STABILITY ANALYSIS ON ENERGETIC ION ALFVEN MODE AND KINETIC BALLOONING MODE IN TOKAMAKS*

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Abstract

Kinetic ballooning analysis has been performed for high performance tokamak discharges by implementing the Shafranov shift Δ' in both positive and negative shear regimes. For discharge parameters pertinent to transport barriers characterized by steep pressure gradient and large Δ', the kinetic ballooning and drift type modes are stabilized at $\Delta' \simeq -0.3 \sim -0.2$. A new MHD type ballooning mode with large growth rates is predicted for excessively large Δ' in negative shear regime. Also investigated is the Alfven mode driven by energetic ions (such as trapped alpha particles) without resorting to the conventional ideal MHD assumption for the bulk ions. Compared with the earlier predictions, a reduction in the growth rate by two orders of magnitude has been seen.

1. INTRODUCTION

As the beta (β) factor in tokamaks progressively increases, the Shafranov shift Δ' enters as an independent parameter to affect stability of various pressure gradient driven modes. Ideal MHD ballooning mode equation for high β tokamak discharges has earlier been formulated [1]. Kinetic ballooning mode equation incorporating a finite Shafranov shift has recently been derived and applied to discharge conditions pertinent to high performance tokamaks [2]. The purpose of the present paper is to report recent findings of the effects of Shafranov shift on the following modes: (a) drift type modes (trapped electron and ITG modes), (b) kinetic ballooning modes in both positive and negative shear regimes, and (c) Alfven mode driven by energetic ions (such as alphas).

In general, Shafranov shift has stabilizing influences on drift and ballooning modes in positive shear. However, in negative shear, MHD-like modes with large growth rates become unstable when $|\Delta'| \gtrsim 0.3$. This new instability imposes an upper limit on β that can be stably confined with negative shear.

In kinetic analysis of the Alfven mode driven by high energy ions, the ideal MHD assumption $E_\parallel = 0$ is avoided in formulating the high energy ion response to the electromagnetic fields. Significant reduction in the growth rate is predicted compared with those found earlier based on MHD-perturbative approach.

The paper is organized as follows. In Section 2, a kinetic mode equation is developed for Alfven type modes in tokamaks by implementing energetic trapped ions and finite Shafranov shift. Results of shooting code analysis for effects of Shafranov shift on various modes (including predominantly electrostatic drift type modes) are shown in Sections 3 and 4. In Section 5, stability of energetic ion driven Alfven mode will be discussed followed by concluding remarks in Section 6.

2. MODE EQUATION

A kinetic ballooning mode equation incorporating a finite Shafranov shift Δ' has recently been formulated [2] for tokamak discharges with nearly circular magnetic surfaces:

$$\frac{d}{d\theta}\left\{J\left(1+\frac{\Lambda^2}{J^4}\right)\frac{d\varphi}{d\theta}\right\}+V\left(\theta\right)\varphi=0, \tag{1}$$

where $J = 1 + \Delta' \cos\theta$,

$$\Lambda = s\theta + J\Delta' \sin\theta + [3\Delta'(s - 1 + \Delta'^2) - \alpha(1 - \Delta'^{5/2})](1 + 2\Delta'^2)\sin\theta, \tag{2}$$

$$V = \frac{\beta_e}{2\epsilon_n^2} J^3 \left\{ (\Omega - 1)(\Omega - f) + \eta_e f - \frac{(\Omega - 1)^2}{1 + \tau(1 - I_i)} \right\}, \quad \Omega = \omega/\omega_{*e}, \quad \tau = T_e/T_i, \tag{3}$$

$$f(\theta) = \frac{2\epsilon_n}{J}\left(\cos\theta + \frac{\Lambda}{J^2}\sin\theta\right), \; I_i = \left\langle \frac{\omega + \widehat{\omega}_{*i}}{\omega + \widehat{\omega}_{Di}} J_0^2\left(\frac{k_\perp v_\perp}{\Omega_i}\right)\right\rangle, \tag{4}$$

and

$$k_\perp = k_\theta \sqrt{1 + \frac{\Lambda^2}{J^4}}. \tag{5}$$

In the presence of energetic trapped ions, the denominator $1 + \tau(1 - I_i)$ is to be modified as

$$1 + \tau(1 - Zn_h)(1 - I_i) + Z^2 \tau_h n_h (1 - I_h), \tag{6}$$

where

$$I_h = \left\langle \frac{\omega + \widehat{\omega}_{*h}}{\omega + \widehat{\omega}_{Dh}} J_0^2\left(\frac{q k_\perp v_\perp}{\sqrt{\varepsilon}\Omega_i}\right)\right\rangle, \; \varepsilon = r/R. \tag{7}$$

Effects of trapped electrons and a finite ion transit frequency can also be implemented in the ballooning mode equation.

3. EFFECTS OF SHAFRANOV SHIFT Δ' ON DRIFT TYPE MODES

The kinetic ballooning mode equation is to be solved with a shooting code in which full ion velocity integrations must be evaluated at each mesh point. A fast shooting code based on the Gaussian-Hermite quadrature approximation has been employed. The boundary conditions are: $d\phi/d\theta = 0$ at $\theta = 0$ (even parity mode) and $\phi(\infty) = 0$.

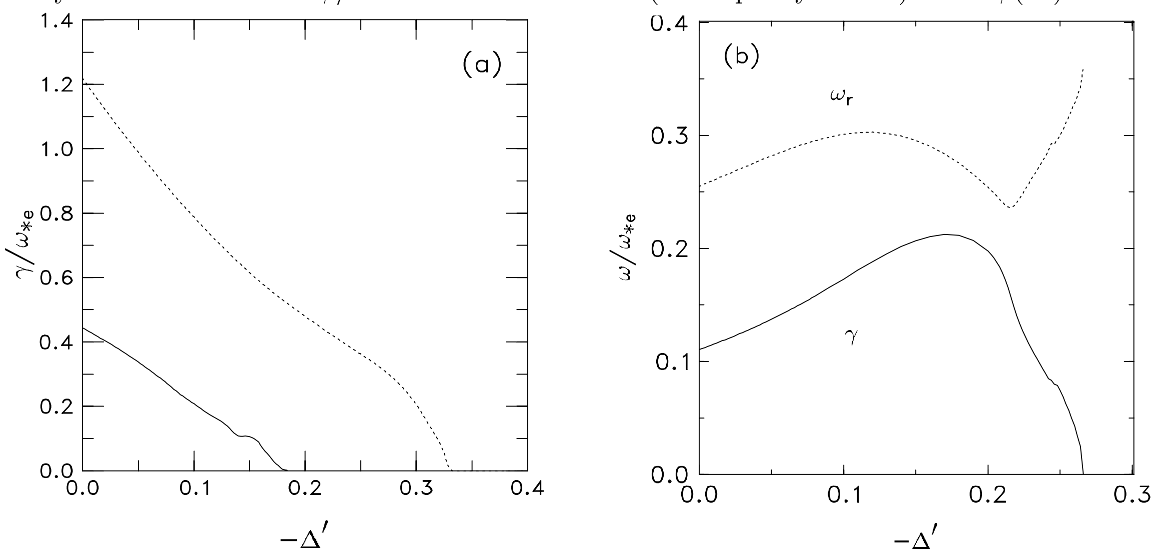

Fig. 1. (a) Growth rates of the KBM in positive shear (dahed line) and in negative shear (solid line) vs. Shafranov shift Δ'. Discharge parameters pertinent to transport barriers in high performance tokamaks are assumed. (b) Stabilization of the trapped electron drift mode by Δ'. $b_0 = 10^{-3}$.

The dashed line in Fig. 1 (a) shows stabilizing influence of Δ' on the kinetic ballooning mode in the MHD second stability regime when $\alpha = 2.0$, $s = 0.4$, $b_0 = (k_\theta \rho_i)^2 = 0.01$, $\tau = T_e/T_i = 1, \varepsilon_n = L_n/R = 0.18$, $\eta_i = \eta_e = 2$. (For $s = 0.4$, the ideal MHD ballooning mode is unstable in the interval $0.36 < \alpha < 1.47$ and the assumed ballooning parameter $\alpha = 2.0$ is clearly above the cutoff.) Complete stabilization occurs at $\Delta' \simeq -0.33$ for the parameters chosen. The solid line in Fig. 1 (a) depicts dependence of the growth rate of KBM in negative shear with parameters pertinent to those observed at the transport barrier in JT60-U [3], $\alpha = 5$, $s = -0.51$, $\tau = 0.67$, $\eta_i = 1.3$, $\eta_e = 1.1$, $L_n/R = 0.13$, $q = 3.9$. The kinetic ballooning mode in negative shear regime is characterized by extended eigenfunctions, relatively small growth rate ($\gamma/\omega_A \lesssim 0.1$ where $\omega_A = V_A/qR$, the Alfven frequency), and mode frequency of the order of the Alfven frequency, $0.5 \lesssim -\omega_r/\omega_A \lesssim 1$. A modest finite ion temperature gradient is required of the instability, $\eta_i \gtrsim 0.5$ which is known to enhance the ion magnetic drift resonance. When $T_i = T_e$, the threshold α is also small, $\alpha \gtrsim 0.1$. However, the instability only exists in a shear window, $-0.5 \lesssim s \lesssim -0.1$. When Δ' is ignored, there is no upper cutoff in the ballooning parameter.

In order to investigate stability of the trapped electron driven drift mode in the region of JT-60U ITB, the mode equation has been modified to implement bounce averaged trapped electron density response and also the ion transit effect perturbatively. Fig. 1 (b) shows the mode frequency and growth rate of the mode $b_0 = 0.001$ as a function of Δ'. The trapped electron drift mode is unstable at extremely long wavelengths, $b_0 \simeq 10^{-3}$ due to deactivation of ion Landau damping. Longer wavelength modes are stabilized at a small Δ' while shorter wavelength mode requires a substantially larger Δ' for stabilization.

4. MHD BALLOONING MODE DRIVEN BY Δ'

The stability analysis depicted in Fig. 2 shows the growth rate (normalized by the diamagnetic frequency) as a function of the Shafranov shift parameter Δ' using ideal MHD, kinetic and two-fluid theory with $s = -0.2$, $\alpha = 1.2$, $b_0 = (k_\theta \rho_i)^2 = 0.01$, $\eta_e = \eta_i = 1$ and $\epsilon_n = L_n/R = 0.1$. All three methods predict destabilization of the ballooning mode at large $|\Delta'|$ with the critical Δ' for the ideal MHD mode occurring at approximately $\Delta' \simeq -0.4$. For smaller shift values, $\Delta' \gtrsim -0.4$, only the kinetic and two-fluid theories are able to recover the mode indicating that the instability is hydrodynamic in nature. A marginal stability diagram can be constructed in (Δ', α) space using MHD theory which shows a steady reduction of the instability region with increasing $|s|$. The eigenfunction structure at $\Delta' \lesssim -0.4$ is well confined, $|\theta| \lesssim \pi$, within the unfavourable region and is MHD-like in appearance. In contrast, the eigenfunction structure of the kinetic mode in the region $\Delta' \gtrsim -0.4$ is greatly extended.

The present finding of a new type of MHD ballooning mode in negative shear may impose a limit on β in tokamaks operated with negative shear. In stability analysis of ballooning mode, the conventional (s, α) diagram is not sufficient particularly at high β because of the entry of Shafranov shift Δ' as an additional, independent parameter. The Shafranov shift is a measure of total plasma energy stored, while the ballooning parameter is a measure of local plasma pressure gradient. Destabilization at large $|\Delta'|$ is thus not an unexpected result.

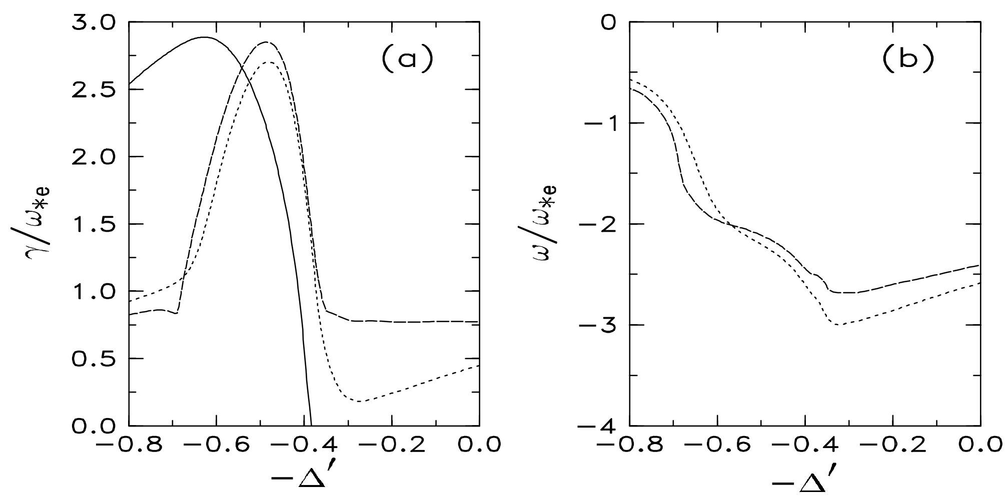

Fig. 2. (a) Growth rates of ideal MHD (solid line), two-fluid (dashed line), and kinetic (dotted line) ballooning modes vs. Δ' in negative shear tokamak discharge. $\alpha = 1.2$, $s = -0.2$, $b_0 = 0.01$, $T_e = T_i$, $\eta_{i,e} = 1$, $q = 3$. (b) Corresponding mode frequencies of the two-fluid (dashed line) and kinetic (dotted line) ballooning modes.

5. STABILITY OF ALFVEN MODE DRIVEN BY ENERGETIC IONS

Results of kinetic ballooning analysis for Alfven mode driven by energetic ions are shown in Fig. 3. The growth rate increases almost linearly with the population of high energy ions as seen Fig. 3 (a). Fig. 3 (b) shows the growth rate previously found by Cheng [4] and that from kinetic analysis for the following parameters: $s = 0.5$, $T_e/T_i = 0.5$, $q = 2$, $\alpha_c = \alpha_h = 0.1$, $L_n/R = 0.2$, $r/R = 0.1$, $\eta_e = \eta_i = \eta_h = 1$. Here, α_c is the ballooning parameter of the bulk plasma and α_h is that of energetic ions. It is apparent that MHD analysis grossly overestimates the growth rate. The maximum growth rate revealed by kinetic analysis occurs at an extremely small ion finite Larmor radius parameter, $b_0 \simeq 10^{-4}$. The underlying assumption of high n mode number thus becomes dubious (if not breaks down) and more accurate assessment of the growth rate will require global mode analysis. Stabilization of the high energy ion mode revealed with the kinetic shooting code is due to the following factors: finite orbit width of trapped energetic ions, accurate assessment of responses of trapped electrons and energetic ions, and avoidance of the ideal MHD approximation.

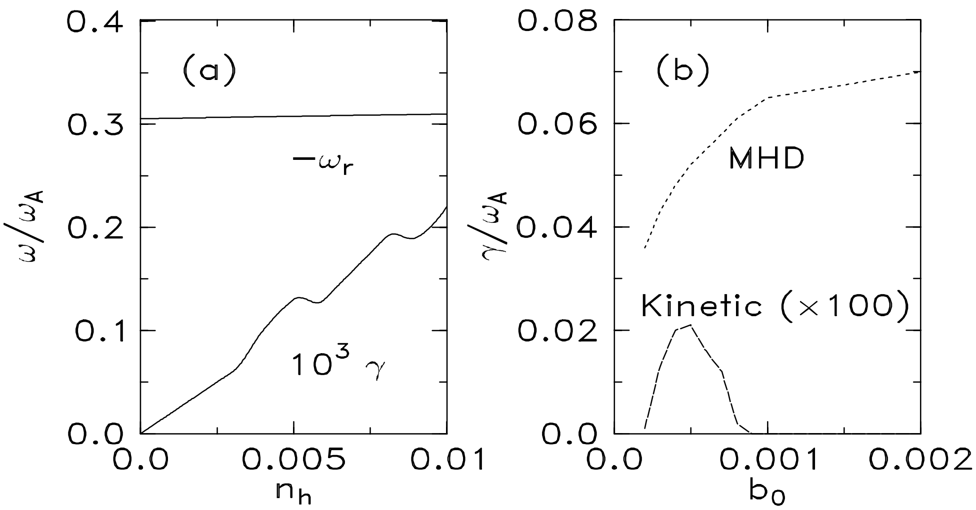

Fig. 3. (a) The frequency ($-\omega_r/\omega_A$, $\omega_A = V_A/qR$) and growth rate (γ/ω_A) of the Alfven mode as functions of the population of high energy ions n_h. $\alpha_{\text{bulk}} = 0.1$. (b) Comaprison of the kinetic growth rate with that based on MHD-perturbative approach [4]. $\alpha_{\text{bulk}} = \alpha_{\text{alpha}} = 0.1$.

6. CONCLUSIONS

In summary, effects of the Shafranov shift Δ' on the drift and kinetic ballooning modes in high performance tokamaks have been studied in terms of a kinetic ballooning mode equation recently formulated. In positive shear, Δ' is in general stabilizing. In negative shear, Δ' can be destabilizing. In particular, MHD ballooning mode is destabilized in the region $\Delta' \lesssim -0.4$ and may impose a limit on β. Stability of Alfven mode in the presence of energetic ions has also been investigated without resorting to MHD approximation for bulk ions. Both bulk ions and energetic ions are treated kinetically.. The growth rate found in the present study is significantly smaller (by two orders of magnitude) than that predicted earlier based on MHD-perturbative approach.

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