Self-field in free-electron laser with planar wiggler and ion-channel guiding

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The theory of self-electric and self-magnetic fields of a relativistic electron beam passing through a planar wiggler and ion-channel guiding are analyzed .We have shown effects of self fields on the motion of electron by numerical study.

The application of an ion-channel as an electron beam-guiding medium not only eliminates the use of solenoid or quadruple magnets [1], but also allows for beam currents higher than the vacuum limit [2]. Passage of a relativistic electron beam through a planar wiggler and ion-channel guiding is analyzed [3]. Equilibrium self-electric and magnetic fields, and single particle trajectories are expressed.[4-6]

In the idealized one-dimensional approximation, consider a relativistic electron moving along z-axis of a planar wiggler magnetic field described by

$$\mathbf{B}_w = \mathbf{e}_y B_w \sin(k_w z) \qquad (1)$$

where $k_w = 2\pi/\lambda_w$ is the wiggler wave number. The transverse electrostatic field generated by an ion-channel guiding may be written as

$$\mathbf{E}_i = 2\pi e n_i r \mathbf{e}_\theta = 2\pi e n_i (x\mathbf{e}_x + y\mathbf{e}_y) \qquad (2)$$

where the n_i is the density of positive ions with charge e. The equation of motion of the electron can be written as

$$\frac{d\mathbf{p}}{dt} = -e\left(\mathbf{E}_i + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) \qquad (3)$$

By using Eqs. (1) and (2), the electron equation of motion may be written as

$$\frac{dv_x}{dt} = -\omega_i^2 x + \Omega_w v_z \sin(k_w z) \qquad (4)$$

$$\frac{dv_y}{dt} = -\omega_i^2 y \qquad (5)$$

$$\frac{dv_z}{dt} = -\Omega_w v_x \sin(k_w z) \qquad (6)$$

where $\omega_i^2 = 2\pi e^2 n_i / m\gamma$ and $\Omega_w = eB_w/\gamma mc$. The steady state solution of Eqs. (4)-(6) with constant axial velocity $v_z \approx v_\parallel = cons$. (to first order in the wiggler amplitude) have been found to be

$$v_x = \frac{\Omega_w k_w v_{\|}^2}{\omega_i^2 - k_w^2 v_{\|}^2}\cos(k_w z) \qquad (7)$$

$$v_y \approx 0 \qquad (8)$$

$$v_z \approx v_{\|} = cons. \qquad (9)$$

with assuming $v_y << v_{\|}$.

In order to model the self-fields of the electron beam, we make the assumption of a homogeneous electron density profile,

$$n_b(r) = \begin{cases} n_b & r \le r_b \\ 0 & r > r_b \end{cases}$$

n_b is the number density of the electrons and r_b is the radius of the beam. By solving Poisson's equation, $\frac{1}{r}\frac{\partial}{\partial r}(rE^{(s)}{}_r) = -4\pi\, e n_b(r)$, the self –electric field obtain

$$\mathbf{E}^{(s)} = -2\pi e n_b r \mathbf{e}_r \qquad (10)$$

and by using the Ampere low, the self magnetic field produced by axial velocity can be expressed by

$$\mathbf{B}^{(s)} = -2\pi e n_b \beta_{\|} r \mathbf{e}_{\boldsymbol{\theta}} \qquad (11)$$

To explain the other component of self-magnetic field, we consider the electron moving along x-axis as an oscillating electron with a velocity along x-axis

$$\mathbf{B}^{(s)} = \mathbf{e}_y \frac{4\pi e n_b}{c}\frac{\Omega_w v_{\|}^2}{\omega_i^2 - k_w^2 v_{\|}^2}\sin(k_w z) \qquad (12)$$

So the total self–magnetic field can be obtained from Eqs. (11) and (12)

$$\mathbf{B}^{(s)} = -2\pi e n_b \beta_{\|}(x\mathbf{e_y} - y\mathbf{e_x}) + \mathbf{e_y}\frac{2\omega_b^2 \beta_{\|}^2}{\omega_i^2 - k_w^2 v_{\|}^2} B_w \sin(k_w z) \qquad (13)$$

where $\beta_{\|} = v_{\|}/c$ and $\omega_b^2 = 2\pi e^2 n_b / \gamma m$. Finally, the total electric fields and total magnetic fields are

$$\mathbf{E} = 2\pi e(n_i - n_b)(x\mathbf{e}_x + y\mathbf{e}_y) \qquad (14)$$

$$\mathbf{B} = -2\pi e n_b \beta_{\|}(x\mathbf{e_y} - y\mathbf{e_x}) + \mathbf{e_y}\lambda B_w \sin(k_w z) \qquad (15)$$

$$\lambda = 1 + \frac{2\omega_b^2 \beta_{\|}^2}{\omega_i^2 - k_w^2 v_{\|}^2} \qquad (16)$$

Substitution Eqs. (14) and (15) in equation of motion, we obtained

$$\frac{dv_x}{dt} = -\left[\omega_i^2 - \omega_b^2(1-\beta_\parallel\beta_z)\right]x + \lambda\Omega_w v_z \sin(k_w z) \qquad (17)$$

$$\frac{dv_y}{dt} = -\left[\omega_i^2 - \omega_b^2(1-\beta_\parallel\beta_z)\right]y \qquad (18)$$

$$\frac{dv_z}{dt} = \omega_b^2\beta_\parallel(x\beta_x + y\beta_y) - \lambda\Omega_w v_x \sin(k_w z) \qquad (19)$$

The axial velocity has an oscillation of second order in the wiggler amplitude, so for the first order of wiggler amplitude and low current density of electron beam, we can consider $v_z \approx v_\parallel \equiv cons.$ and the steady -state solution in the presence of self-fields can be written as

$$v_x = \frac{\lambda\Omega_w v_\parallel^2 k_w}{\omega_i^2 - \frac{\omega_b^2}{\gamma_\parallel^2} - k_w^2 v_\parallel^2}\cos(k_w z) \qquad (20)$$

$$v_y \approx 0, \quad v_{y0} << v_\parallel \qquad (21)$$

In comparison with Eq. (7), the correction factors may be expressed in the form

$$(B_w)_{eff} = \lambda B_w \qquad (22)$$

and

$$(n_i)_{eff} = \mu n_i \qquad (23)$$

where $\mu = 1 - n_b^2 / n_i^2\gamma_\parallel^2$.

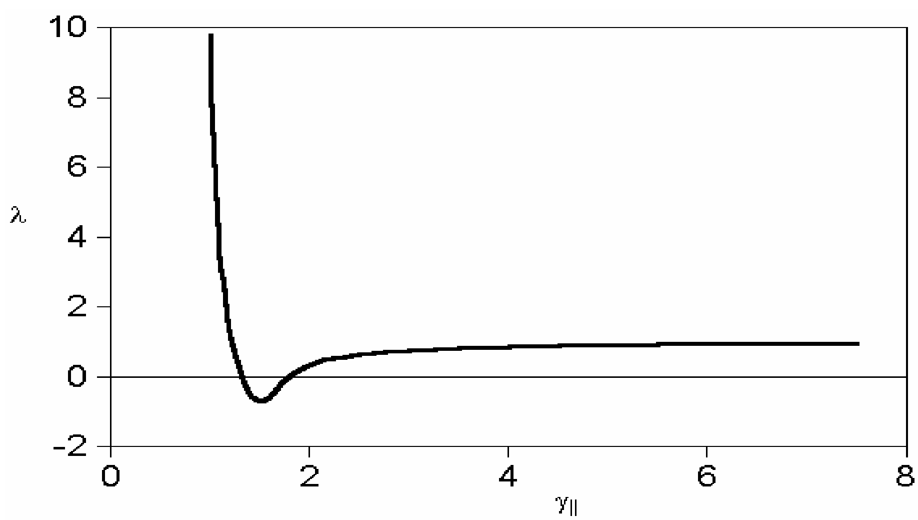

Figure 1: Correction factor λ for the wiggler magnetic field

as a function of the axial Lorentz factor $\gamma_\parallel$ for $n_i / n_b = 0.5$

The numerical calculation is done for finding correction factors λ and μ with beam electron density $n_b = 10^{12} cm^{-3}$ and wiggler wavelength $\lambda_w = 2cm$. Figure 1 shows correction factor λ as function of the Lorentz factor $\gamma_{\|}$ for $n_i / n_b = 0.2,\ 0.4$ and figure 2 shows correction factor μ for $n_i / n_b = 0.5$ as afunction of Lorentz factor. It is concluded that the self-field corrections are large at low values of $\gamma_{\|}$ but negligible when $\gamma_{\|}$ is sufficiently large.

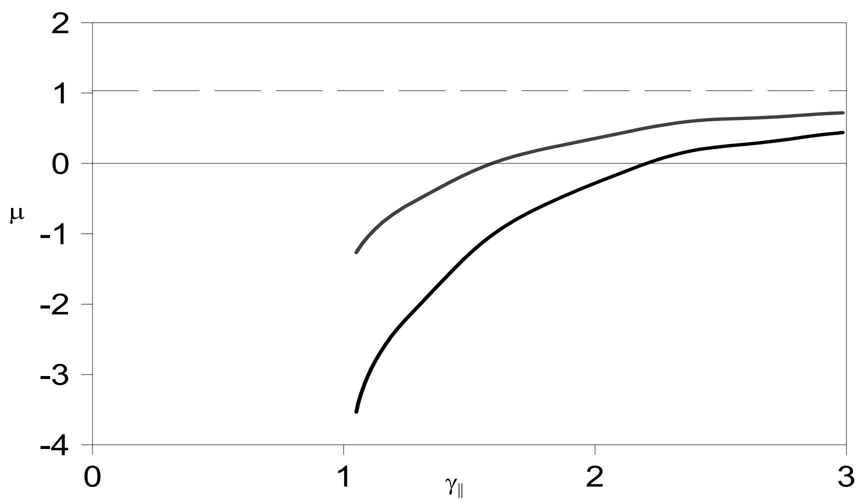

Figure 2: Correction factor μ as a function of $\gamma_{\|}$.

Upper line is for $n_i / n_b = 0.4$ and the other is for $n_i / n_b = 0.2$.

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