's co and counter injecting beams to independently control rotation and β. Typically, discharges were established in H-mode at a particular injected torque and β_N (under neutral beam feedback control), and the 3-D field was then ramped, triggering a 2/1 tearing mode. The DIII-D "I coils" were used to apply these fields – two toroidal rows of six 'picture frame' coils located inside the vessel, above and below the outboard midplane. The fields were applied with a fixed toroidal phase, initially set for optimal correction of the intrinsic error field in DIII-D, before ramping through zero current to progressively increase the net applied field. In the early stages of the field ramp, a low amplitude 10Hz toroidally rotating field was also applied to measure the plasma response, and so its linear stability, to such driving perturbations.

The plasmas used were based on previous ITER-baseline-like H modes with matched shape, β_N and relaxed profiles [9]. These were found to be β_N limited by rotating 2/1 tearing modes, identified to be predominantly Δ′ triggered, with the β_N limit also falling with decreasing rotation, likewise interpreted as a Δ′ effect [9,10]. This is an excellent regime to explore Δ′ and rotation dependencies in 3-D field sensitivity. Parameters were scanned over a wide range of beam torque and β_N values (Fig. 1) up to the natural tearing β_N limit (in red). It is found that even with substantial 3-D field and low torque, the limit still comes from a *rotating* 2/1 mode (e.g. Fig. 2 with 7 Gauss in flux averaged boundary field and just 0.3 Nm beam torque). This indicates a different process from conventional locked mode penetration, with the 3-D field acting to change the underlying tearing stability, most likely through rotation braking.

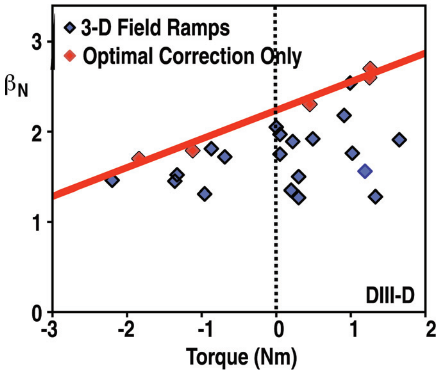

FIG 1. Parameter space explored with DIII-D 3-D field ramps (blue), and natural NTM β_N limits (red).

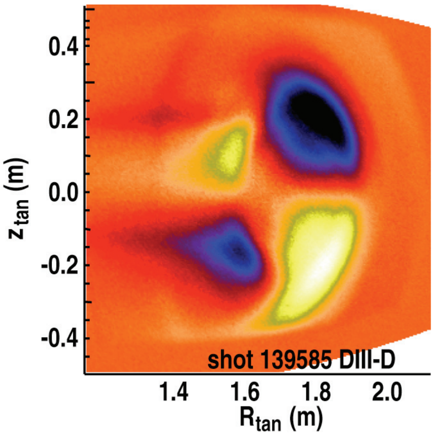

FIG 2. Fourier decomposed fast camera image of rotating 2/1 mode at 1.8kHz triggered by n=1 field.

Field thresholds to trigger modes are plotted in Fig. 3. Note points at the lowest field values, ~1.3 G, represent those obtained with optimal error correction. Results are categorized into 3 bands of applied neutral beam torque (torque rather than rotation is used as the relevant drive that must be overcome to induce braking). This highlights two key trends. For a given beam torque, as the tearing β limit is approached, the required field to induce a mode falls off linearly, reaching the optimum correction level of 1.3 G. Also, for a given β_N, the field threshold falls as torque and rotation (which remains in the direction of the plasma current) fall. The behavior suggests the field interaction may be enhanced by proximity to tearing instability. Indeed, it can be parameterized purely in terms of this proximity: fitting the optimal error correction tearing β_N limits (red points) from Fig. 1 against torque [$\beta_{N\text{-}TM\text{-}limit} = 2.22+0.32T_{NBI}$], it is found that the mode thresh-olds of Fig. 3 scale directly with proximity in β_N to this limit (Fig. 4), with no significant additional rotation dependence or improvement to fits achievable by further fitting.

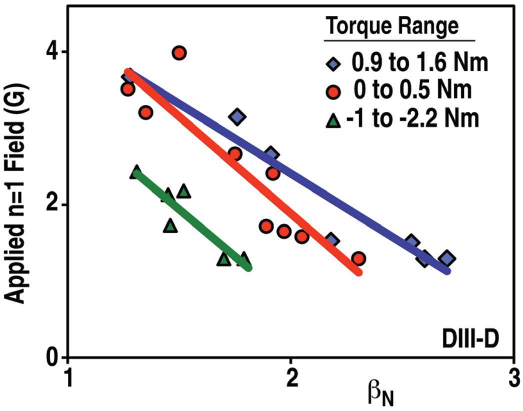

FIG 3. Applied field required to induce 2/1 mode vs plasma parameters.

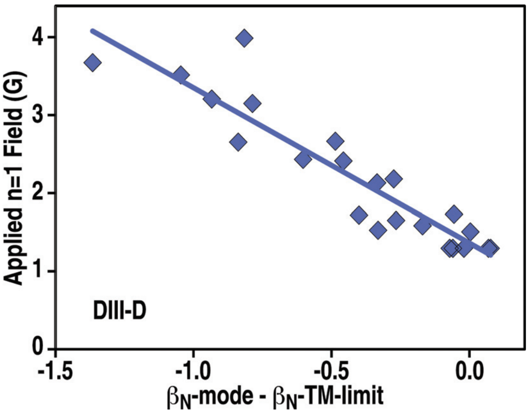

FIG 4. Applied field required to induce 2/1 vs proximity to natural tearing β_N limit.

These results identify an important operational limit, but to understand how to optimize against this limit it is also important to understand the process: Is the response based purely on proximity to tearing β_N limit? Is rotation important to shield out fields? Is tearing stability being decreased or the 3-D field being amplified? Insight is gained by magnetic probing, with a small 10 Hz field. This shows (Fig. 5) plasma response increasing sub-stantially with β_N – thus less external field is needed to cause braking and trigger modes at high β_N, because the plasma amplifies the field more, as in [7]. Also, at constant β_N with values <2, the plasma response rises as rotation (measured in the vicinity of the q=2 surface) falls. However, in contrast to the field threshold behavior above, the field amplification does not correlate well with simple proximity to the fitted neoclassical tearing mode (NTM) β_N limit [Fig. 6(a)]. Instead we see separate dependencies on β_N and rotation [Figs. 6(b,c)], with a regression scaling of $\beta_N^{2.25}\,\Omega^{-0.47}$ giving a better correlation.

The rotation dependence in particular in the above results highlights the resistive response – as shielding weakens at low rotation, this allows greater tearing of the "suppressed" island at the rational surface. Proximity to tearing instability is an important part of this response, as studies away from the tearing β_N limit show no such response [12]. The β_N dependence provides further support to this picture. β_N associated amplification is of course expected, in part, from an increased ideal response of the plasma. However, the dependence appears stronger than expectations from previous magnetic probing results [5], given that these plasmas are well below the ideal β_N limit. Thus we conclude that increases in the plasma *resistive response* to 3-D fields as Δ' stability and rotation shielding are weakened, give rise to a lowering of 3-D field thresholds. Thus raising rotation and improving tearing stability through the current profile may be key factors in optimizing plasma resilience to 3-D fields.

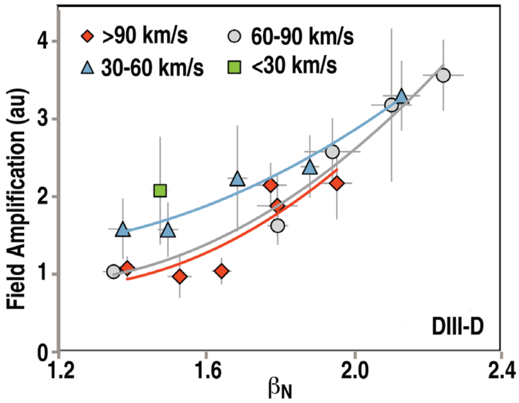

FIG 5. Plasma response to probing field with quadratic regression lines highlight differences.

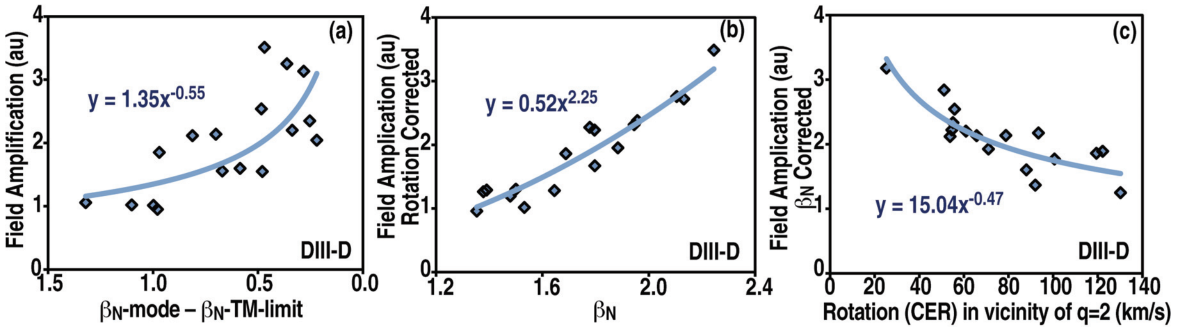

FIG. 6. Plasma response plotted (a) vs proximity to tearing β_N limit, or as function of β_N and rotation (b and c) correcting for other parameter using 2-D regression fit.

3. Understanding the Interaction of 3-D Fields with the Tearing β Limit on NSTX

To provide a consistent basis for extrapolating tolerable error field thresholds, it is important to understand the interaction of the 3-D field with the intrinsic tearing mode instability. Is a resonant field needed? How does braking alter the rotating tearing mode stability? How does this physics connect with the conventional penetration to locked mode?

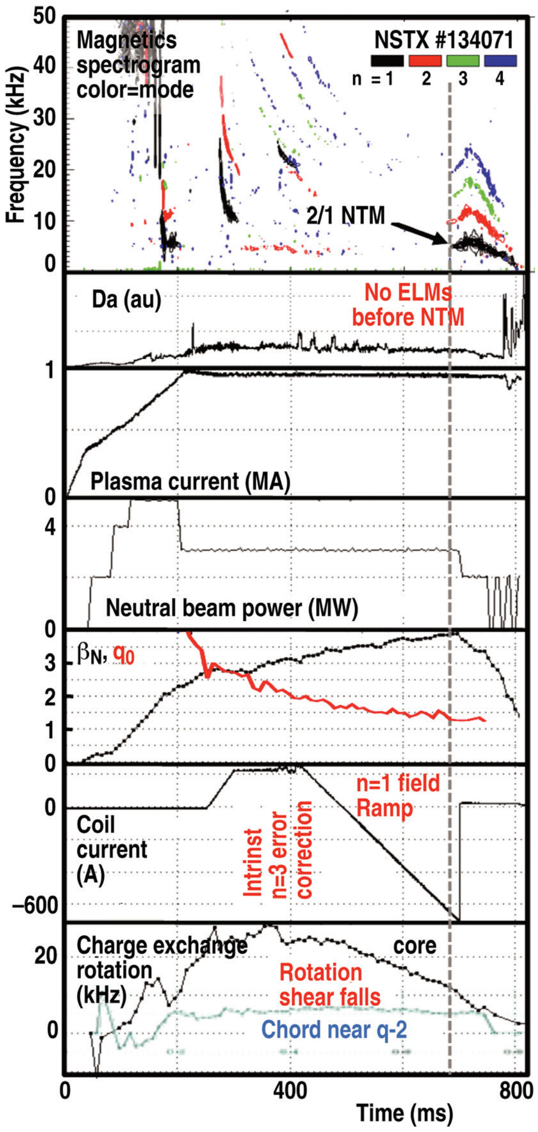

FIG. 7. Typical NSTX discharge run.

NSTX is a useful tool to explore these issues, as its time evolving profiles can help deconvolve rotation from rotation shear effects in NTM stability [13], and it can apply both n=1 and n=3 fields to explore questions of resonance. Here plasmas were established in H mode with steadily rising β as profiles evolved, with central safety factor falling towards unity (Fig. 7). At the start of the discharge, 3-D fields were applied principally to compensate a known n=3 intrinsic error field. Then at 400 ms an additional field ramp was applied comprising of n=1 or n=3 fields, or some fixed ratio thereof. Inter-shot lithium evaporation was used to minimize edge localized modes (ELMs), removing these as potential triggers of the modes.

The resulting mode thresholds are plotted in Fig. 8 in terms of local NTM bootstrap drive (as profiles are evolving, β_N is no longer a good proxy for this). Rotation shear is normalized for inverse Alfvén time and magnetic shear scale length, as proposed in Ref. [10]. This data shows a weak but significant trend with the rotation shear based parameter, while no significant trend in rotation, indicating the 3-D field acting through rotation shear on underlying tearing mode stability, in a similar manner to observations of trends in natural NTM β limits on NSTX [13] and elsewhere [10].

The action of these fields on the plasma rotation is explored in Figs. 9 and 10. Here modes were accessed with different levels of field by varying the field ramp; a faster ramp would trigger the mode earlier in the discharge, at lower NTM drive but with increased applied field. As it is hard to quantify n=1 and n=3 fields on the same basis, we use the simplest measure of amplitude possible: the current in the coils. Figure 9 shows that for both types of field there is a transition from rotating to locked mode formation at similar levels of n=1 or n=3 applied field. Exploring the rotation response through the birth frequency of the triggered mode (Fig. 10), we find both types of field exhibit progressive and similar magnitude braking, with a best fit taking the form: $\Omega_{21}=6958-(2.26I_{n=1}+2.52I_{n=3})^2$. These data suggest the main action of the fields is through braking, either by changing the rotating mode's stability, accessing it at lower drive through decreased rotation shear as in Fig. 8, or by reaching a critical 50% of natural rotation level, when the plasma transitions directly to a locked mode.

The n=3 field effects are surprising, because this fields is non-resonant and should not drive a localized q=2 response that leads to a loss of torque balance and penetration as expected for n=1 fields (Sec. 1). A explanation may lie in the proximity of these plasmas to tearing instability coupled with the lowering of rotation by n=3 fields – this potentially increases the sensitivity of the plasma to any residual low amplitude n=1 field enabling the conventional penetration process, a process also observed in [7]. The similarity of n=1 and n=3 braking effects might also be explained if we consider that the n=1 field will contain both resonant and non-resonant components. As non-resonant braking increases with plasma rotation, while resonant braking falls, it is possible that in the high rotation plasmas used here, the n=1 field acts predominantly through non-resonant braking, until rotation is lowered sufficiently for resonant effects to take over.

From this analysis, a key point emerges: the main action of 3-Ds field on tearing stability for both rotating and locked mode onset is through magnetic braking. Thus, the threshold for mode formation can be considered as one in rotation itself. This is reminiscent of the original Fitzpatrick model [1], which identified a 50% rotation threshold for mode penetration. Here the criterion might be generalized slightly, as less braking is needed to lower the rotating mode threshold. This perhaps provides a common basis for extrapolating mode field thresholds, at least for given profiles and β_N, irrespective of the precise onset mechanism – it is the criteria for achieving significant braking to trigger a mode that matters.

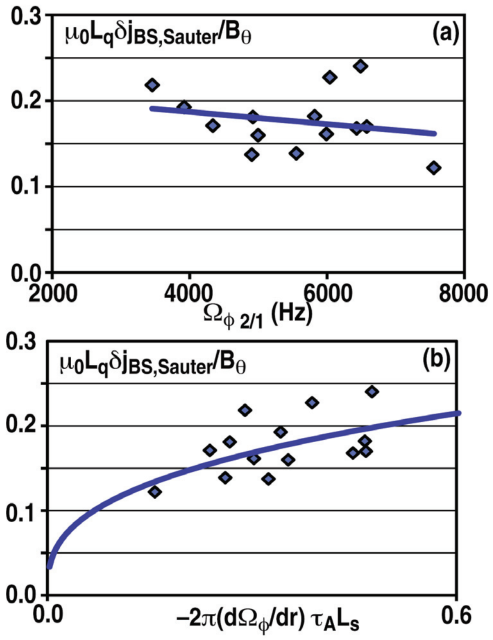

FIG 8. Local q=2 electron bootstrap drive for NTM stability as defined in [10] at NTM onset vs rotation (a) and rotation shear (b) on NSTX as n=1 & n=3 fields varied.

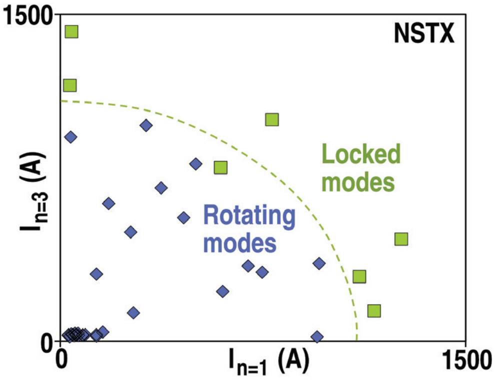

FIG. 9. Occurrence of modes with n=1 and n=3 fields.

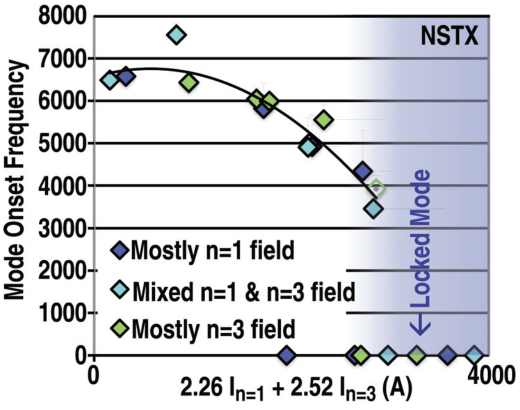

FIG. 10. Braking effect of n=1 & n=3 fields based on optimal correlation 2-D fit as described in text.

4. Scaling of Field Thresholds & Extrapolation to ITER

Having established a common mode threshold mechanism based on rotation braking, it is tempting to attempt an error field threshold extrapolation akin to the approach used for Ohmic regimes [3]. This utilized a dimensional argument to infer machine size scaling from density and toroidal field dependencies. It implicitly included rotation as a hidden variable – a self-generated parameter that plays a key role but is already manifest in the measured threshold scalings, and so included in the extrapolation. However, the origins of plasma rotation are complex, and mechanisms governing rotation in H-mode may be quite different from Ohmic regimes, and so scale differently. Nevertheless, this can be dealt with consistently for ITER, as its relatively low torque injection (predominantly heated by fusion α's and much higher energy beams) allows rotation to again be treated as a self-generated hidden variable, implicit in the scalings. Thus measurements of error field threshold scalings are needed in torque free H-modes, and have been obtained by utilizing the balanced beam capability of DIII-D with discharges as described in Sec. 2, fixing β_N to the ITER baseline value of 1.8 while density and toroidal field (at fixed q_{95}=4.4) were scanned from shot to shot.

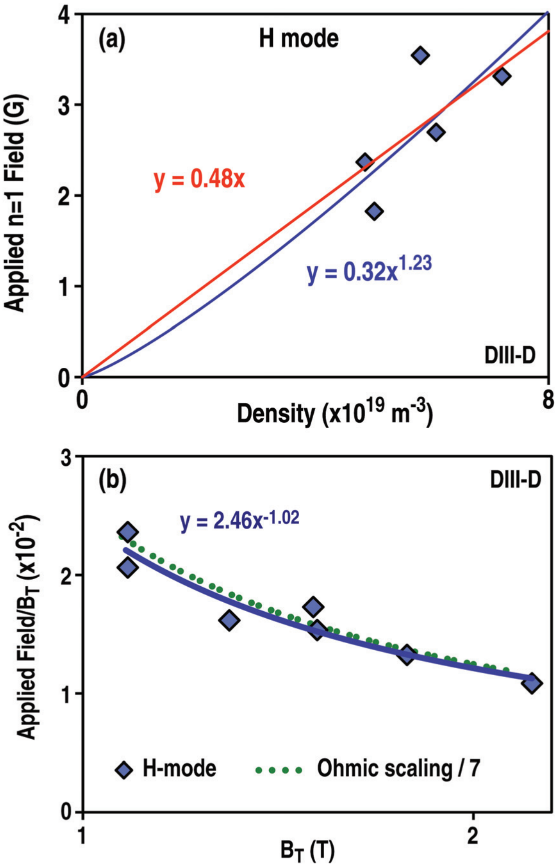

FIG. 11. Density (a) and toroidal field (b) scaling of n=1 mode field threshold in torque free H-modes on DIII-D at constant β_N =1.8 and q_{95}=4.4.

The scalings are plotted in Fig. 11, correcting for slight variations in proximity to β_N limit (from slight β_N and torque variations) using the approach described for Fig. 4. The density scan shows, perhaps, a slightly steeper dependence than the linear scaling found in Ohmic regimes, though this is within error bars of a proportional scaling. The toroidal field is almost identical in exponent to the Ohmic scalings, except that in order to achieve an overlay, the Ohmic scaling has had to be divided by a factor of 7 for the parameters of these plasmas. Using the dimensional constraint of [3] for scaling of the form $B_{pen}/B_T \sim \mathrm{R}^{\alpha R}\,\mathrm{n_e}^{\alpha n}\,\mathrm{B_T}^{\alpha B}$, and assuming linear density scaling (given the sparsity of the data) yields a machine size scaling exponent of $\alpha_R = 2\alpha_n + 1.25\alpha_B = 0.725$. Thus the H mode field threshold scalings are broadly consistent with Ohmic scalings, apart from a lower baseline and increased sensitivity in proximity to the tearing β_N limit. Combining the dependencies with the scalings observed in Sec. 2, an overall scaling for torque free H-modes is obtained:

$$\frac{B_{pen}}{B_T} = \left(1.72 - [\beta_N - 1.8]\right) \times \frac{\left(n_e/10^{20}\,m^{-3}\right)\left(R/6.2m\right)^{0.725}}{\left(B_T/5.3T\right)^{1.02}} \times 10^{-4}$$

where the fit has been couched relative to ITER baseline parameters, and B_{pen} is the q=2 resonant component of RMS flux averaged field at the boundary, as described in Sec. 2.1. Thus a field threshold of 1.4x10^{-4} in DIII-D β_N =1.8 torque free plasmas in Fig. 4 extrapolates to 1.7x10^{-4} in ITER baseline H-modes. This is significantly lower even than the thresholds expected for ITER's low density Ohmic regime – projections for ITER-FDR were

1.25 G/T in terms of vacuum 2/1 field [3], but rose to $1.8\text{x}10^{-4}$ for the final ITER design, equivalent to $2.8\text{x}10^{-4}$ in the measure used here. Further work is needed to confirm density scalings, and test at the lower q_{95} of ITER, but if these do not yield a significant (and somewhat unexpected) improvement to counteract the low absolute levels observed in DIII-D, then this suggests a pressing need to re-evaluate whether ITER's error field correction system can meet the challenge, particularly as it was designed to minimize m=1-3 perturbations, whereas the current understanding [6] is that higher m fields matter most.

5. Conclusions & Implications for Future Devices

New effects have been identified in the interaction of 3-D fields with tokamak plasmas, with the response to such fields enhanced at lower rotation or in proximity to the tearing mode β_N limits. This leads to regimes previously considered robust to the effects of such fields, such as the ITER baseline, becoming highly sensitive to them. Observations suggest this interaction occurs through an increased resistive response. This enables new mechanisms of mode formation, such as through magnetic braking changing the underlying tearing stability, as well as potentially enhancing the conventional error field penetration process. The criterion for mode formation appears related to the degree of braking induced in the plasma. When close to the tearing β limit, decreased rotation shear can destabilize the rotating mode, while at lower β higher levels of field are needed, to stop rotation and drive locked modes directly. New scalings have been obtained for the tolerable level of field in ITER-baseline-like torque-free plasmas at β_N=1.8, though with elevated q_{95} (4.4 cf 3.1 in ITER). These show dependencies similar to Ohmic regimes, but with a much lower absolute value, and with an additional linear dependence on proximity to tearing β_N limit (which itself depends on rotation). Fields have been expressed in terms of the dominant eigenmode that resonates with the q=2 surface, including ideal plasma response, providing a basis for evaluating thresholds from any form of resonant error field, by calculating its overlap integral with the dominant eigenmode for driving a 2/1 response at q=2 [13]. The results highlight the importance of now carefully re-evaluating error field correction capability for ITER. They also have wider ramifications for tokamak regimes, indicating a potential for increased 3-D field sensitivity when plasmas are close to tearing instability, especially with low injected torque, but also the possibility to raise thresholds through increases in rotation or improving underlying tearing stability, for example by tuning the current profile or applying torque.

This work was supported in part by the US Department of Energy under DE-FC02-04ER54698, DE-AC02-09CH11466, DE-FG02-04ER54461, and DE-FG02-07ER54917.

References

[1] FITZPATRICK, R., et al., Phys. Fluids **B** (1991) 644; Nucl. Fusion **33** (1993) 1049
[2] BUTTERY, R.J., et al., Nucl. Fusion **40** (2000) 807
[3] BUTTERY, R.J., et al. Nucl. Fusion **39** (1999) 1827
[4] BOOZER, A.H.A.H. Boozer, Phys. Rev. lett. **86** (2001) 5059
[5] GAROFALO, A.M., et al., Phys. Plasmas **9** (2002) 1997
[6] PARK, J.-K., et al., Phys. Rev. Lett. **99** (2007) 195003
[7] REIMERDES, H., et al., Nucl. Fusion **49** (2009) 115001
[8] LA HAYE, R.J., et al., Nucl. Fusion **32** (1992) 2119
[9] BUTTERY, R.J., et al., Phys. Plasmas **15** (2008) 056115
[10] LA HAYE, R.J., et al., Phys. Plasmas **17** (2010) 056110
[11] PARK, J.-K., et al., this conference, EXS/P5-12
[12] REIMERDES, H., this conference, EXS/5-4
[13] GERHARDT, S.P., et al., Nucl. Fusion **49** (2009) 032003