Scaling of Density Peaking for Plasma with Pellet Injection

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Abstract. This work aims to investigate the plasma behaviors during pellet injection in ITER plasma using 1.5D BALDUR integrated predictive modeling code together with the Neutral Gas Shielding (NGS) pellet ablation model, with grad-B drift effect is included. The result from the simulation shows that the density increases but the temperature decreases significantly after the pellet injection is used. A scaling of electron plasma density peaking resulting from pellet is developed based on the simulation results. To determine relevant parameters the multiple regression analysis is utilized in order to obtain those numerical values concerning the plasma electron density peaking, then the scaling of density peaking with pellet injection can expressed as follows:

$$\frac{n_e}{n_l} = 0.544 \cdot r_p^{0.306} v_p^{0.215} I_p^{-0.013} M_p^{0.024}$$

when ,n_l , I_p , r_p, v_p , M_p are the plasma electron density, line average plasma electron density, the effective spherical pellet radius, the pellet velocity, the atomic mass number of the ablatant, respectively.

1. Introduction

The injection of pellets into fusion plasmas has recently gained significant interest due to at least two reasons. First, pellets are considered to be suitable for fueling of reactor plasmas, and, second, pellets are used to mitigate the Edge Localized Modes (ELMs) and thus reduce the power load on the divertors. To fully understand the mechanisms of these processes, the proper knowledge of the profile of the material deposited by pellets is of crucial importance. It has been observed that the predictions of the International Thermonuclear Experimental Reactor (ITER) using different integrated predictive modeling codes that the density profile in ITER is flat or relatively small peaking, which results in pessimistic nuclear fusion performance in some predictions. The pessimistic performance causes a concern in the fusion community. If the central plasma density in ITER is increased, the fusion performance will also increase. In general, peaking the plasma central density can be achieved by several methods, such as pellet injection, which has been successfully developed and widely used in many existing tokamak experiments. Density peaking using pellet injection has many advantages compared to other methods; such as the penetration depth into plasma core and the enhancement of impurity. However, density peaking with pellet injection yields a complicated plasma scenario since the pellet's interactions with the plasma involve many complicated physical processes. As a result, it is essential to investigate the behavior of pellet in ITER plasma in order to properly use the pellet to enhance plasma and fusion performance in ITER. It has been proposed from many tokamaks that pellet injection from the high field side can penetrate deep into the plasma core than injection from the low field side due to grad-B effect. Owing to ITER tokamaks has a large size of minor radius then grad-B effect would be significant impact when pellet is injected.

2. BALDUR Integrated Predictive Modeling Code

The BALDUR integrated predictive modeling code [1] is a multifluid transport code which solves time-dependent 1.5-dimensional radial diffusion equations for plasma profile.

2.1 Core Plasma Transport Model

BALDUR transport code can be described by a linear combination of neoclassical and anomalous transport. The anomalous transport (MMM95 model) is a linear combination of theory-based transport models which consists of the Weiland model for the ion temperature gradient (ITG) and trapped electron modes (TEM), the Guzdar–Drake model for drift-resistive ballooning modes, as well as a smaller contribution from kinetic ballooning modes. The Weiland model for drift modes such as ITG and TEM modes usually provides the largest contribution to the MMM95 transport model in most of the plasma core. The Weiland model is derived by linearizing the fluid equations, with magnetic drifts for each plasma species. Eigenvalues and eigenvectors computed from these fluid equations are then used to compute a quasilinear approximation for the thermal and particle transport fluxes. The Weiland model includes many differengnt physical phenomena such as effects of trapped electrons, $T_i \neq T_e$, impurities, fast ions, and finite β. The resistive ballooning model in MMM95 transport model is based on the 1993 *ExB* drift-resistive ballooning mode model by Guzdar–Drake, in which the transport is proportional to the pressure gradient and collisionality. The contribution from the resistive ballooning model usually dominates the transport near the plasma edge. Finally, the kinetic ballooning model is a semi-empirical model, which usually provides a small contribution to the total diffusivity throughout the plasma, except near the magnetic axis. The electron and ion diffusion coefficients can be written as the sums of the components as follows

$$\chi_e = \chi_e^{NEO} + \chi_e^{ITG/TEM} + \chi_e^{RB} + \chi_e^{KB} \tag{1}$$

$$\chi_i = \chi_i^{NEO} + \chi_i^{ITG/TEM} + \chi_i^{RB} + \chi_i^{KB} \tag{2}$$

2.2. Pellet Ablation and Pellet Drift Models

A pellet injection module is integrated with the core transport code to calculate pellet ablation rates and drift displacements in the major-radius direction. A neutral gas shielding model [2] is applied to calculate the ablation rates of the injected pellet during its passage through the background plasma, and the ablation rate is expressed in terms of power functions as follows:

$$\frac{dN}{dt} = 1.12 \times 10^{16} n_e^{0.333} T_e^{1.64} r_p^{1.33} M_i^{-0.333} \tag{3}$$

where T_e and n_e are the electron temperature in eV and the electron density in cm^{-3}, respectively, r_p is the pellet radius in cm and M_i is the atomic mass of pellet material (M_i = 2 and 3 is used for deuterium in this modeling). There exist several models to calculate the pellet drift motions, such as the simplified MHD pellet displacement model [3] and the PRL model [4]. In BALDUR, the NGS [2] scaling model which is based on the grad-B induced pellet [5] is taken into account as a pellet drift model to calculate the drift displacement □drift of the ablated plasma particle:

$$\Delta_{drift} = c_1 v_p^{c_2} r_p^{c_3} n_{e0}^{c_4} T_{e0}^{c_5} (\left\| \theta \right| - c_6 \right| + c_7)^{c_8} \times (1 - \Lambda)^{c_9} a_0^{c_{10}} R_0^{c_{11}} B_t^{c_{12}} k^{c_{13}} \tag{4}$$

where n_{e0} and T_{e0} are the electron density and temperature at the plasma core axis, respectively. This drift model scales the drift displacement drift with pellet velocity v_p, injection angle θ, minor radius a_0, major radius R_0, toroidal magnetic field strength at the axis B_t and plasma elongation κ. The impact parameter Λ of the pellet trajectory is normally ignored in the drift calculation. The coefficients and power parameters c_s (s = 1, 2,..., 13) in equation (4) are described in [4].Using the two models described above for the pellet injection module, the deposition of pellet fuel can be estimated with the background plasma profiles. The 1.5 dimensional equilibrium configurations of the background plasma are calculated by an equilibrium solver coupled with the core transport code. In the numerical calculation, cell positions, pellet path lengths between neighboring cells, and magnetic field strength are set as the initial conditions before injecting a pellet. The path and velocity of the pellet that determine its spending time in each cell are assumed to be straight and constant, respectively, during its penetration through the plasma. The density and temperature profiles of the background plasma are self-consistently calculated by coupling the pellet model with the core transport code. Once a pellet enters the plasma, the pellet ablation rate and the drift displacement at each cell are computed by equations (3) and (4). The time evolution of the pellet ablation rate is governed by the fourth-order Runge–Kutta method.

3. Results and Discussion

Self-consistent modeling of discharges with NGS pellet ablation model and grade-B drift in the ITER tokamak, is carried out using BALDUR code [1] yielding the time evolution of temperature, density and current profiles. The simulations are carried for the standard H-mode scenario discharges with pellet injection from HFS (R = 6.2 m, a =2.0 m, κ_{95} = 1.85, δ_{95} = 0.3, B_T = 5.3 T, Z_{eff} = 1.4) with variation of the relevant parameters.

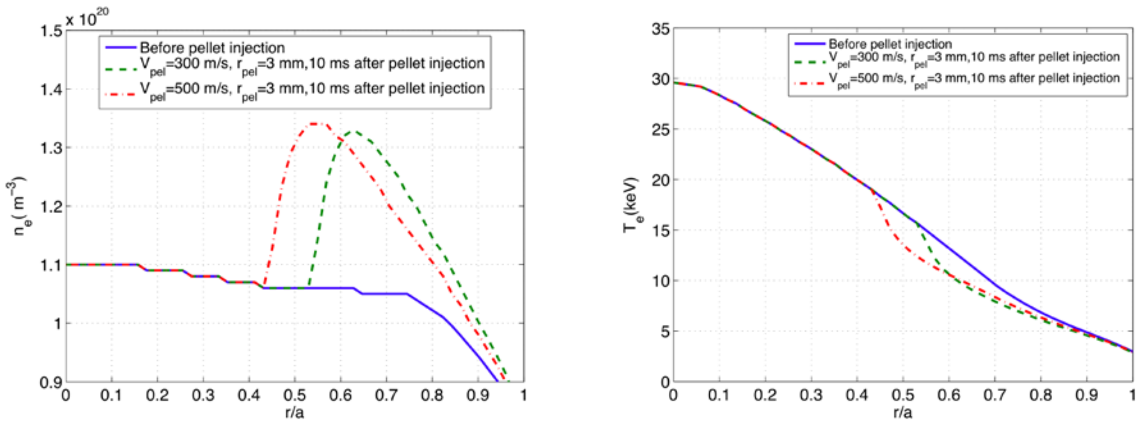

FIG. I. Plasma electron density (left) and plasma electron temperature (right) as functions of the normalized minor radius. Two pellet velocities (300 and 500 m/s) are used.

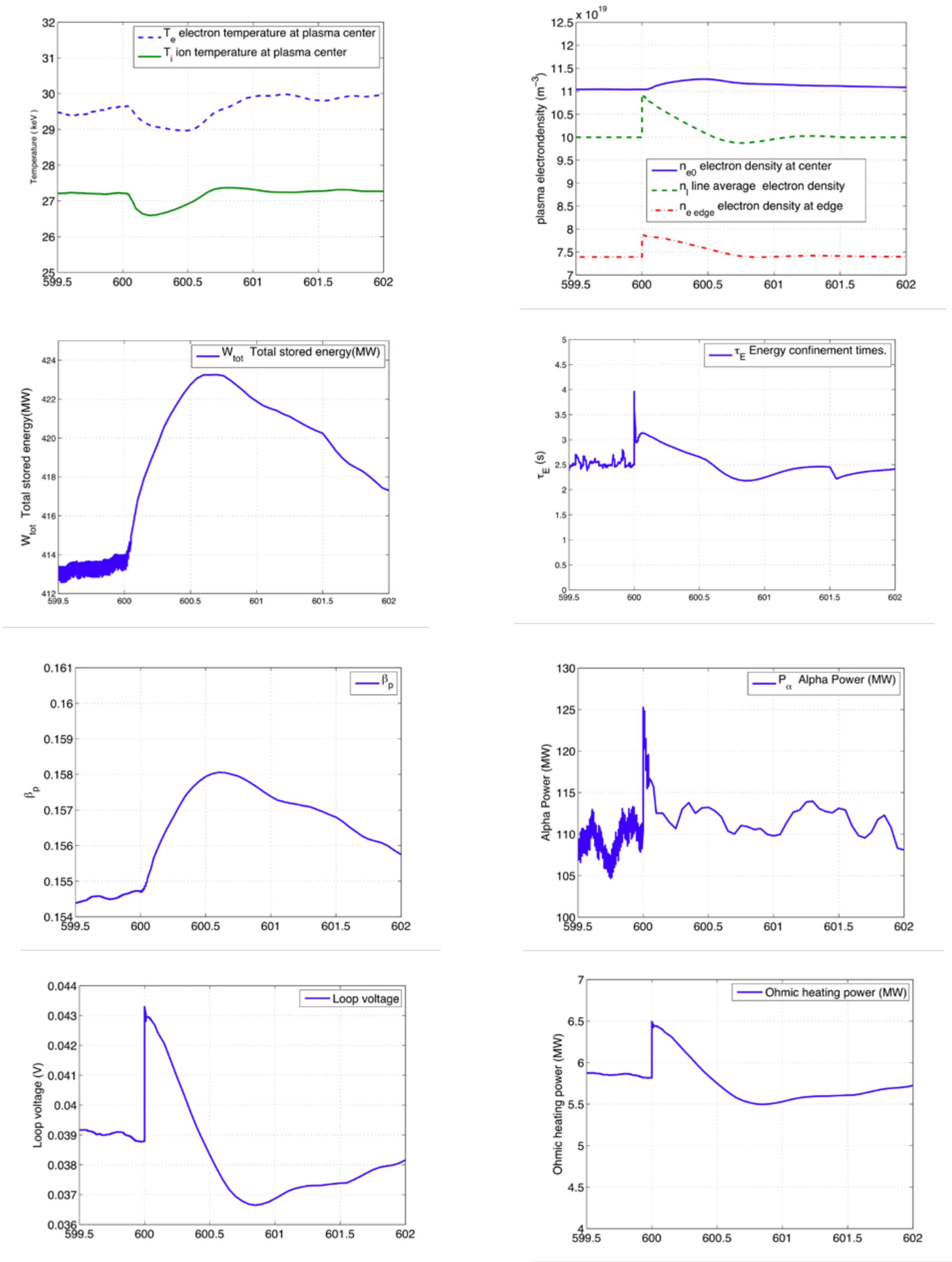

FIG. 2. (From left to right, top to bottom) Time evolution of plasma temperature, plasma density, total stored energy, energy confinement time, poloidal beta, alpha power, loop voltage, and ohmic heating power. The pellet is injected at t = 600 s.

Profiles obtained from the simulation in *FIG.I* show that the temperature decreases significantly after the pellet injection due to the dilution of hot plasma electrons and ions with cold gas from the evaporating pellet. In FIG II, electron temperature decreases and recovers to the pre-injection level in 0.5 sec. The density increases after the pellet injection and returns to the original value in 0.5 sec. The plasma stored energy and energy confinement time also increases. Polaroid beta (β_p) shown in *FIG.II* for a pellet injection the values and time histories of β_p obtained this way are in good agreement with calculations of total plasma energy from T_e and n_e profiles. The loop voltage increases by about 5% after injection and then falls to or below its previous values in 0.5 sec. Ohmic heating power is important parameter which relating to pellet injection due to when pellet is injected plasma temperature is decreased then ohmic heating power ($P_{\Omega}=\eta j^2$) increasing. Finally the goal of nuclear fusion is improve fusion Q from FIG.II alpha power is increased after pellet injection it mean to fusion Q also increases (Q= $(5\times P_{\alpha})/P_{aux}$).

4. Multivariable Statistical Analysis

In order to further improve our understanding of the pellet ablation physics and to facilitate future scaling studies for the next generation of tokamak a scaling of electron plasma density peaking with pellet injection is developed based on the simulation results, the resulting is used analysis with the statistically and the following form is assumed

$$\frac{n_e}{n_l} = C\cdot\ r_p^a v_p^b I_p^c M_p^d \tag{5}$$

where n_e, n_l , I_p, r_p, v_p, *and* M_p are the plasma electron density, line averaged plasma electron density, the plasma current, the effective spherical pellet radius, the pellet velocity, and the atomic mass number of the ablatant, respectively. It is possible to linearize the above equation by taking the natural log of both sides, thus yielding an equation whose coefficients can be determined through a multiple linear regression technique.

4.1 Multiple Linear Regressions

Multiple linear regression is used for studying the relationship between several predictor variables and a response variable. The model is of the form

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon \\ \varepsilon &\sim N(0,\sigma^2) \end{aligned} \tag{6}$$

where β_0 is the y-intercept and the parameters β_i, i=1, 2, .., k, are called partial coefficients and ε is the error term. The error term represents the unexplained variation in the dependent variable. The mean of the random variable ε is assumed to be zero.
To fit the model, we assume that the residuals

$$res_i = y_i - (\beta_0 + \beta_1 x_1^2 + \beta_2 x_2^3 + \ldots + \beta_k x_k^2) \tag{7}$$

follow the normal distribution with the mean equal to 0 and the variance equal to σ_i^2. Then the maximum likelihood estimates of the parameter β_i can be obtained by minimizing the chi-square,

$$\chi^2 = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{\sigma_i^2} \tag{8}$$

The parameters are estimated using a weighted least-square method. After fitting, the model can be evaluated using hypothesis tests. If the regression assumptions hold, the t-tests can be performed for the regression coefficients with the null hypotheses and the alternative hypotheses:

$$H_0: \ \beta_j = 0$$
$$H_\alpha: \ \beta_j \neq 0 \tag{9}$$

The t-values can be computed as:

$$t = \frac{\hat{\beta}_j - 0}{s_{\hat{\beta}_j}} \tag{10}$$

With the *t*-value, we can decide whether or not to reject the corresponding null hypothesis. Usually, for a given confidence level α, we can reject H_0 when $|t| > t \ \alpha/2$. The probability that H_0 in the *t* test above is true (Prob>|*t*|) can be computed as

.
$$prob = 2(1 - tcdf(|\ t\ |, df_{Error})) \tag{11}$$

Where *tcdf*(*t*, *df*) computes the lower tail probability for the Student't *t* distribution with *df* degree of freedom. From the *t*-value, we can calculate the (1-α)$\times$100% confidence interval for each parameter by:

$$\hat{\beta}_j - t_{(\alpha/2, n^*-k)} \varepsilon_{\hat{\beta}_j} \le \hat{\beta}_j \le \hat{\beta}_j + t_{(\alpha/2, n^*-k)} \varepsilon_{\hat{\beta}_j} \tag{12}$$

The Confidence Interval Half-Width can be computed as

$$CI = \frac{UCL - LCL}{2} \tag{13}$$

where UCL and LCL is the Upper Confidence Interval and Lower Confidence Interval, respectively. The relevant input parameters for the simulations is shows in TABLE I.

TABLE I: SIMULATION PARAMETERS FOR OHMIC, NBI, AND ICRH PLASMAS.

Parameters	**Values**
Pellet radius r_p	2, 3, 4 mm.
Pellet velocity v_p	0.1, 0.2, 0.3, 0.4, 0.5 m/s
Plasma current I_p	12, 15, 18 MA
Pellet species M_i	2 and 3 amu
Heating Power P_{heat}	35, 40, 45, 50, 60 MW

After fitting the model and evaluated using hypothesis tests, the fit statistics are summarized in TABLE II and the results of the regression analysis show in TABLE III.

TABLE II: SUMMARY OF STATISTICS DATA.

Number of Points	1350
Degrees of Freedom	1345
Residual Sum of Squares	9.75919
R value	0.9484
R- Square	0.89946
Adj. R-Square	0.89916
Root-MSE(SD)	0.08518
Norm of Residuals	3.12397

TABLE III: RESULTS OF REGRESSION ANALYSIS OF THE SIMULATION DATA WITH NGS MODEL AND GRAD-B DRIFT SCALING.

		Value	**Standard Error**	***t*-Value**	**Prob >|*t*|**	**95% LCL**	**95% UCL**	**CI Half-Width**
n_p/n_l	Intercept	0.544	0.020	26.033	0	0.503	0.585	0.041
	R_p	0.306	0.002	107.785	0	0.300	0.3116	0.005
	v_p	0.215	0.016	13.167	0	0.248	0.2480	0.032
	I_p	-0.013	9.464E-4	-14.589	0	-0.011	-0.011	0.001
	M_i	0.024	0.004	5.337	0	0.033	0.033	0.009

The detail of statistical parameters and regression analysis can be found in [6]. From TABLE III the regression analysis for the database shows remarkably good agreement between the peak density and pellet radius. If the pellet radius is increased, the peak density increases. The velocity also increases. When the plasma current is increased, the peak density decreases .When the pellet species is changed from deuterium to tritium the plasma peak density increases. For predict the plasma peak density to facilitate future studies for the next generation of tokamak a scaling of electron plasma density peaking with pellet injection can be written in the following form

$$\frac{n_e}{n_l} = 0.544 \cdot r_p^{0.306} v_p^{0.215} I_p^{-0.013} M_p^{0.024} \tag{14}$$

where n_e, n_l , I_p ,r_p, v_p , M_p are described in Eq. 5.

5. Conclusion

A scaling of electron plasma density peaking with pellet injection is established with BALDUR simulation results using NGS model with grad-B drift effect included. These simulations cover a wide range of plasma operating space and pellet parameters. A statistical analysis of the data has been conducted on the plasma electron density, line average plasma electron density, the effective spherical pellet radius, the pellet velocity, and the atomic mass number of the ablatant. Plasma peak density scaling is established in term of engineering scaling law. Then scaling of electron plasma density peaking with pellet injection is possible to use in predicting the performance of future fusion devices.

6. References

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