EFFECT OF ROTATION ON IDEAL AND RESISTIVE MHD MODES*

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Abstract

Experimental observations suggest that toroidal plasma rotation can play a crucial role in determining the stability of a tokamak. Conventional MHD stability analysis ignores rotation. Results from analytic and numerical analysis of a number of tokamak stability issues including toroidal plasma rotation are reported. Depending on the plasma and rotation profiles, its net effect can be either stabilizing or destabilizing; it also has substantial effect on the observed mode structure. The localized interchange, the double kink mode, low n resistive MHD modes and the effect of sheared rotation on magnetic island structure are discussed.

1. INTRODUCTION AND SUMMARY

The effect of rotation and rotational shear on the ideal and resistive MHD stabilities has been studied. For the ideal MHD modes, a large rotation shear introduces Kelvin-Helmholtz drive to localized plasma interchanges (Mercier modes) and can reduce the pressure gradient threshold of the ideal localized interchange. However, a small rotation shear modifies the magnetic well and when the rotational pressure gradient is larger than the kinetic pressure gradient, can have a stabilizing effect on the localized interchange. For the double kink mode, rotation shear enhances the side band coupling and provides further destabilization of the mode. A numerical study utilizing the MARS code show that shear flow in general provides stabilization to resistive MHD modes. A self-adjoint variational principle with a constraint relation is shown to be related to the ideal MHD stability of the plasma. This affords easy adoption by present ideal MHD codes for the study of general plasma equilibria with flow. Finally, the effect of rotational shear coupled with the plasma viscosity can significantly affect the shape of the magnetic islands.

2. ANALYTIC STUDY

Plasma rotation modifies the plasma equilibrium, providing an extra source of free energy for known plasma instabilities to enhance their growth rate through the Kelvin-Helmholtz process. These give rise to two different effects on plasma stability. We first study the Kelvin-Helmholtz modification to the Mercier criterion, by focusing on the region close to the rational surface and assuming the plasma to be imparted with a small toroidal rotation but with a large shearing rate. Frieman and Rotenberg's variational principle [1] is applied to localized plasma motion around the mode rational surface. Modification of the localized interchange stability criterion is obtained by maximizing the growth rate. Rotational shear couples to both the Alfvén and sound waves, reducing the stabilizing effect of these waves. These two couplings give rise to two different new terms in the modified localized interchange (Mercier) stability criterion $D_{I_{rot}} < 0$ [2].

$$D_{I_{rot}} = D_I + \frac{1}{4}\left(M^{*2} + AM^{*4}\right) + C\beta M^{*2} \quad . \tag{1}$$

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*Work supported by U.S. Department of Energy Grants DE-FG03-95ER54309 and DE-FG02-92ER54139, and Contract DE-AC03-89ER51114.
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Here, $D_1 < 0$ is the interchange stability criterion for a general plasma equilibrium without flow [3], A is a geometric factor due to plasma surface up-down asymmetry, C is a geometric factor related to the coupling of the sound wave with the Alfvén wave, and M^* is the Alfvén wave Mach number based on the shear flow. This criterion is a direct generalization of that of Bondeson et al. [4] to the case of toroidal geometry and both of the two new terms indicate destabilization. As pointed out by Bondeson et al. [5], the term proportional to C is expected to be strongly modified by kinetic effects and should result in Landau damping. Numerical evaluation of the term $1/4(M^{*2} + AM^{*4})$ indicates that in present day tokamaks, this effect is not large enough to destabilize the interchange mode in the strongly rotating NCS region.

The above result, derived for general geometry, is incomplete and neglects the effect of the centrifugal force. This modification to plasma equilibrium effect is studied by using the equations of motion directly [6] for a low β large aspect ratio circular tokamak. The centrifugal force, which always points in the direction of the major radius; when averaged over the plasma surface can act as a magnetic well. It modifies the localized Mercier stability condition to

$$\frac{s^2}{4} + \frac{r}{R}\alpha_p\left(1 - \frac{1}{q^2}\right) + D_\Omega > 0 \ , \qquad (2)$$

where $s = (r/q)(\partial q/\partial r)$, $\alpha_p = -(2R_0 q^2/B^2)(\partial P/\partial r)$, and $D_\Omega = \alpha_p (r/R)[M^2/(\eta_T + 1)]$ $[(\alpha_\Omega/\alpha_p)(\eta_T + 1) - 1 + M^2(\eta_T - 1)]$. Here, M is the Mach number of the toroidal rotation, $\alpha_\Omega = -(2R_0 q^2/B^2)\ (\partial \rho R_0^2 \Omega^2/2)/\partial r$ and $\eta_T = (\rho\partial T)/(T\partial\rho)$. Effects of finite toroidal rotation on the Mercier criterion thus depends on the sign of D_Ω. The centrifugal pressure gradient will be stabilizing if $\alpha_\Omega/\alpha_p \geq [1/(\eta_T + 1)] - M^2\ [(\eta_T - 1)/(\eta_T + 1)]$.

We turn our attention next to plasma global modes. Because the plasma rotation frequency is smaller than the Alfvén frequency, substantial modifications of the ideal MHD stability of the tokamak only occurs in weak magnetic shear tokamak plasmas with a q value close to being rational. To facilitate analytic tractability, we specialize to a large aspect ratio circular plasma with low β and examine the double kink mode [7] which is localized at a local minimum in q between two rational surfaces. The analysis of Ref. [7] for the double kink in a large aspect ratio circle is extended to include toroidal rotation. The result [6] from the minimization of δW_{rot} given by Waelbroeck [8] is

$$\delta W_{rot} = \delta W_{GHH} + \delta W_\Omega \ , \qquad (3)$$

where $\delta W_{GHH} \propto (8/15)(m^2 - 1)\Delta^2 + (r/R)\,\alpha_p(1 - 1/q^2_{min}) - \Lambda\alpha^2\Delta^{1/2}$, $\delta W_\Omega = D_\Omega + T_\Omega$, and $T_\Omega = -\Delta^{1/2}\Lambda(\alpha_\Omega^2 + 2\alpha_\Omega\alpha_p)$. Here m is the poloidal mode number, $\Delta = (1 - nq_{min}/m)$, Λ is a number which encapsulates information regarding the poloidal sideband harmonics $m \pm 1$. The first two terms in δW_{GHH} which contribute to the stability of the plasma are the effects due to field-line bending, and the magnetic well. The third term which is destabilizing comes from the effect of side band coupling. New effects appear in the two terms in δW_Ω. D_Ω (discussed above) acts as a modification to the magnetic well. T_Ω enhances the side band coupling and is in general destabilizing. Thus, the overall effect of rotation on the double kink mode is more destabilizing than the effect on the interchange mode. The stability diagram in $\alpha_\Omega - M^2$ space for a case where $\delta W_{GHH} = 0$, $r/R = 0.1$, $\alpha_p = 0.1$, and $q^2(q_{min}\Delta/2r_{min}q)^{1/2}\Lambda = 0.5$ is shown in Fig. 1.

3. NUMERICAL STUDY

For the numerical study, the MHD equilibria are computed with the inclusion of sheared toroidal rotation effects. The low mode number, MHD stability of these rotating equilibria is determined by an extended version of the MARS code [9], which solves for the complex growth rates of these modes. New effects included in this extended version of the MARS code include inertial effects of the equilibrium and toroidal rotation shear within the plasma. Of these two

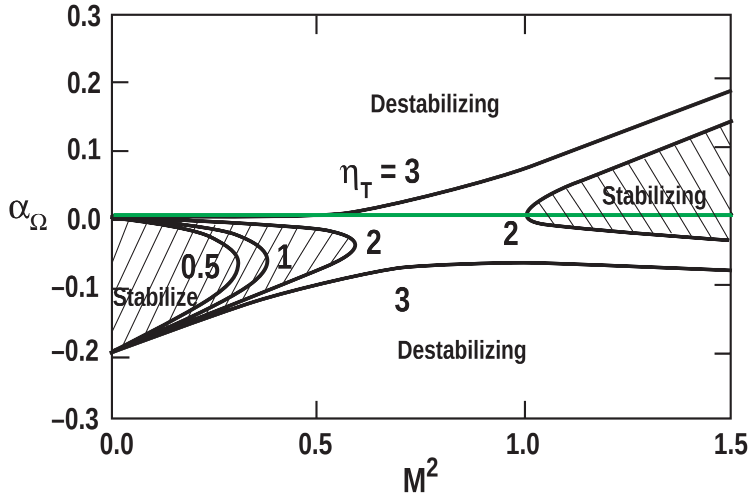

FIG. 1. The effect of rotation on the double kink mode where $q^2(q_{min}\Delta / 2r_{min}q)^{1/2}\Delta = 0.5$*,* $r / R = 0.1$*,* $\alpha_p = 0.1$*. At small* M^2*, only the region in which* $\alpha_\Omega < 0$ *shows a stabilizing effect.*

effects, the stability of the plasma is affected most by the Doppler shift of the rotation frequency between different flux surfaces due to rotational shear.

Results from the above analytic study indicates that the effect of rotation on ideal MHD modes is generally destabilizing. Here we concentrate on resistive MHD modes. Another practical reason for focusing on resistive modes is that relatively large resistivity can be utilized to attain the required resolution around singular surfaces. The tokamak geometry is chosen with an aspect ratio 2.5, elongation 1.8, triangularity 0.8. The q profile is assumed to be of an NCS plasma with $q_0 = 2.9$, $q_{min} = 1.9$, $q_{95} = 5.1$, and the q_{min} is located at $\sqrt{\psi} = 0.5$, where ψ is the normalized flux function. β_n is assumed to range between 0. and 3.0. An external conducting wall is assumed to be located at 1.3 times plasma radius. At low $\beta_n \le 0.5$ the classical double tearing mode for this plasma is readily stabilized by an increase in β_n or flow shear. At higher β_n the plasma develops a resistive interchange and a global resistive double kink. The plasma is found to be close to marginal stability around $\beta_n = 2.5$, depending on external wall distance and resistivity of the plasma. Plasma shear flow can have a substantial stabilization effect on the resistive kink mode. Close to marginal stability the resistive interchange and the external resistive kink may interact with each other. For instance, with $\beta_N = 2.5$, and with the external wall at $r_w = 1.325$ plasma radius, the growth rate ($\gamma\tau_A$, here τ_A is the Alfvén transit time) of the resistive external kink is reduced from 2×10^{-2} to 8.6×10^{-3} by sheared toroidal rotation with profile $\Omega = \Omega_0(1-\psi)$ and the central Mach number $\Omega_0 = 0.3$. When the external wall distance is slightly reduced to $r_w = 1.3$, the growth rate of the mode is reduced to 1.36×10^{-3}, and the mode structure is modified to exhibit peaking at the inner $q = 2$ surface characteristic of the resistive interchange.

4. A SELF ADJOINT VARIATIONAL

Frieman and Rotenberg [1] showed that for an ideal plasma with flow $\vec{v}$, the perturbed Lagrangian displacement $\vec{\xi}$ satisfies the equation of motion $\rho\gamma^2\vec{\xi} + 2i\rho\gamma\vec{v}\cdot\vec{\nabla}\vec{\xi} = \vec{F}(\vec{\xi})$; here ρ is the mass density, $\vec{F}$ the force operator, and $\vec{v}\cdot\vec{\nabla}$ is the Doppler frequency shift operator. This equation is associated with a variational which determines the complex growth rate γ. Due to its non-self-adjointness, in contrast to the MHD energy principle without flow, this variational principle has not been used extensively. However, the real growth rate g, which is related to γ through $\gamma = \pm g - i\lambda_b$, satisfies an auxiliary equation of motion

$$\rho g^2\vec{\xi} = \vec{F}\left(\vec{\xi}\right) + \rho\left[\lambda_b^2\vec{\xi} + 2i\lambda_b\left(\vec{v}\cdot\vec{\nabla}\right)\vec{\xi}\right], \tag{4}$$

with the constraint for the displacement $\vec{\xi}$ as

$$\lambda_{\rm b} = -i\,\frac{\int \rho \vec{\xi}^* (\vec{v}\cdot\vec{\nabla})\vec{\xi}\,dr}{\int \rho \vec{\xi}^* \cdot \vec{\xi}\,dr} \quad . \qquad (5)$$

It is obvious from Eq. (4) that g^2 satisfies a self-adjoint variational principle $\delta W_{\rm rot}$ which is obtained by multiplying Eq. (4) with $\vec{\xi}^*$ and integrating over the plasma volume. Also, Eq. (5) is satisfied when g^2 is minimized with respect to $\lambda_{\rm b}$, while maximized with respect to $\vec{\xi}$ in Eq. (4). At marginal stability or for a plasma without flow, the $\vec{\xi}$ determined by Eq. (4) coincides with that given by the exact equation of motion. The constraint condition Eq. (5) determines the observed rotation frequency. At this frequency, the displacement $\vec{\xi}$ satisfies the requirement of (angular) momentum balance. Since the method for finding the eigenvalues of self-adjoint energy functionals for ideal MHD has been well established and incorporated into many ideal MHD codes, the present formulation Eq. (5) could be easily incorporated into these codes to study the ideal MHD stability of tokamaks with rotation.

5. MAGNETIC ISLAND DEFORMATION DUE TO SHEAR FLOW AND VISCOSITY

In tokamaks, one of the criteria for identifying the presence of a magnetic island is the characteristic 180 degrees phase shift of the temperature fluctuations across the magnetic island. However, in a rotating tokamak, this phase shift has been observed to deviate from 180 degrees, giving rise to an ''anomaly'' and difficulty in interpretation. This anomaly is explained quantitatively in terms of the combined effect of plasma viscosity and flow shear across the magnetic island. In two dimensional geometry, in which the magnetic field is represented by $\vec{B} = \vec{z}\times\vec{\nabla}\psi + B_z\vec{z}$, and the flow velocity represented by $\vec{V} = \vec{z}\times\vec{\nabla}U$, the two dimensional plasma equilibrium equation for the flux function ψ is given by [10]

$$\Delta\psi + F(\psi) - \mu_0 \rho U' \Delta U = \mu_0 \nu \frac{\nabla\psi \cdot z \times \nabla \Delta U}{(\nabla\psi)^2} \quad . \qquad (6)$$

Here, $F(\psi) = B_z B_z' + \mu_0 H'$, with H being the enthalpy function and ν the kinematic viscosity. By inspection, it is seen that the term on the right hand side of Eq. (6) gives rise to an effect of distorting the plasma surface as shown in Fig. 1. The boundary condition or this equation should reflect the coupling of magnetic fluctuations on neighboring singular flux surfaces. An analytic estimate, which was verified by detailed extensive numerical solutions of Eq. (6) shows that the phase shift anomaly is given by $8k\nu(dV/dr)w\mu_0 G/\delta B^2$, where k is the wavelength of the magnetic perturbation along the lengthwise direction of the magnetic island, w is width of the magnetic island, dV/dr is the gradient of flow on the island separatrix and G is a quantity approximately 1. For a rotating island located at the outer resonant surface of a DIII–D NCS plasma where $R = 1.7$ m, $B_0 = 2$ T, with $dV/dr = 10$ km/s , $\delta B = 5$ G, $\nu/\rho = 1$ m^2/s, an island wave number of $k = 7$/m gives rise to a phase shift anomaly of 20 degrees across the magnetic island in agreement with experimental observation.

REFERENCES

[1] FRIEMAN, E., ROTENBERG, M, Rev. Mod. Phys. **32** (1960) 898.
[2] CHU, M.S., Phys. Plasmas **5** (1998) 183.
[3] GREENE, J.M., JOHNSON, J.L., Phys. Fluids **18** (1975) 510.
[4] BONDESON, A., IACONO, R., BHATTACHARGEE, A., Phys. Fluids **30** (1987) 2180.
[5] BONDESON, A., IACONO, R., Phys. Fluids B **1** (1989) 1431.
[6] ZHENG, L.J., CHEN, L., CHU, M.S., ''Effect of toroidal rotation on the localized modes in low-β circular tokamaks'', submitted to Physics of Plasmas (1997).
[7] GIMBLETT, C.G., HASTIE, R.J., HENDER, T.C., Phys. Plasmas **3** (1996) 3369.
[8] WAELBROECK, F.L., Phys. Plasmas **3** (1996) 1047.
[9] BONDESON, A., VLAD, G., LUTJENS, H., Phys. Fluids B 4 (1992) 1889.
[10] REN, C., CHU, M.S., CALLEN, J.D., ''Magnetic island deformation due to sheared flow and viscosity'', submitted to Physics of Plasmas (1998).