OPTIMISATION OF STELLARATOR SYSTEMS: POSSIBLE WAYS

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Abstract

The results of our search for advanced helical (stellarator) systems with a small number of field periods over the last five years are presented. The comparison of stellarator systems with toroidal (helical or axial) and poloidal directions of the contours with B = constant on the magnetic surface as well as systems with Helias and Heliac-like orientation of the magnetic surfaces cross-sections with respect to the principal normal to the magnetic axis is undertaken. Particular attention is paid to some attractive features of the systems with constant B-lines in the poloidal direction.

1. INTRODUCTION

The approach to stellarator improvement based on the optimisation of the boundary magnetic surface which was used in the W7-X project [1] has led to the understanding of possibilities to control the lines B = constant on the magnetic surface (B-lines) [2] to avoid the unfavourable trajectories of the charged particles and ensuing strong neoclassical transport. The discovery by Nührenberg and Zille [2] of the possibility of hidden quasisymmetries (QS) in stellarator systems has initiated a search for less restrictive criteria of optimization, e.g. local omnigeneity (Refs. [3-5]) and pseudosymmetry (PS) (Refs. [6, 7]). It was shown also that Helias and Heliac-like types of QS (or PS) configurations are possible which differ in the number of the near axis elliptical magnetic surface cross-sections turns with respect to the B-lines within one system period (Refs. [8, 9]). There could be toroidal (helical or axial) and poloidal directions of the constant B-contours on the magnetic surface in the QS and in the PS systems. Unexpectedly good properties of the poloidal QS (PS) systems were also discovered during computations (Refs. [5, 10, 11]).

Here we present the survey of our results in the field on stellarator optimisation. We will have in mind only the case of helical systems with rather smooth small shear where the nonuniformity of the magnetic strength $B = B_o(s)\{1 - k(s)x + ...\}$ is determined mainly by the magnetic axis curvature $k(s)$ (for the toroidal direction of the constant B-lines, where $B_o(s)$ = constant) or by the magnetic field nonuniformity on the magnetic axis $B_o(s)$ (for the poloidal direction of the constant B-lines). We describe the characteristic features of the QS configurations with the conditions and consequences of the quasisymmetry (Section 2). The less restrictive case of the pseudosymmetry (PS) is considered in Section 3. Here a considerable body of related results including computational ones are collected in a table.

2. THE CONDITION OF QUASISYMMETRY

As was shown by A. Boozer [12], for the guiding centre equations of motion to have an additional conserved integral, the modulus of the magnetic field B should be independent of one of the angular variables of the Boozer flux coordinates a, $_B$,, $_B$: $B = B(a, _B)$ or $B = B(a, _B)$. This property is called quasisymmetry (QS) and the 3-D toroidal systems possessing such property are referred to as QS systems. From the viewpoint of the guiding centre motion, the QS system is equivalent to its fully symmetric counterpart:
(a) the lines B = constant do not form islands on the magnetic surfaces,
(b) there are no locally trapped and trapped-transition particles,
(c) the bounce averaged trajectories lie on magnetic surfaces,
(d) the "banana" width in terms of flux coordinates is not changed when the particles drift along the direction of QS (i.e. along the line B = constant).

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The only difference between a QS system and the fully symmetric case is the "banana" width change in real space of a particle when it drifts along the direction of QS. This difference, however, does not lead to an increase in transport. Thus, one could believe that the QS condition is the most stringent.

One should keep in mind that the QS-condition can be satisfied only in some region that is near the magnetic axis or near a single magnetic surface (Ref. [13]). In reality, 3-D QS systems are not truly realisable. Strictly speaking, the QS systems we consider constitute an approximation to this ideal condition.

2.1. Toroidal QS

By analogy with fully axial and helical symmetry, the corresponding types of QS systems could be envisaged too. In the smooth systems under consideration, the direction of QS is defined by the magnetic axis behaviour. If the magnetic axis principal normal rotates from one cross-section to another, quasihelical-symmetry (QHS) can be realised. If the principal normal "oscillates" only, quasi-axisymmetry (QAS) is possible. These two types can be termed as toroidally quasisymmetric systems, $B = B(a, \theta_B)$ with θ_B being the tokamak-like or helical-like poloidal coordinate, respectively.

Quasihelical symmetry. The realisation of QHS means an elimination of all harmonics of the magnetic field strength that violate the QS condition, including the "toroidal" inhomogeneity of B which is responsible for the tokamak-like part of the Pfirsch-Schlueter current. This increases the equilibrium limitation on the plasma pressure to a rather large value which is inherent in the helically symmetric configurations. It is worth noting that the condition of the elimination of only the toroidal inhomogeneity is less stringent than the condition of QHS.

Quasiaxial symmetry. In the QAS systems, the helical inhomogeneity of B is eliminated (in Boozer coordinates). The equilibrium β–limit here is moderate like that in a tokamak.

Both the QHS and QAS configurations can have different number n_1 of elliptical cross-sections turns in one system period with respect to the magnetic axis principal normal. One can identify these types as Helias-like ($n_1 \neq 0$) or Heliac-like ($n_1 = 0$) configurations. The physical difference between these types is as follows. The QS Helias-like configurations have some difficulties with the creation of a magnetic well because of the change of the magnetic surface cross-section orientation with respect to the magnetic axis principal normal as one follows the system around. As was shown in Ref. [2], the QHS-like system with $n_1 = -1/2$ has no magnetic well, thus some deviation from QHS should be introduced to provide stability. From another viewpoint, the Helias-like systems are smoother than the Heliac-like devices at fixed number of field periods. As a result, Helias-like systems are more favourable from the viewpoint of the stabilisation of very localised ballooning modes (in the extended poloidal representation) [14] (see Table 1).

2.2. Poloidal QS

The systems with poloidal direction of constant B-lines, like in a mirror device with straight axis (quasimirror symmetry, QMS), theoretically exhibit even more attractive features. At zero longitudinal net current, such systems have no secondary currents and there is no neoclassical transport here (the guiding centre trajectories lie on magnetic surfaces). Note that the QMS condition cannot be satisfied near the magnetic axis because of the curvature-induced poloidal nonuniformity of the magnetic field. This condition could be satisfied only at some distance from the magnetic axis when the mirror type nonuniformity of B surpasses the curvature effect [13].

3. LESS RESTRICTIVE CONDITIONS FOR ENHANCED PLASMA CONFINEMENT

Two approaches can be considered here that satisfy less restrictive condition for improved plasma confinement
˘ Fulfillment of only a part of the above-mentioned QS conditions (a) - (c).
˘ Fulfillment of these conditions only for that fraction of the particle distribution that follow the "most dangerous" guiding centre trajectories.

3.1. Pseudosymmetric systems

In the low-collisionality regime, the locally trapped particles constitute the most dangerous fraction. Their elimination should considerably decrease the low-collisional transport. To eliminate these particles, one should exclude the cases where the B-contours intersect the magnetic field line (**B**-line) at two rather closely spaced points. It means an absence of the B-line *tangent* to the **B**-line (otherwise the neighbouring B-lines will intersect the magnetic field line twice). This condition can be formulated as the independence of B on one of the angular variables of an arbitrary (not necessarily Boozer) flux coordinate system, $B=B(a,)$ or $B=B(a,)$ with straight magnetic field lines. Such systems were termed in Ref. [6] as pseudosymmetric (PS). The omission of criteria (c) and (d) required for QS opens an additional degree of freedom in the system which can be exploited to explore the possibilities to satisfy the PS condition in the entire plasma volume [6].

As in QS systems, the PS systems could have toroidal (axial and helical) and poloidal directions of the contours B = constant. They could be of both Helias and Heliac type as well. In contrast with the QS case, the condition of the poloidal PS can be fulfilled in the vicinity of the magnetic axis if the positions of the magnetic field extrema on the axis coincide with its zero curvature points.

The PS condition allows to eliminate the dipole secondary current in systems with helical and poloidal directions of constant B-lines and still permits to optimise the system towards stability. The results of numerical calculations of local mode stability in a few systems with poloidal direction of constant B-contours are shown in the Table 1. One can see from this table that the transition of the constant B-lines from the helical to the poloidal direction increases the plasma pressure limit significantly.

Thus, the systems with the poloidal direction of constant B-lines demonstrate some attractive features.

We see that the −limit increases with the number of periods. Another possibility to increase the −limit could be the hybrid of two mirror systems with PS stellarator-type equilibrium connectors [15]. With a low magnetic field in the mirror sections, such system could display attractive properties. Our first near-axis investigations show the possibility to close the secondary currents inside the connectors simultaneously with the fulfillment of the PS condition and with creation of the magnetic well.

TABLE 1. THE LOCAL MODE BETA-LIMITS [11,16,17]

Configuration	Mercier < >-limit	Ballooning mode < >-limit
Helical direction of the B-lines:		
$N = 4$, Heliac	3%	1%
$N = 4$, Helias	1%	1%
Poloidal direction of the B-lines:		
$N = 4$, Heliac	3%	3%
$N = 5$, Heliac	6%	6%
$N = 5$, Helias	6%	5%

3.2. Local omnigeneity

To improve the PS configurations, the condition of omnigeneity can be imposed in addition. As was shown in Refs. [4, 6], the global omnigeneity (on the whole magnetic surface) is equivalent to the QS condition. Instead, local omnigeneity (near the B_{min}-lines, i.e. for the deeply trapped particles) can be used for confinement improvement [3]. The deeply trapped particles being the most dangerous in general, appear to be the best confined in the locally omnigenous systems [5].

The local omnigeneity could be useful as well without the PS condition. As was shown in Ref. [5], the trapped-transition particles may not be so dangerous. Thus, the optimisation of the system without the PS condition also represents a path towards improved stellarator systems.

4. CONCLUSION

The understanding of the features of corrugated stellarator systems with a poloidal direction of the constant B-lines constitutes the main result of the ways to improve stellarator-like devices in the work that we have addressed in this paper. Starting from the concept of quasisymmetry optimisation of the mod-B behaviour, our computational results identify unfamiliar systems with nonuniform magnetic field that exhibit favourable properties (enhanced plasma confinement compatible with high enough). The straight mirrors combined with such optimized PS-stellarator connectors represent an attractive object for future research.

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