Relativistic current driven nonlinear Langmuir structure in plasmas

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The nonlinear stationary states of a relativistic electron beam moving in a homogeneous positive background are calculated for the full range of amplitudes of a longitudinal self-induced electric field in the collision less limit. The parameter that controls the system is the ratio of the electrostatic energy of the fluctuations over the kinetic energy of the beam (κ). In the collision less limit it might be that the number density of electrons and velocity of the beam in the stationary case are constant. If $\kappa > 2$ it is shown that no wave breaking occurs. Instead, the electric field becomes discontinuous at certain points and the electrons delay there forming periodic electrostatic (Langmuir) structures centered on negatively charged planes. The size and charge of the above structures as well as their wavelength, which now depends on the ratio of the electrostatic energy of the fluctuations over the kinetic energy of the beam, are derived.

I. INTRODUCTION

In the present paper we propose to construct the basis of a nonlinear relativistic theory of plasma oscillations by studying the movements of electrons in one of the electron relativistic beams forming the velocity distribution for the full range of amplitudes of the induced electric field. In particular, it is assumed that a monochromatic relativistic beam of cold electrons having fluid velocity v_e (x) and
number density n_e (x) is moving in a fixed homogeneous positive ion background of number density n_0. In the collision less limit it might be that v_e=v_0, n_e= n_0 (both constant) so that the relativistic beam would from a constant electric current $j = en_0 v_0$. On the other hand, an infinite set of other possibilities exist according to which the electrons accelerate and retard periodically in space so that their density decreases and increases, respectively, in accordance with the equation of continuity.
These periodic departures from charge neutrality induce in turn a longitudinal electric field E(x) which exerts the necessary force on the electrons so that the whole process is kept in a stationary state.

It turns out that in this case the basic parameter that controls the system is

$$\kappa_R = \left[\frac{E_m}{4\pi n_0 m_0 \gamma_0 v_0^2} \right]^{\frac{1}{2}} \qquad (1)$$

here E_m is the amplitude of the electric field and v_0, n_0, respectively, are the electron velocity and density at the points in space where electrons are submitted to maximum Force : E(x)= $\pm E_m$. If $\kappa_R \ll 1$ the variables v_e (x), n_e(x), $\phi(x)$, and E(x) vary harmonically in space in accordance with linear relativistic theory so that the wavelength of nonlinear relativistic oscillations is independent of $\kappa_R \ll 1$. In the case $\kappa_R = \sqrt{2/(1+\gamma_0)}$, ($\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$), it is found that the electric field gradient is infinitely steep periodically in space so that according to Poison's law the electron density becomes also infinitely large forming relativistic periodic electrostatic (relativistic Langmuir) structures. One could show that the electrons stop momentarily at the center of these structures and then continue their motion again by Study of individual electron movement. Similar explosive behavior of electric field gradient and electron density has been also found in the time dependent non relativistic problem. In the cases $0 \le \kappa_R \le \sqrt{2/(1+\gamma_0)}$, $\sqrt{2/(1+\gamma_0)} \le \kappa_R < \infty$ it is shown that in addition to the infinite electric field gradient and electron density, occurring periodically at certain singular points, the electric field becomes also discontinuous on these points. Therefore, according to Poisson's law a negatively charged plane is formed at the center of each relativistic Langmuir structure. Also in this case the wavelength of the relativistic Langmuir structures depends on κ_R and the study of the individual electron movement could be shown that the electrons delay for certain time at each charged plane before moving on. At the limit $\kappa_R \to +\infty$ the relativistic Langmuir structures collapse to a 1-D (one – dimensional) crystal. The size and charge of the above structures in each of them have been exactly calculated from the solution of the nonlinear relativistic equations governing the system and are consistent with the concepts of continuity and global neutrality over the wavelength of the relativistic beam. It should be emphasized that the formation of charged planes within the beam is not an adhoc assumption but it is a result of the discontinuity of the electric field occurring in the $E-\phi$ phase space of the system. In the first part of the article, the linear relativistic theory of the present problem is developed. In the second part the basic nonlinear relativistic equations are solved and the macroscopic variables, modulated in terms of the amplitude of the electric field, are calculated.

II. LINEAR RELATIVISTIC THEORY

Let us consider the system of nonlinear relativistic Equation:

$$\frac{\partial \mathbf{p}_e}{\partial t} + (\mathbf{v}_e.\nabla)\mathbf{p}_e = -eE \qquad (2)$$

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e \mathbf{v}_e) = 0 \qquad (3)$$

$$\nabla.\mathbf{E} = 4\pi e(n_0 - n_e) \qquad (4)$$

and linearize it by allowing $p_0 \neq 0$ or $v_0 \neq 0$ (i.e., an electric current is now present) but keeping E_{ex}=0 (zero external electric field)

$$v_0 \frac{dP_{1e}}{dx} = -e(-\frac{d\phi}{dx})$$

$$n_0 \frac{dv_{1e}}{dx} + v_0 \frac{dn_{1e}}{dx} = 0 \qquad (5)$$

$$\frac{dE_1}{dx} = -4\pi e n_{1e}$$

Using traveling wave solutions of the from $e^{-i(\omega t - \kappa x)}$ (monochromatic) in stationary state $(\frac{\partial}{\partial t} = \circ, \frac{\partial}{\partial x} = \frac{d}{dx})$ we obtain:

$$\frac{d\phi}{dP_{1e}} = \frac{v_0}{e}$$

$$\frac{dn_{1e}}{dx} = -\frac{n_0}{v_{1e}}\frac{dv_{1e}}{dx} \qquad (6)$$

$$\frac{dE_1}{dx} = -4\pi e n_{1e}$$

by solution Eq (5) we obtain differential equation for E_1:

$$\frac{d^2E_1}{dx^2}+\left[\frac{\frac{4\pi n_0 e^2}{m_0}}{v_0^2}\right](1-\frac{v_{1e}^2}{c^2})^{3/2}E_1=0 \quad (7)$$

The solution of which is given by dispersion relation:

$$\omega_p'^2=\frac{\omega_p^2}{v_0^2}(1-\frac{v_{1e}^2}{c^2})^{3/2} \quad (8)$$

We see that Eq. (6) admits stationary waves of wavelength $\lambda=2\pi s'$;

$$s'=\frac{v_\circ}{\omega_p'}=\frac{4}{\sqrt{\gamma_\circ}}\frac{v_\circ}{\omega_p}=\left(\frac{m_\circ\gamma_\circ v_\circ^2}{16\pi n_\circ e^2}\right)^{\frac{1}{2}} \quad (9)$$

Note that within the context of the linear relativistic theory, stationary waves cannot exist in the absence of electric current $v_\circ=\circ$ because relation $\omega=\omega_p{}'$ represent only traveling standing waves.

It can be shown by solving the steady state of Eq. (4) that in the linear limit $(\kappa_R \langle\langle 1)$ the fluid variables of the stationary waves are harmonic in space (Fig. 1).

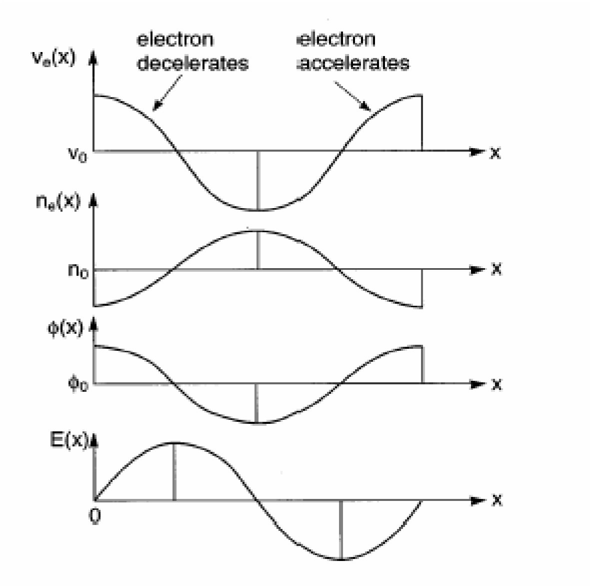

FIG.1.Harmonic behavior of fluid variables at the small fluctuation limit

$$E=E_m\sin\sqrt{\frac{\gamma_0}{4S'^2}}x$$

$$\phi(x)=\phi_0+\sqrt{\frac{4S'^2}{\gamma_0}}E_m\cos(\sqrt{\frac{\gamma_0}{4S'^2}}x)$$

$$P_{1e}=P_0(1+\frac{\kappa_R}{\sqrt{\gamma_0}}\cos\sqrt{\frac{\gamma_0}{4S'^2}}x) \quad (10)$$

$$n_{1e}=n_0(1-\kappa_R\sqrt{\gamma_0}\cos\sqrt{\frac{\gamma_0}{4S'^2}}x)$$

$$v_{1e}=v_0(1+\kappa_R\sqrt{\gamma_0}\cos\sqrt{\frac{\gamma_0}{4S'^2}}x)$$

Where $\phi(x)$ represents the electric potential

($E = -\frac{d\phi}{dx}$), E_m is the amplitude of the electric field. κ_R; s' are defined by Eqs. (1), (9), respectively. It could be that, in Eq. (9) $\phi_\circ$, is an arbitrary additive potential and v_0, n_0 are average values of v_e, n_e along x. However; as we will see in the next section, such definition of n_0, v_0, n_0 is not tenable in the context of a nonlinear relativistic theory. The movement of an individual electron in the relativistic linear limit can be calculated by integrating Eq. (9):

$$v_{1e} = \frac{dx_{1e}}{dt} = v_0\left[1 + \kappa_R\sqrt{\gamma_0}\cos\sqrt{\frac{\gamma_0}{4S'^2}}x\right] \quad (11)$$

The solution of which is given by

$$x_e(t) = 2\sqrt{\frac{4S'^2}{\gamma_0}}\arctan\left\{\sqrt{\frac{1+\kappa_R}{1-\kappa_R}}\tan\left(\frac{\sqrt{1-\kappa_R^2}}{2\sqrt{\frac{4S'^2}{\gamma_0}}}v_0 t\right)\right\} \quad (12)$$

Where $x_e(0) = 0$. Expanding x_e in terms of κ_R and using $\omega'_p = \frac{v_0}{s'}$ we further obtain

$$x_e(t) = v_0 t + \sqrt{\frac{4S'^2}{\gamma_0}}\kappa_R\sin(\omega_p t) + O(\kappa_R^2) \quad (13)$$

3. NONLINEAR RELATIVISTIC THEORY

The stationary states of an electron relativistic beam can be calculated by solving the steady state of the nonlinear Eqs. (2)

$$v_e\frac{\partial P_e}{\partial x} = -eE$$

$$\frac{\partial}{\partial x}(n_e v_e) = 0 \quad (14)$$

$$\frac{\partial E_1}{\partial x} = 4\pi e(n_0 - n_e)$$

Integrating the first two fluid equations and using $E = -\frac{d\phi}{dx}$ we obtain:

$$\frac{m_0c^2}{\sqrt{1-\frac{v_e^2}{c^2}}} - \frac{m_0c^2}{\sqrt{1-\frac{v_0^2}{c^2}}} = e(\phi(x) - \phi_0) \quad (15)$$

where ϕ_0, n_0, v_0 are constants that will be defined below and m_0 the mass of electron in rest. Also it assumed that $v_{e\geq 0}$ i.e., the direction of the beam never reverses. Introducing Eqs. (14) into Poisson's equation we obtain an equation for ϕ,

$$\frac{d^2\phi}{dx^2} = -4\pi e n_0\left(1 - \frac{v_0}{c}\frac{\alpha}{\sqrt{\alpha^2 - 1}}\right)$$

where $\alpha = \frac{1}{\sqrt{1 - v_0^2/c^2}} + \frac{e}{m_0c^2}(\phi - \phi_0)$

Multiplying both sides by $d\phi/dx$ we have

$$\frac{dE^2}{d\phi} = -8\pi e n_0\left(1 - \frac{v_0}{c}\frac{\alpha}{\sqrt{\alpha^2 - 1}}\right) \quad (16)$$

where $\alpha \equiv \alpha(\phi(x))$.A few conclusions can be drawn from Eq. (16): (i) $E^2=E^2_m$ (max. value) at $\phi=\phi_0$ where [from Eqs. (14)] (ii) inversely ϕ_0 ,v_0 ,n_0, can be defined respectively, to be the relativistic electrostatic potential the electron velocity and the electron number density at the positions where electrons are submitted to maximum force. We notice that the latter definition of ϕ_0 ,v_0 ,n_0 is also valid for the linear limit where ϕ_0 ,v_0 ,n_0 are identified as average value of ϕ_e ,v_e ,n_e along x. integrating Eq. (16) over the limits we (E_m,ϕ_0) ; (E ,ϕ) find

$$E^2 - E_m^2 = -8\pi n_0 m_0 \gamma_0 v_0^2 [1 + \frac{a_R}{2}(\phi - \phi_0) - \sqrt{1 + a_R(\phi - \phi_0) + \frac{v_0^2 a_R}{4c^2}(\phi - \phi_0)^2}\,] \quad (17)$$

where $E_0^2 = 4\pi n_0 m_0 \gamma_0 v_0^2$. Using dimensionless variables $E^* = E_m / E_0$, $Z_R = a_R(\phi - \phi_0)$ $\kappa_R = \frac{E_m}{E_0}$,Eq.(17)gives a family of curves in the phase space $Z_R - E^*$ modulated by the parameter κ_R (Fig.2).

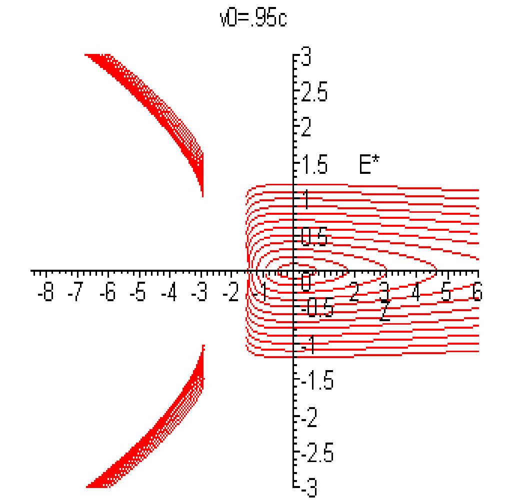

FIG.2.Electric field-electrostatic phase space

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