

CONTROL OF NEOCLASSICAL TEARING MODES IN LARGE TOKAMAKS

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Abstract

Some self-consistent effects pertaining to feedback control of neoclassical tearing modes in high temperature large tokamaks are investigated. For the ECRH scheme of local electron heating, it is shown that the self-consistent bootstrap currents created by the driven pressure gradients within the island are comparable to those due to the usually considered resistivity change mechanism. Similar self-consistent currents can also arise from pressure gradients created by density and energy deposition from neutral beams, thereby offering a new possibility of neoclassical mode control. The stabilising current in such an application of neutral beams is estimated. It is further shown that such a feedback scheme can be made even more effective through appropriate modulation of the beam source to match the phase variation arising from the island rotation.

I. INTRODUCTION:

Recent long pulse experiments for high beta tokamak discharges have demonstrated the difficulty of attaining the ideal magnetohydrodynamic (MHD) limit of plasma pressure due to the onset of low (m, n) resistive modes[1-5]. These instabilities which produce magnetic islands at the low order rational surfaces appear to be well described by the neoclassical tearing mechanism[1]. In this mechanism, a seed magnetic island at a low (m, n) rational surface flattens the equilibrium pressure gradient locally, thereby switching off the bootstrap current; this results in a θ -dependent negative current perturbation on the given rational surface which drives up the amplitude of the magnetic island by the Rutherford nonlinear growth mechanism. If the island is allowed to grow and saturate at a large width, it can significantly degrade the overall performance of the discharge. Neoclassical tearing modes are thus a major concern for future steady state high beta devices like ITER and means of controlling them are a subject of much current theoretical and experimental interest. At present two schemes are considered particularly attractive for control of neoclassical tearing modes, namely, through the use of electron cyclotron current drive(ECCD)[6, 7] and through resonant heating by electron cyclotron waves[8, 9]. In these methods, waves at the resonant electron cyclotron frequency are used to drive a current (directly or indirectly by local electron heating) at the O -point of the magnetic island, thereby suppressing the drive due to current density perturbation induced by the neoclassical mechanism. The existing theories of feedback control of neoclassical tearing modes however neglect the self-consistent bootstrap currents created by the driven pressure gradients within the island[8, 9, 10]. This is justified on symmetry arguments since the model equilibrium magnetic fields used in these theories retain the lowest order even term in the magnetic shear parameter. In this paper we reexamine this issue by retaining asymmetric terms in the magnetic shear and explicitly calculate the self-consistent bootstrap current perturbation at the O point due to the pressure gradients within the island created by a heat source (such as ECRH). We find such a contribution to be quite significant and comparable to the usual current perturbation calculated from the resistivity change mechanism. Their combined contribution in the island evolution equation helps to substantially reduce the saturation width of the island. The self-consistent contribution turns out to be particularly significant for high beta plasmas.

We next consider a new possible mechanism of neoclassical tearing mode control - specifically through the application of neutral beams. We show that the beams can act as an effective density and energy source which can also drive pressure gradients within the island and hence provide an additional stabilizing self-consistent bootstrap current perturbation. The control mechanism can be made even more effective through appropriate modulation of the neutral beam to match the phase variation arising from the island rotation. We estimate the requirements of a neutral beam source which may be used for such a stabilization scheme of neoclassical tearing modes in a large device like ITER.

II. SELF-CONSISTENT BOOTSTRAP CURRENTS DURING LOCAL HEATING:

In the conventional Rutherford theory[11], the nonlinear evolution equation for the island width is derived from the asymptotic matching condition of the current in the inner layer to the exterior parameter Δ' (the logarithmic jump in the vector potential across the magnetic surface), namely,

$$\frac{1}{2}\Delta'\psi_1 = \mu_0 R \int_{-\infty}^{\infty} d\rho \oint \frac{d\alpha}{2\pi} \cos(m\alpha) J_{\parallel} \quad (1)$$

where J_{\parallel} is the parallel current in the inner layer, ψ_1 is the perturbed flux function, R is the major radius, ρ is the radial coordinate and $\alpha \equiv \theta - \iota_0 \xi$ is the helical resonant angle formed from θ the poloidal angle and ξ the toroidal angle and ι_0 is the rotational transform at the rational surface. The equilibrium magnetic field is represented as $\vec{B} = \vec{\nabla}\Phi \times \vec{\nabla}\alpha + \vec{\nabla}\xi \times \vec{\nabla}\psi$ where $\vec{\nabla}\Phi \times \vec{\nabla}\alpha = I\vec{\nabla}\xi$ is the toroidal magnetic field. In the presence of a magnetic island the helical flux function is given by

$$\psi = \int d\Phi(\iota - \iota_0) - \psi_1 \cos(m\alpha) \quad (2)$$

where ψ_1 is the perturbation amplitude of the island. For $m \geq 2$ the constant ψ approximation holds so that ψ_1 is a weak function of the toroidal flux. By Taylor expanding the expression in the integral of (2) the flux function describing the magnetic surface close to the rational surface can be written down as,

$$\Psi = \frac{\psi}{q_s \psi'_s \iota'_o} = \frac{x^2}{2} + \frac{\iota''_o}{\iota'_o} \frac{x^3}{6} - \Psi_1 \cos(m\alpha) \quad (3)$$

where $\Psi_1 = \psi_1 / q_s \psi'_s \iota'_o = \psi_1 q_s / \psi'_s q'_s$ with $\psi'_s = \psi'(\rho_s) = (d\psi/d\rho)_{\rho=\rho_s}$ and $x = \rho - \rho_s$ is a measure of the distance away from the rational surface. If ψ_1 is taken to have the same sign as ι'_o , then the O-point of the island is located at $m\alpha=0$ and the X-point at $m\alpha = \pm\pi$. The full width of the island is then approximately given by $W = 4\sqrt{\Psi_1}$. Note that the term proportional to x^3 in (3) is smaller than the x^2 term and is generally neglected. We have retained it for its odd parity which becomes important when contributions due to the symmetric term average to zero. Finally the parallel current J_{\parallel} is assumed to satisfy $\vec{B} \cdot \vec{\nabla} J_{\parallel} = 0$ in the vicinity of the island and hence is taken to be a function of the flux surface, $J_{\parallel} = J_{\parallel}(\Psi)$.

In applying the Rutherford prescription to the neoclassical regime, an appropriate modification of the Ohm's law is made to model the dynamics in the inner layer. As discussed in [12], the current contributions in this region can be classified in terms of their origin as,

$$J_{\parallel}(\Psi) = J_{ind} + J_{bs} + J_{aux} \quad (4)$$

where

$$J_{ind} = \frac{1}{\eta} \langle E_{\parallel} \rangle = \frac{1}{\eta} \frac{1}{R} \frac{d\psi_1}{dt} \langle \cos(m\alpha) \rangle \quad (5)$$

is the contribution from the inductive electric field and

$$J_{bs} = \frac{1}{B} \langle \vec{B} \cdot \vec{\nabla} \cdot \pi_{\parallel e} \rangle = -\frac{1}{B} \frac{\mu_e}{\nu_e} \frac{I}{\psi'_s} \langle \frac{dP}{dx} \rangle \quad (6)$$

is the neoclassical contribution giving rise to the perturbed bootstrap current. Here μ_e is the electron viscous damping rate and ν_e is the electron collision frequency. Finally the last term J_{aux} allows for an externally controlled driven current which we will discuss shortly. To calculate the bootstrap current contribution (6) one needs to determine the appropriate pressure profile in the inner region. For sufficiently large magnetic islands, the pressure profile in the vicinity of the rational surface is also a perturbed flux function, $p = p(\Psi)$. It can be obtained by solving an appropriate diffusion equation which assumes a crossfield diffusion process with a pressure source located away from the rational surface and is given by[12],

$$\frac{dp}{d\Psi} = p'_s \frac{\Theta(\Psi - \Psi_1)}{\oint \frac{d\alpha}{2\pi} \sqrt{2\Psi + \frac{W^2}{8} \cos(m\alpha)}} \quad (7)$$

where, $p'_s = \frac{dp}{d\rho} |_{eq}$ is the equilibrium value of the pressure gradient in the absence of the magnetic island and Θ is a step function. This model thus incorporates a flat spot inside the island separatrix that shuts off the bootstrap current locally and thereby drives the island unstable. The basic idea in most control schemes is therefore to drive an external current J_{aux} in this region by direct or indirect means. The localized heating scheme using ECRH is an indirect scheme in which the heating induced self-consistent temperature variations cause variations in the parallel current profile through the resistive Ohm's law. The magnitude of this perturbed current can be easily estimated as,

$$\frac{\delta J_{\parallel}}{J_{\parallel 0}} = -\frac{\delta \eta}{\eta_0} = \frac{3}{2} \frac{\delta T_e}{T_{e0}} \quad (8)$$

where $J_{\parallel 0} = E_{\parallel} / \eta_0$ is the externally driven Ohmic current. To estimate the temperature perturbations within the island one needs to calculate the temperature profile induced by the local heating source. Solving a model diffusion equation,

$$\vec{\nabla} \cdot [\chi_{\perp} n \vec{\nabla} T_e] = -S_T \quad (9)$$

where χ_{\perp} is the cross-field diffusivity and S_T is the local heating source, the temperature profile inside the island has the form,

$$\frac{dT_{e0}}{d\Psi} = -\frac{\int_{-1}^{\Omega} d\Omega' \oint d\alpha \frac{S_T(\Omega, \alpha)}{\sqrt{\Omega' + \cos(m\alpha)}}}{\oint d\alpha n \chi_{\perp} \sqrt{\Omega' + \cos(m\alpha)}} \quad (10)$$

where $\Omega = \frac{16\Psi}{W^2}$, is the normalized flux surface level and n is the density. For a simple step function heating profile $S_T = S_{T0} \Theta(\Omega_c - \Omega)$ with $\Omega_c > 1$ and uniform χ_{\perp} , the temperature gradient inside the island is given by,

$$\frac{dT_e}{d\Psi} = -\frac{S_{T0}}{n\chi_{\perp}} \quad (11)$$

Using the above estimate the temperature perturbation is seen to scale as $W^2 S_{T0} / 8n\chi_{\perp}$, so that the corresponding perturbed current inside the island is,

$$J_{aux} = \delta J_{\parallel} = \frac{3}{16} \frac{W^2 S_{T0} J_{\parallel 0}}{n T_{e0} \chi_{\perp}} \quad (12)$$

Substituting the above discussed expressions for J_{ind} , J_{bs} , J_{aux} in (1) one can obtain an island evolution equation in which it has been shown[9, 10, 12] that the J_{aux} contribution arising from

the variation in the resistive Ohm's law has a stabilising influence on the island growth and leads to a reduced saturated width. However these calculations ignore another contribution of the driven temperature (pressure) gradient (10), namely, the self-consistent bootstrap current arising from it. This contribution normally vanishes in the flux averaging process when the equilibrium flux expression retains only the lowest term of the Taylor expansion. When the asymmetric x^3 term is retained in Ψ we obtain a finite contribution to the bootstrap current from the driven pressure gradient term within the island. Specifically our pressure profile model for the calculation of the neoclassical contribution has the form,

$$\begin{aligned} \left\langle \frac{dP}{dx} \right\rangle &= \left\langle \frac{d\Psi}{dx} \right\rangle \frac{dP}{d\Psi} \\ &= \left\langle x + \frac{\iota_o''}{\iota_o'} \frac{x^2}{2} \right\rangle \left[\frac{\text{sign}(x)p'_s \Theta(\Psi - \Psi_1)}{\oint \frac{d\alpha}{2\pi} \sqrt{2\Psi + \frac{W^2}{8} \cos(m\alpha)}} - \frac{S_{T0}}{\chi_\perp} \Theta(\Psi_1 - \Psi) \right] \end{aligned} \quad (13)$$

Evaluation of (1) with this pressure model and the $J_{||}$ contributions listed in (4) gives us the following time evolution equation for the island width,

$$0.82 \frac{dw}{dt} = \frac{1}{\tau_r} (\Delta' \rho_s + \frac{D_{nc}}{w} - w D_{heat} - w D_{bs}) \quad (14)$$

where $w = W/\rho_s$ is the island width normalized to the local minor radius and $\tau_r = \mu_o \rho_s^2 / \eta$. The various coefficients are given as,

$$D_{nc} = -4.6 \sqrt{\epsilon} \frac{2\mu_o p'_s R^2 q_s}{\psi_s'^2 q'_s}, \quad (15)$$

$$D_{heat} = \frac{16}{5\pi} \frac{R \mu_o J_{||0} q_s}{\psi_s'^2 q'_s} \frac{S_{T0} \rho_s^2}{n T_e \chi_\perp}, \quad (16)$$

$$D_{bs} = 0.14 \sqrt{\epsilon} \frac{\mu_o \rho_s^2 R^2 q_s}{\psi_s'^2 q'_s} \frac{S_{T0} \iota_o''}{\chi_\perp \iota_o'} \quad (17)$$

where $\epsilon = \rho_s/R$ is the local inverse aspect ratio. The term proportional to D_{bs} is the additional self-consistent bootstrap contribution arising from the pressure gradient within the island induced by the local heating source. Its functional form is similar to the usual resistivity modified current contribution term proportional to D_{heat} and its effect is also stabilising. The relative magnitudes of the two contributions are also comparable as can be seen from a comparison of the two coefficients.

$$G = \frac{D_{bs}}{D_{heat}} = 0.14 \sqrt{\epsilon} \frac{R \iota_o'' n T_e}{\psi_s' \iota_o' J_{||0}} \quad (18)$$

Typically this ratio is of the order of $\beta_p/\sqrt{\epsilon}$ which can be of order unity or larger particularly for high beta plasmas. From (14) the saturated island width is now given by,

$$w_{sat} = \frac{D_{nc}}{-\Delta' \rho_s} \frac{2}{1 + \sqrt{1 + Y}} \quad (19)$$

where $Y = 4 \frac{D_{heat} D_{nc}}{(-\Delta' \rho_s)^2} (1 + G)$ is a measure of the consolidated effect of localized heating. As is clear the additional bootstrap current contribution helps to reduce the size of the saturated island width by enhancing the effect of D_{heat} .

III. FEEDBACK CONTROL USING NEUTRAL BEAMS:

The beneficial aspects of the self-consistent bootstrap currents from driven pressure gradients within the island layer have led us to examine other options of locally altering island pressure profiles. Modulated neutral beams are an attractive possibility since they can deliver controlled sources of density, momentum and energy into the plasma at appropriate phase and amplitude to alter the local plasma properties. With the present available beam energies they can effectively penetrate to the $q = 2$ to $q = 1$ surfaces in order to influence the evolution of the (1, 1), (3, 2) and (2, 1) tearing modes. Such a scheme has already been proposed for the control of kink and kink-ballooning precursor modes of major disruption[13] and more recently for the control of resistive drift tearing modes[14]. Based on our model calculations in the previous section we sketch a possible scheme for application of neutral beams in the control of neoclassical tearing modes.

The basic advantage of the neutral beam based scheme is that the contributions to J_{aux} now result both from the local heating effect and the deposition of external density. The pressure profile within the layer can be decomposed as,

$$\frac{dp}{d\rho} = \frac{d}{d\rho}(nT_e) = n\frac{dT_e}{d\rho} + T_e\frac{dn}{d\rho} \quad (20)$$

The density gradient $dn/d\rho$ can be calculated using a similar model calculation discussed in the previous section i.e. by solving an appropriate particle diffusion equation,

$$\vec{\nabla} \cdot (D\vec{\nabla}n) = -S_n \quad (21)$$

For a uniform D and $S_n = S_{n0}\Theta(\Omega_c - \Omega)$ with $\Omega_c > 1$ the above equation is once again easily solved. The total pressure gradient inside the island is now given as,

$$\frac{dp}{d\Psi} = -\left(\frac{S_{T0}}{\chi_{\perp}} + \frac{S_{n0}T_e}{D}\right) = -\frac{S_{T0}}{\chi_{\perp}}\left(1 + \frac{S_{n0}T_e\chi_{\perp}}{S_{T0}D}\right) \quad (22)$$

The corresponding island evolution equation has the same form as before but with enhanced coefficients and is given by,

$$0.82\frac{dw}{dt} = \frac{1}{\tau_r}[\Delta'\rho_s + \frac{D_{nc}}{w} - wD_{heat}(1 + G_1)] \quad (23)$$

where, $G_1 = G(1 + \frac{S_{n0}T_e\chi_{\perp}}{S_{T0}D})$ incorporates the effect of both the density and heat injection and can be used to compare the effectiveness of the neutral beam scheme to the conventional ECRH scheme for the stabilisation of the neoclassical tearing modes. A rough measure of this comparison can be obtained by equating the effective source contributions in both schemes, namely,

$$\frac{S_{T,ECRH}}{\chi_{\perp}} = \frac{S_{T,NB}}{\chi_{\perp}} + \frac{S_{n,NB}}{D} \quad (24)$$

Since the source functions S_T, S_n are proportional to the beam or rf power (24) leads to the following relation between the respective power requirements,

$$P_{ECRH} = \frac{\eta_{NB}P_{NB}}{\eta_{ECRH}} \left[1 + \frac{S_{n,NB}\chi_{\perp}}{S_{T,NB}D}T_e \right] \quad (25)$$

where the efficiency factors $\eta_{NB,ECRH}$ have been introduced to account for the coupling efficiencies of the beam and RF waves to the neoclassical tearing mode. The efficiency of the geometrical coupling of the neutral beam is proportional to the modal coefficient of the double Fourier series expansion of the toroidal dimension L_T and poloidal dimension L_p of the beam

footprint over the spatial periods $2\pi R$ and $2\pi a$ where a is the minor radius of the tokamak. Assuming $L_T \ll 2\pi R$ and $L_p \ll 2\pi a$, the coupling coefficient η_{NB} can be written as,

$$\eta_{NB} \approx \frac{L_T L_p}{4\pi^2 R a} \mathcal{R}_{NB} \quad (26)$$

where \mathcal{R}_{NB} is determined by the convolution of the beam deposition profile and the spatial structure of the mode in that direction. An analogous expression holds for the geometric coupling of the ECRH waves. The ratio of the two coupling terms η_{NB}/η_{ECRH} is therefore proportional to $\mathcal{R}_{NB}/\mathcal{R}_{ECRH}$. For a radial injection the coupling of the beam to the mode is relatively weak[14] and the above ratio can be quite small. However for a poloidal injection of the beam the convolution would be along a chord (beam line) in the poloidal plane and therefore comparable to the convolution factor for the ECRH scheme. As to the other terms in (24), the ratio χ_{\perp}/D can typically vary from unity to rather small values at the tokamak plasma edge. For a large device like ITER we can approximately take this ratio to be of order unity near the $q = 2$ surface. Further, writing

$$S_{T,NB} = E_b S_{N,NB} \left(1 + \frac{\sigma_{q-x}}{\sigma_{ion}} \right) \quad (27)$$

where σ_{q-x} and σ_{ion} are the charge exchange and ionisation cross sections for the beam and E_b is the energy of the beam, we see that the ratio $\eta_{ECRH} P_{ECRH}/\eta_{NB} P_{NB}$ is of the order of $1 + (T_e/E_b)(1 + \sigma_{q-x}/\sigma_{ion})^{-1}$ thus making the two schemes comparable in terms of power requirements. The beam energy required to penetrate to the $q = 2$ surface in ITER, which is a distance of about 50 cm from the plasma edge[15], is approximately 100 keV[16] and well within the present energy range of beams.

Our calculations so far have been restricted to a static island configuration where the oscillating time variation of the tearing mode has been ignored. As is well known, the mode has a real frequency which is of the order of the drift frequency and which causes the island to rotate in time[17]. This poses a problem for any static control scheme and increases their overhead by the fraction of time that the island is away from their influence. The neutral beam feedback scheme can be made more effective by modulating the neutral beam source to match the phase variation arising from the island rotation. To illustrate this scheme, we incorporate a real frequency term in our model evolution equation (23) and rewrite it as follows,

$$\frac{d\Psi_1}{dt} = \left(\frac{C}{w} + \frac{D}{w^2} - i\omega_0 - F \right) \Psi_1 \quad (28)$$

where ω_0 is the drift tearing frequency and

$$\Psi = \Psi_1 e^{-i\omega t} \quad (29)$$

is represented in the complex notation with $\omega = \omega_R + i\gamma$, C and D are real constants proportional to the Δ' and D_{nc} terms and w is treated as a constant. The coefficient F is proportional to the neutral beam source and is modelled as a complex term $F = F_0 \exp(i\phi)$ where ϕ is the phase factor. Eqn.(28) is in the standard form of a linear feedback scheme[13, 14], from which we can easily get the following two conditions,

$$\omega_R = \omega_0 + F_0 \cos(\phi) \quad (30)$$

and

$$\gamma = \gamma_0 - F_0 \sin(\phi) \quad (31)$$

where $\gamma = (C/w + D/w^2)$. These two relations demonstrate the nature of control possible with the help of the phase parameter ϕ and the strength of the amplitude F_0 . At saturation $\gamma = 0$ and the size of the island is determined from (31). This fixes the quantity $F_0 \sin(\phi)$ but allows

some freedom in the choice of the independent parameters F_0 and ϕ . In particular ϕ can be chosen so as to reduce the net rotation frequency of the mode as defined by (30). This can facilitate the tracking of the mode and improve the quality of the feedback control.

IV. DISCUSSION AND CONCLUSION:

Our principal result in this paper is the demonstration that driven pressure gradients within the island region (due to ECRH for example) can lead to finite self-consistent bootstrap currents within the island that reduce the growth of a neoclassical tearing mode. This contribution that has been neglected in earlier calculations from symmetry arguments, survives when the next order magnetic shear terms are retained in the equilibrium magnetic field and its magnitude is comparable to the usual current perturbation calculated from the resistivity change mechanism. Their combined contribution in the island evolution equation substantially reduces the saturation width of the island. This enhances the effectiveness of the ECRH scheme for neoclassical tearing mode control and also raises the possibility of other alternate schemes that can build on this effect. We suggest the use of modulated neutral beams as one such scheme in which pressure gradients within the island can be effectively altered by controlled delivery of both density and energy at appropriate phase and amplitude. Our preliminary estimates show that such a scheme is feasible and of comparable efficacy to the ECRH scheme in terms of power requirements and other parameters and therefore warrants a more detailed study.

Finally we would like to remark that another consequence of neutral beam injection (particularly in the unbalanced parallel injection mode) is the production of large scale toroidal rotation in the plasma. The concomitant change in the equilibrium pressure profiles can have a significant influence on the neoclassical tearing mode evolution and have not been studied so far. Large flows can change not only the magnitude of the Δ' parameter in the external region but also bring about mode coupling between various resonant surfaces due to the centrifugal force induced poloidal asymmetry in the equilibrium pressure profile. A detailed calculation of this effect including the appropriate inner layer modifications in the dynamics of the neoclassical tearing mode is presently in progress.

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