Kinetic Global Analysis of Alfvén Eigenmodes in Toroidal Plasmas

A. Fukuyama 1), T. Akutsu 1)

1) Department of Nuclear Engineering, Kyoto University, Kyoto, Japan

email contact of main author: fukuyamanucleng.kyoto-u.ac.jp

Abstract. Systematic study on low to medium n (toroidal mode number) Alfvén eigenmodes (AE) in tokamaks has been carried out. Linear stability of AE in the presence of energetic ions was studied using the kinetic full-wave code TASK/WM. We have reproduced the destabilizing effect of toroidal co-rotation on TAE for JT-60U parameters. We have found the existence of reversed-shear-induced Alfvén eigenmode (RSAE) which localizes near the q minimum in a reversed magnetic shear configuration. Two kinds of mode structures are identified for energetic particle mode (EPM) below the TAE frequency gap. For a helical plasma, the existence of GAE in the central region and TAE in the off-axis region was confirmed.

1. Introduction

Various low-frequency modes excited by energetic ions have been observed experimentally in tokamak plasmas. Most of them are attributed to the Alfvén eigenmodes (AE) which have been extensively explored theoretically and numerically [**?**]. The linear stability and the nonlinear effect on energetic ion confinement are key issues of the analyses. Recent stability analyses [**?**,**?**,**?**] have revealed the importance of mode-conversion to the kinetic Alfvén waves and the sensitive dependence on profiles of safety factor, edge density, rotation velocity and energetic ion pressure. Therefore, careful study of AE in present-day tokamaks, next step devices and fusion reactor is indeed required.

In this paper, we present the results of systematic study of low to medium n (toroidal mode number) AE using the kinetic full-wave code TASK/WM [**?**] and analyze the coupling to lower-frequency modes such as drift waves and MHD modes. In this code, Maxwell's equation with kinetic dielectric tensor is solved in magnetic flux coordinates. The response of energetic ions is calculated from the drift kinetic equation. With poloidal and toroidal mode expansion, parallel wave number is correctly taken into account to describe the wave-particle resonant interaction. Eigenmode with complex wave frequency is obtained by maximizing the wave amplitude for given external current proportional to the plasma density.

2. Effect of Toroidal Rotation

The effect of rotation on the excitation of toroidicity-induced AE (TAE) has been observed on JT-60U [**?**]. When the plasma rotates in the toroidal direction with velocity u, the TAE frequency shifts downward as

$$\omega = \frac{v_{\rm A}}{2qR}\left(1 - \frac{u^2}{v_{\rm A}^2}\right) \tag{1}$$

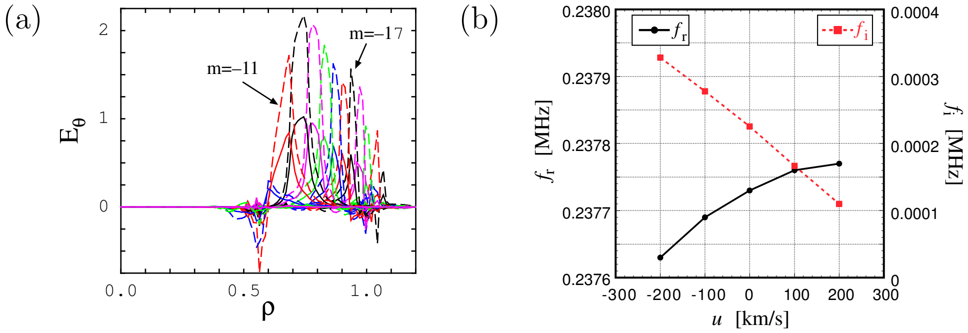

FIG. 1. (a) Radial structure of eigenmode with $n = 7$. (b) Rotation velocity dependence of eigen frequency and damping rate.

where $v_{\rm A}$ is the Alfvén velocity, q is the safety factor and R is the major radius. Since this frequency reduction is usually very small, the order of 10^{-4}, dominant frequency shift comes from the change of resonant q,

$$q = -\frac{m + 1/2}{n} - \frac{1}{2n}\frac{u}{v_{\rm A}} \tag{2}$$

for toroidal and poloidal mode numbers n and m. We have examined the case for JT-60U parameters: $R = 3.4\,\rm m$, minor radius $a = 1.1\,\rm m$, elongation $\kappa = 1.2$, triangularity $\delta = 0.3$, toroidal magnetic field $B_0 = 3.8\,\rm T$, plasma current $I_{\rm p} = 3\,\rm MA$, and electron density $n_{\rm e}(0) = 0.3 \times 10^{20}\,\rm m^{-3}$.

FIG. 1(a) shows the eigen mode structure of the $n = 7$ TAE mode for $m = -21 \sim -7$. The eigen frequency linearly depends on u. The damping rate shown in FIG. 1(b) indicates the destabilizing effect of co-rotation as is observed on JT-60U [?]. We should note that the dependence on u is very sensitive to the damping at the Alfvén resonance near the plasma edge.

3. AE in Reversed Magnetic Shear Configuration

In a tokamak plasma with a reversed magnetic shear, the damping rate of TAE is usually enhanced owing to the short distance from the Alfvén resonance. In a certain range of $q_{\rm min}$, however, weakly damped AE exists near the lower and upper ends of the frequency gap [?].

In order to elucidate the effect of reversed shear, we utilized a simple configuration; circular cross section, flat profiles of the density and temperature and spline q profile. The parameters simulating the present-day tokamaks are major radius, $R_0 = 3\,\rm m$, minor radius, $a = 1\,\rm m$, wall radius, $b = 1.2\,\rm m$, toroidal field, $B_0 = 3\,\rm T$, electron density, $n_{\rm e} = 10^{20}\,\rm m^{-3}$, electron and ion temperatures, $T_{\rm e} = T_{\rm i} = 3\,\rm keV$, ion species, deuterium, central safety factor, $q(0) = 3$, edge safety factor, $q(a) = 5$, and radius of $q_{\rm min}$, $r_{\rm min}/a = 0.5$.

The radial structure of the Alfvén continuum frequency is sensitive to the value of $q_{\rm min}$ as shown in FIG. 2. When $q_{\rm min} = 2.4$, two frequency gaps exist near the radii with $q = 2.5$. They come close to each other with the increase of $q_{\rm min}$ and are merged at $q_{\rm min} = 2.5$. Further increase of $q_{\rm min}$ widens the frequency gap.

In the case of $q_{\rm min} = 2.4$ shown in FIG. 3(a), the eigenmode with least damping rate has

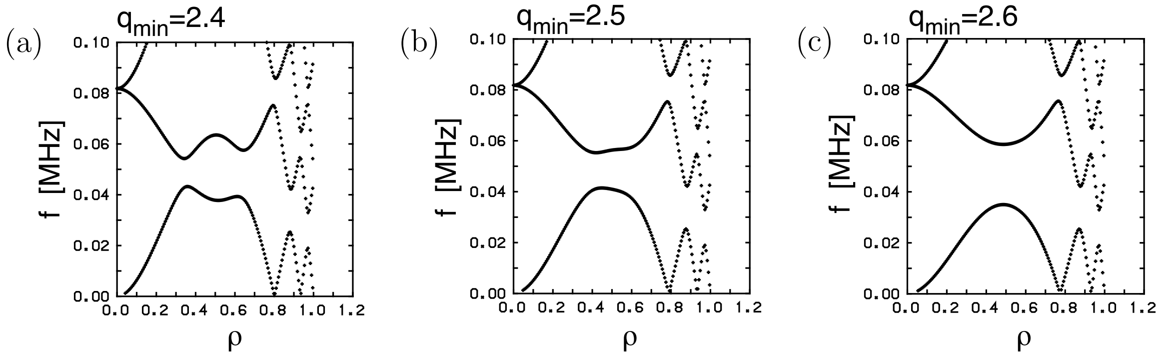

FIG. 2. Radial dependence of the Alfvén resonance frequency for (a) $q_{\rm min} = 2.4$, (b) $q_{\rm min} = 2.5$, and (c) $q_{\rm min} = 2.6$.

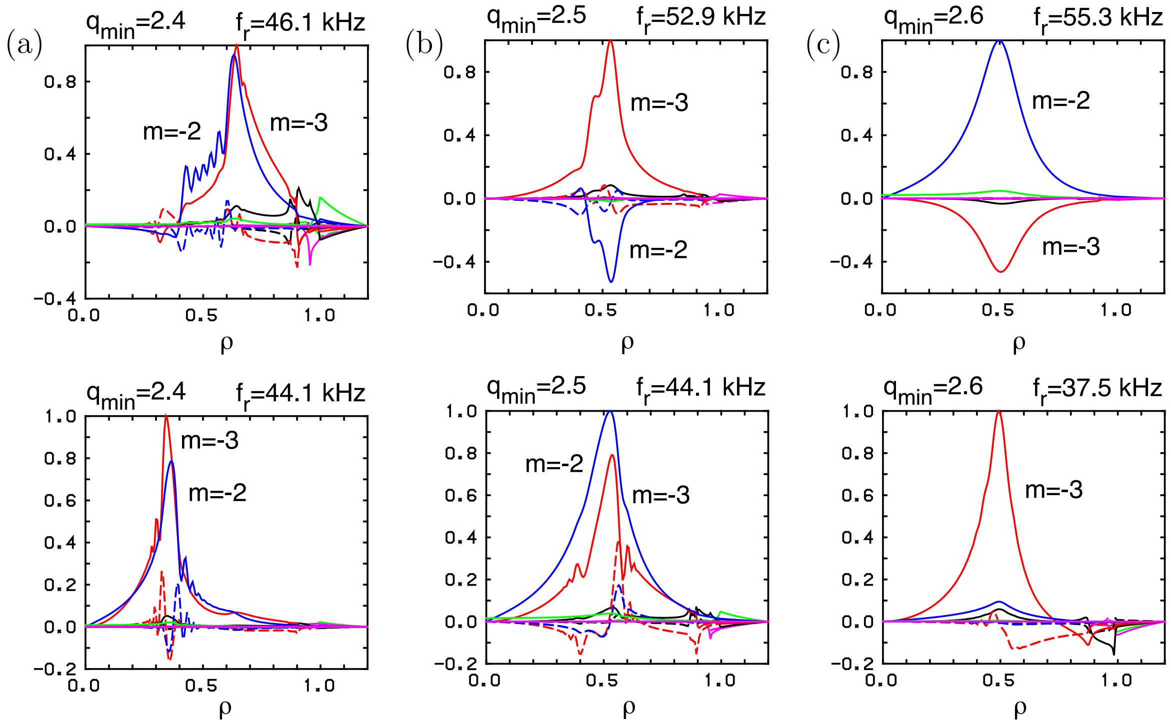

FIG. 3. Radial structure of eigen modes for (a) two TAEs with slightly different frequencies for $q_{\rm min} = 2.4$, (b) higher frequency RSAE and merged TAE for $q_{\rm min} = 2.5$, and (c) higher and lower frequency RSAE for $q_{\rm min} = 2.6$.

both $m = -2$ and $m = -3$ components with comparable amplitudes (FIG. 3(a)). This behavior of TAE disappears for $q_{\rm min} = 2.6$ shown in FIG. 3(b) and (c). There are two eigenmodes with a higher frequency, 55.3 kHz, and a lower frequency, 37.5 kHz. Since the $m = -2$ component is dominant for the high frequency mode and the $m = -3$ component for the low frequency mode, these are irrelevant to the poloidal mode coupling due to the toroidal effect.

FIG. 4 indicates the $q_{\rm min}$ dependence of the eigen frequency. In a tokamak plasma with reversed magnetic shear, TAE with relatively large damping rate exists for $\ell < q_{\rm min} < \ell + 1/2$ (ℓ: integer) and weakly damped AE for $\ell + 1/2 < q_{\rm min} < \ell + 1$. The latter, which we call reversed-shear-induced AE (RSAE), is similar to the global Alfvén eigenmode (GAE), but is localized near the location of $q_{\rm min}$. The frequency of RSAE is very sensitive to $q_{\rm min}$

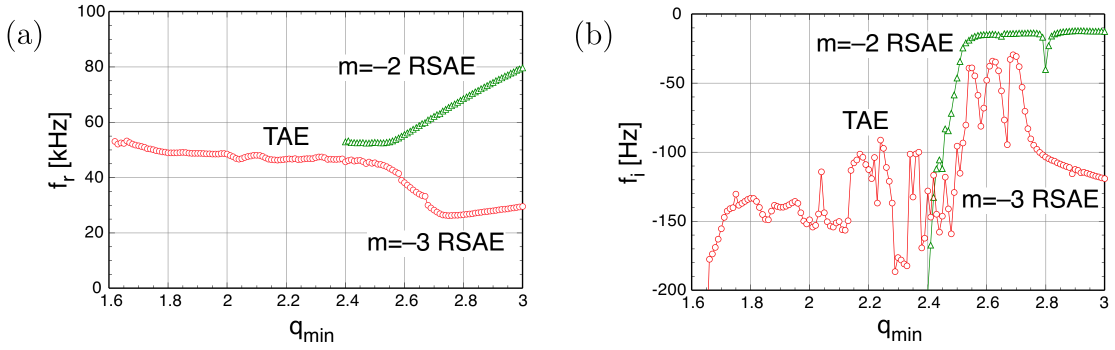

FIG. 4. $q_{\rm min}$ dependence of eigenmode frequency for tokamak plasma with reversed shear. (a) Real frequency and (b) damping rate.

and this behavior explains the slow frequency sweep observed on JT-60U [?].

We have examined the destabilization by energetic ions. Linear increase of the imaginary part of the wave frequency was observed with the increase of energetic ion pressure. The excitation depends on the location of steepest pressure gradient. The mode structure is also strongly affected by the profiles of density and safety factor near the plasma edge through the gap structure of the Alfvén resonance frequency. Dependences on other parameters, central q, density jump at the internal transport barrier and energetic ion density, were also studied.

4. Energetic Particle Mode (EPM)

Energetic ions with a large pressure gradient can excite EPM with frequency below the TAE frequency gap (FIG. ??). With β of energetic ions about 0.5%, modes in the range between 40 kHz and 80 kHz become unstable for $n = 1$. The contour of wave amplitude shown in FIG. ??(b) indicates the location of eigen frequency on complex frequency space. We found that there are two kinds of modes in this frequency range. The most unstable mode A is purely $m = -1$ and localizes within $m = -1$ Alfvén resonance surface. The mode B is similar to the mode A, while the mode C couples with $m = -2$ and other modes like TAE and is stable.

In the low frequency range, the eigen frequency becomes comparable to drift frequency, bounce frequency and toroidal precession frequency. Taking account of the particle orbit in a real magnetic geometry, we have formulated the kinetic dielectric tensor. The eigenmode analysis with this dielectric tensor will enable us to study the coupling to drift Alfvén waves and fishbone instabilities which have been studied mostly with MHD model.

5. AE in Helical Plasmas

Using three-dimensional MHD equilibrium, we have analyzed AE in toroidal helical plasmas. The existence of GAE in the central region and TAE in the off-axis region was confirmed. The analysis of AE with helical mode coupling requires careful selection of poloidal modes for computation. The result of eigenmode analysis was compared with those of previous ideal MHD analyses.

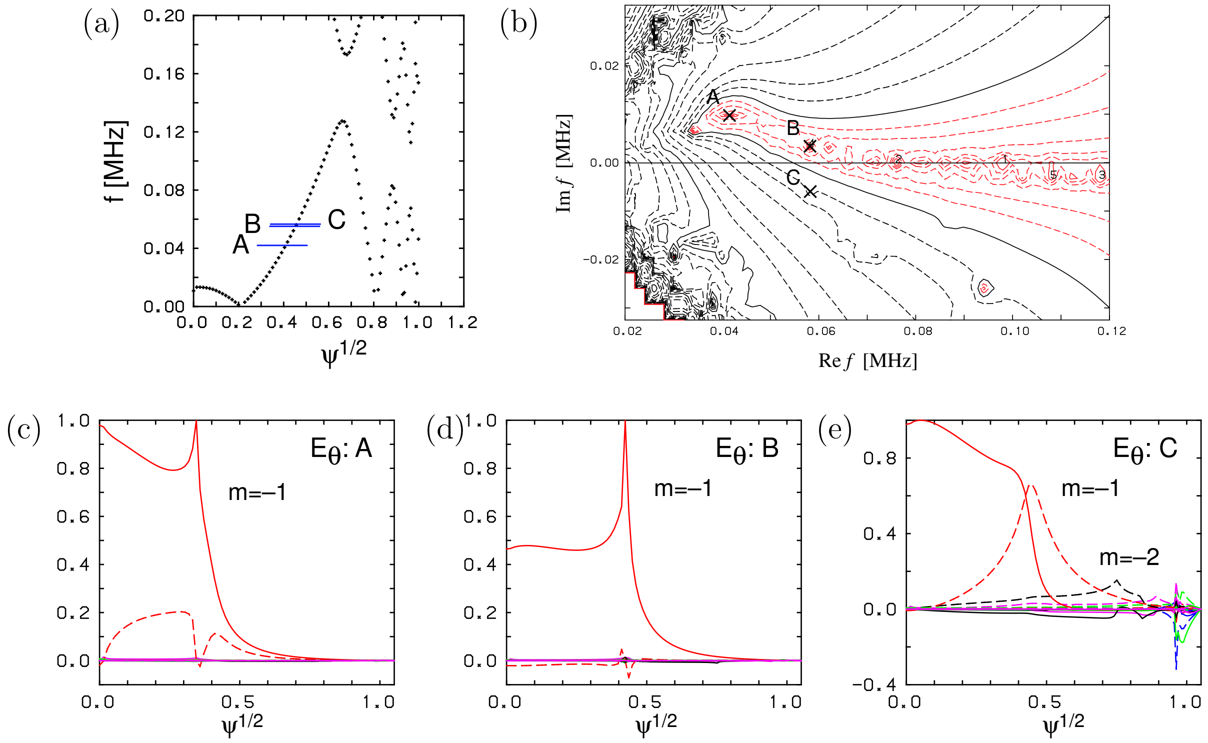

FIG. 5. (a) Alfvén resonance frequency as a function of radius and the frequencies of EPM eigenmodes. (b) Contour of wave amplitude in complex frequency space. Radial eigen function of (c) unstable mode A: $f_\mathrm{r} = 41.8\,\mathrm{kHz}$, $f_\mathrm{i} = 8.1\,\mathrm{kHz}$, (d) unstable mode B: $f_\mathrm{r} = 57.7\,\mathrm{kHz}$, $f_\mathrm{i} = 3.6\,\mathrm{kHz}$, and (c) stable mode C: $f_\mathrm{r} = 58.0\,\mathrm{kHz}$, $f_\mathrm{i} = -6.0\,\mathrm{kHz}$

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