INSTABILITY THRESHOLD OF NEOCLASSICAL TEARING MODE, DOUBLE TEARING MODE AND OFF-AXIS SAWTEETH CRASH IN TOKAMAKS

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Abstract

The neoclassical and double tearing modes have been analyzed with related new phenomena in the reversed magnetic shear tokamak plasmas. The instability threshold, and the linear and nonlinear evolution are derived for the neoclassical tearing modes. It is found that the perturbed bootstrap current in the resistive layer has a stabilizing effect while the equilibrium bootstrap current in the outer region can destabilize the modes. The dispersion relation is derived for the double tearing mode. It is found that the onset of “annular crash” is due to the fast reconnection of the hot and cold islands, triggered by the interaction of both branches. The onset of “core crash” is mainly due to the coalescence between the hot islands, triggered by the explosive growth of the inner branch.

1. INTRODUCTION

Recently, the high confinement regime has been observed in the reversed magnetic shear (RMS) plasma with nonmonotonic q profile in several tokamaks [1]. The high bootstrap current fraction, high β (plasma pressure/magnetic field pressure) and high confinement time were obtained in such high confinement regime. Therefore, the study of the neoclassical and double tearing modes become very important issues for achieving the advanced tokamak operation. The neoclassical tearing modes, the off-axis sawteeth activity and double tearing reconnections were observed respectively in TFTR (Tokamak Fusion Test Reactor) supershot plasma [2] and RMS experiments [3]. It is important to understand how the neoclassical and double tearing modes are destabilized and what are the mechanisms for the related new phenomena such as the destabilization of $m/n = 3/2,\ 4/3,\ 5/4$ modes, the onset of “annular crash” and “core crash” of the off-axis sawteeth.

2. INSTABILITY THRESHOLD OF NEOCLASSICAL TEARING MODE

The nonlinear neoclassical (bootstrap current driven) tearing modes has been predicted in the original theories [4] and observed in the recent TFTR supershot experiments [2,5]. The premise of the original theories is to assume that a negative perturbed bootstrap current is induced when the equilibrium bootstrap current vanishes inside the separatrix due to the flatten of equilibrium pressure. This negative perturbed bootstrap current plays a destabilizing role for the neoclassical tearing modes. However, the recent TFTR experiment results imply that the *perturbed* bootstrap current should be positive and play a stabilizing role since the *perturbed* temperature gradient is negative when the *equilibrium* temperature is flatten across the island [2,5]. The question is what is the destabilization mechanism of the neoclassical tearing modes.

It is well known that the tearing instability in a tokamak is driven by the radial gradient of the *equilibrium* current density, and determined by the critical parameter Δ'_0 which should be calculated from the ideal kink equation in the outer region [6]. Therefore, the *equilibrium* bootstrap current *in the outer region* should play an important role since it provides an additional free energy to drive instability. Δ'_{nc} should be derived from the outer solutions rather than the inner solution.

In the outer region away from the resistive layer at $r = r_s$ defined by the safety factor $q(r_s) = m/n$, the resistivity and the inertia can be neglected. An analytic expression for Δ' of neoclassical tearing instability can be derived from the outer solutions [7]. One has

$$\Delta' = \Delta'_0 + \Delta'_{nc} = -\pi(\lambda / r_s)\cot(-\pi a) \quad (1)$$

where Δ', $a = m - 1/2 - \sqrt{(m-1/2)^2 + \lambda}$ and $\lambda = -[(qr_s / B_\theta)(dj/dr)/(dq/dr)](r_s)$ reduce, respectively, to Δ'_0, a_0 and λ_0 when $j_{b\zeta}(r) = 0$. Obviously, Δ' depends the total current $j(r)$ which is the sum of the Spitzer current $j_s(r)$ and the bootstrap current $j_{b\zeta}(r)$. For the case of $\lambda > \lambda_0$, the bootstrap current increases the instability of the mode when the classical tearing mode is unstable ($\Delta'_0 > 0$), and provides a drive to destabilize the neoclassical tearing instability when the classical tearing mode is stable ($\Delta'_0 > 0$). The Δ' reduces to the Δ'_0 for the classical tearing mode when $j_{b\zeta}(r) = 0$.

Consequently, one can obtain a criterion from Eq. (1) for the instability threshold of the neoclassical tearing mode

$$\left|(qr_s dj/dr)/(B_\theta dq/dr)\right| > m - 1/4 \quad (2)$$

Usually, for the normal tokamak plasma profiles with $dj/dr < 0$, $dq/dr > 0$, the higher m modes are not easily to be destabilized because it is more difficult to satisfy the criterion (2) for the higher m modes. In the TFTR high performance supershots, the pressure profile is highly peaked so that the bootstrap current modifies evidently the total current profile. In that case, the higher m modes such as $m/n = 3/2,\ 4/3,\ 5/4$ can be destabilized.

Actually, the expression of the parameter λ can be simplified according to the force equilibrium equation $\nabla P = \mathbf{J} \times \mathbf{B}$. It is easy to verify that one has $dp/dr = -B_\theta(r) j_s(r)$ because the bootstrap current is parallel to the magnetic field. Therefore, the total current profile can be expressed as $j(r) = c(1 + 1.46\sqrt{r/R_0})\, j_s(r)$, here c is a constant which can be determined by supposing the total current is fixed whatever $j_b(r) = 0$ or $j_b(r) \neq 0$ for a given $q_s(0)$ value.

In the inner region around the resistive layer at $r = r_s$, the resistivity and the inertia should play an important role. It is easy to obtain that the linear growth rate of the neoclassical tearing mode is

$$\gamma = [(1-k_1)/(1-k_2)]^{3/5} (\Delta'/\Delta'_0)^{4/5} \gamma_0 \quad (3)$$

where $k_1 = 1.46\sqrt{\varepsilon_s}$, $k_2 = 1.95\sqrt{\varepsilon_s}$ ($\varepsilon_s = r_s / R_0$), γ_0 is the linear growth rate of the classical tearing mode. Obviously, the perturbed bootstrap current has a stabilizing effect on the neoclassical tearing mode meanwhile the neoclassical enhancement of the resistivity has a destabilizing effect. The linear growth rate γ decreases by a factor of $(1-k_1)^{3/5}$ due to the perturbed bootstrap current, and increases by a factor of $(1-k_2)^{-3/5}$. Totally, the linear growth rate γ increases by a factor of $(1-k_1)/(1-k_2)]^{3/5}$ besides a factor of $(\Delta'/\Delta'_0)^{4/5}$.

In the nonlinear phase, for the simplicity, only the quasilinear current effect is included [9] while the quasilinear effect of the magnetic field is not considered here [10]. Then, a nonlinear evolution equation for the perturbed magnetic flux is obtained:

$$\partial\tilde{\psi} / \partial t = (\sqrt{2}\bar{s} / qc_0^2)^{1/2}(1-k_1)\eta_{nc}\Delta'\tilde{\psi}^{1/2} \quad (4)$$

where $c_0 = \pi\,\Gamma(3/4)/\Gamma(1/4)$, $\bar{s} = d\ln q(r_s)/d\ln r$ is the shear parameter. Defining the island half-width as $w = 2(q\tilde{\psi}/\bar{s})^{1/2}$, it is easy to get a evolution equation for w

$$dw/dt = (2^{1/4}/c_0)[(1-k_1)/(1-k)_2]\eta\Delta'(w) \quad (5)$$

Similar to the linear case, the perturbed bootstrap current plays a stabilizing effect while the resistivity increment plays a destabilizing effect. The growth of the island width is enhanced by a factor of $(1-k_1)/(1-k_2)$ besides a factor of Δ'/Δ'_0.

It is interesting to compare our theoretical analyses with the TFTR experiment results [4]. Supposing the Spitzer current density is a peaked profile $j_s(r) = j_0/(1+r^2)^2$, then the total current density is a hollow profile. One can observe that the neoclassical tearing modes are more unstable than the corresponding tearing mode for the positive magnetic shear. The parameter λ is higher than the parameter λ_0 at any r_s/a value for $dq/dr > 0$. When $q_s(r)$ profile becomes $q(r)$ profile during the postbeam phase, the rational surfaces of $m/n = 5/4,\ 4/3,\ 3/2$ modes move forward to the plasma center while the parameter λ increases as a rational surface approaches the center. For $q(a) = 5$, and $q_s(0) = 0.94$, according to the criterion (2), the $m/n = 5/4,\ 4/3,\ 3/2$ classical tearing modes are stable. However, the $m/n = 5/4,\ 4/3,\ 3/2$ neoclassical tearing modes are unstable. When $q_s(0)$ increases until the rational surface of the $m/n = 5/4$ neoclassical tearing mode disappears, the $m/n = 4/3,\ 3/2$ classical tearing modes are stable whereas the $m/n = 4/3,\ 3/2$ neoclassical tearing modes are unstable. When $q_s(0)$ continues to increase until the rational surface of the $m/n = 4/3$ neoclassical tearing mode disappears, the $m/n = 3/2$ classical tearing mode is stable whereas the $m/n = 3/2$ neoclassical tearing mode is unstable.

3. DOUBLE TEARING MODE AND OFF-AXIS SAWTEETH CRASH

For the double tearing mode, the dispersion relation is constructed based on the criterion parameter Δ'_0 so that it turns out that the off-axis sawteeth crashes are trigged by a m/n = 2/1 double tearing reconnection. The dispersion relation is obtained as

$$[\Delta(\omega) - \Delta_1'^{(0)} - A_{12}]\,[\Delta(\omega) - \Delta_2'^{(0)} - A_{21}] - A_{22} = 0 \quad (6)$$

where A_{12}, A_{21} and A_{22} are calculated from the MHD solutions. Assume that the nonmonotonic q profile has a form $q(r) = q(0)\alpha(1+hr^{2p})^{1+1/p}/(\alpha + hr^{2p})$, here $q(0)$, a, h, and p are free parameters. In the TFTR experiments, the "annular crash" occurs as $q(0)$ increases to a high value among 4 ~ 5 while $q_{\min}$ decreases to below 2, the two singular surfaces of the double tearing mode, r_1 and r_2, are approaching each other. The interaction between the two branches becomes strongest since the growth rates of the two branches are in the same order. The bulging of the hot and cold islands gives rise to current sheet formation and fast reconnection between r_1 and r_2. Such complete reconnection leads to the "annular crash". It is similar to the result of the numerical simulations [10,11], which are the application of Kadomtsev model to the double tearing reconnections. A new equilibrium will be reached as the helical flux function (as well as q profile) is flat in the reconnection region.

When $q(0)$ decreases rapidly toward to 2 while $q_{\min}$ decreases slightly, the "core crash" occurs in the TFTR experiments. The interaction between the two branches becomes quite week because the two singular surfaces r_1 and r_2 are well separated and the growth rate of the inner branch is much larger than that of the outer branch. The linear growth of inner branch increases explosively as r_1 approaching to plasma core because of $\Delta' \propto f(m,\lambda)/r_1$ [7]. Therefore, As $q(0)$ decreases to 2.2, the $\Delta_1'^{(0)}$ increases as well as $\Delta_2'^{(0)}$ while A_{22} decreases as $q(0)$ decreases. Therefore, the interaction of the two branchesis even weaker so that $\Delta(\omega)$ is almost same as $\Delta'^{(0)}$. Obviously, the interaction of the two branches could not be the main reason to trigger the onset of the "core crash". On one hand, the linear growth of inner branch increases explosively as r_1 approaching to plasma core because $\Delta' \propto f(m,\lambda)/r_1$ becomes very large. Therefore, the quasilinear modification of the magnetic field becomes dominant comparing with the Rutherford behavior, and the width of hot islands expands rapidly as the total magnetic shear becomes quite small at r_1 [9]. On the other hand, the attractive force between parallel currents induced by the hot islands accelerates the reconnection triggered by the explosive growth of the inner branch and the rapid expansion of the hot islands which are close each other. A fast reconnection in the plasma core due to coalescence between the hot islands leads to the "core crash."

4. CONCLUSION

The instability threshold and the linear and nonlinear evolution for the neoclassical tearing mode are derived by including both effects of the *equilibrium* and *perturbed* bootstrap currents. It is found that the effect of the *equilibrium* bootstrap current *in the outer region* can destabilize the neoclassical tearing instability through providing additional free energy. Meanwhile, the *perturbed* bootstrap current *in the inner region* plays a stabilizing effect which is overcome by the destabilizing effect of neoclassical enhancement of the resistivity. A preliminary comparison is made between the theoretical results and TFTR supershot experiments [2,5].

The relation between Δ' for the double tearing mode and $\Delta'^{(0)}$ for single tearing mode is resulted from the solution structure of the ideal external kink equation. The dispersion relation of the double tearing mode is obtained with help of the asymptotic matching condition and the analytic expression of $\Delta'^{(0)}$. The onset of "annular crash" is due to the fast reconnection of the hot and cold islands, triggered by the interaction of both branches of the double tearing mode for which Kadomtsev model can be applied to explain. The onset of "core crash" is mainly due to the coalescence between the hot islands, in which the fast reconnection is triggered by the explosive growth of the inner branch and the rapid expansion of the hot islands.

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