Effect of sheared equilibrium plasma rotation on the stability of tearing modes

R. Coelho 1)

1) Associação EURATOM/IST, Centro de Fusão Nuclear, 1049-001 Lisbon, Portugal

e-mail contact of main author: rcoelho@cfn.ist.utl.pt

Abstract. The effect of sheared equilibrium plasma rotation on the stability of tearing modes in an ohmic (low plasma β) regime is investigated. It is found, by means of numerical MHD simulations in a cylindrical geometry, that plasma rotation in the equivalent toroidal direction can result both on the increase or decrease of the instability growth rate. Anomalous perpendicular plasma viscosity and plasma rotation shear at the modes' rational surface play a key role on assessing the effect of shear flow. While destabilising for low viscosity plasmas (ratio of the resistive to viscous diffusion time scales $\tau_R/\tau_V<<1$), for viscous plasmas ($\tau_R/\tau_V>1$) shear flow reduces the growth rate. Above a given threshold in the rotation shear (that depends on the ratio τ_R/τ_V) a tearing mode, unstable in the absence of rotation, can be stabilised.

1. Introduction

Equilibrium plasma rotation can have a considerable effect on the plasma performance and operational limits of present day tokamaks. When considerable auxiliary neutral beam power (NBI) is used, a dominantly sheared toroidal plasma rotation (affecting also the $\mathbf{E}\times\mathbf{B}$ velocity), imparted by the beams, can benefit both plasma confinement and stability. While on one hand confinement is improved owing to a reduction in the scale length of turbulence [1], on the other hand rotation opposes magnetic field line reconnection and mode amplification driven by the static tokamak error-field [2] and contributes to the stabilisation of the resistive wall mode [3]. Non-uniform plasma flow is also known to have a significant effect on the stability of the classical resistive tearing mode problem. Numerical studies have shown that sheared poloidal plasma flows can either increase or decrease the instability growth rate, depending on the plasma viscosity regime and the magnetic and shear flow equilibria [4,5]. Non-linear tearing mode evolution may also be strongly affected by the equilibrium plasma flow. In fact, marginally unstable tearing modes in an inviscid plasma are firstly linearly destabilised by the shear flow and in the non linear regime are characterised by a propagating island with a width oscillating in time [6].

In this work we emphasise on the role of toroidal plasma flows on the stability of tearing modes. In particular, we investigate the effect of toroidal rotation on the reconnection rates in both inviscid and viscous dominated plasma regimes in a small/medium aspect ratio low-β cylindrical tokamak (cylinder with periodic boundaries in the azimuthal $\mathbf{z}$ direction). The basic ordering parameter that allows reducing the MHD vector equations in terms of scalar fields was originally the inverse aspect ratio of the torus [7]. A more physically meaningful expansion parameter that leads to reduced MHD models is $k_{//}/k_{\perp}$ as the expansion parameter, which was introduced as a means to eliminate the fast time scale associated with motions perpendicular to the magnetic field [8]. In this case the most important perturbations are those with long parallel wavelength and it is reasonable to expect that a full inclusion of parallel perturbations of the velocity and magnetic field be necessary. A single helicity symmetric cylindrical model [9,10], valid for all aspect ratios but with imposed incompressible plasma motion for numerical simplicity, is therefore adopted.

2. Numerical model

We assume a low-β, incompressible plasma with constant density (ρ) and anomalous perpendicular viscosity (ν). Neglecting Larmor radius effects, the relevant single fluid MHD equations used to study the growth and dynamics of tearing modes are

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V} \qquad (1)$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \qquad (2)$$

together with the Faraday's and Ampere's law. The magnetic and velocity field are divergence free and $d/dt \equiv \partial/\partial t + \mathbf{V} \cdot \nabla$. In the conventional low-β large aspect ratio ($A=R_0/a$) ordering of the reduced MHD model [7], the $\mathbf{z}$ component of the magnetic field and velocity perturbations is of order A^{-2}, allowing for a representation of the magnetic and velocity fields in terms of one scalar potential each. In a scenario where sheared toroidal plasma rotation is important, the $\mathbf{z}$ component of the velocity perturbation is no longer expected to be negligible ($\rho \tilde{V}_r dV_{0z}/dr$ term in Eq.(1) is important), even in the large aspect ratio limit. A description for both the magnetic and velocity fields, valid for all aspect ratios and for an incompressible plasma, is given by (see Ref 9 with $\mathbf{b} \to \mathbf{b}/|\mathbf{b}|$ where $\mathbf{b} = nr/mR_0\boldsymbol{\theta} + \mathbf{z}$)

$$\mathbf{B} = \nabla\psi_0 \times \mathbf{z} + B_{0z}\mathbf{b} + g(r)\nabla\tilde{\psi} \times \mathbf{b} + \tilde{B}_h \mathbf{b} \qquad (3)$$

$$\mathbf{V} = \mathbf{V}_0 = V_{0\theta}\boldsymbol{\theta} + V_{0z}\mathbf{z} + g(r)\nabla\tilde{u} \times \mathbf{b} + \tilde{V}_h \mathbf{b} \qquad (4)$$

Time evolution equations for the scalars $(\tilde{\psi}, \tilde{B}_h, \tilde{u}, \tilde{V}_h)$ are obtained by performing the appropriate projections of Eqs.(1), Faraday's law and the equation for the vorticity $\mathbf{W} = \nabla \times \mathbf{V}$. In this simplified cylindrical geometry toroidal coupling and curvature corrections are not considered. The following set of equations is obtained [10]

$$\frac{\partial\tilde{\psi}}{\partial t} = [\tilde{u}, \tilde{\psi}] - \mathbf{V}_0 \cdot \nabla\tilde{\psi} + \mathbf{B}_0 \cdot \nabla\tilde{u} - \eta g^{-1}\tilde{J}_h \qquad (5)$$

$$g^{-1}\frac{\partial\tilde{B}_h}{\partial t} = \mathbf{B} \cdot \nabla(g^{-1}V_h) - \mathbf{V} \cdot \nabla(g^{-1}B_h) + \frac{2n}{mR}(\mathbf{V} \times \mathbf{B})_z + \nabla \cdot \left[\frac{\eta}{\mu_0}\nabla(g^{-1}\tilde{B}_h)\right] - \frac{2n}{mR}\eta\tilde{J}_z \qquad (6)$$

$$\begin{aligned} -\rho\frac{\partial}{\partial t}(\nabla_g^2\tilde{u}) &= -\rho g^{-1}\left[(\mathbf{V} \cdot \nabla)W_h + \frac{\partial}{\partial\theta}\left(\frac{n}{mR}\tilde{V}_h\right)^2\right] \\ &\quad + \frac{g^{-1}}{\mu_0}\left[(\mathbf{B} \cdot \nabla)(\mu_0 J_h) + \frac{\partial}{\partial\theta}\left(\frac{n}{mR}\tilde{B}_h\right)^2\right] + \nu\nabla_g^2\left(g^{-1}\tilde{W}_h\right) \end{aligned} \qquad (7)$$

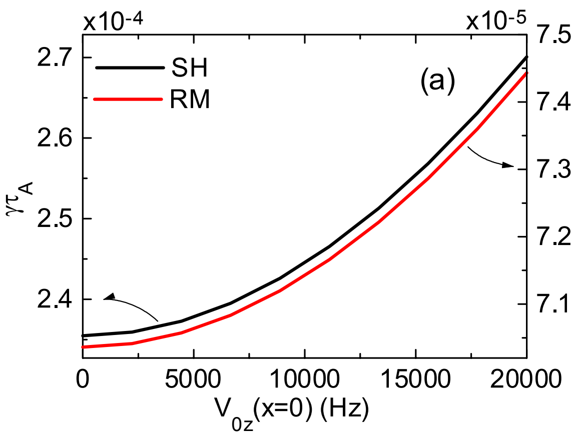
$$\rho g^{-1}\frac{\partial\tilde{V}_h}{\partial t} = -\rho\mathbf{V} \cdot \nabla(g^{-1}V_h) + \frac{1}{\mu_0}\mathbf{B} \cdot \nabla(g^{-1}B_h) + \nu\left[\nabla_g^2(g^{-1}\tilde{V}_h) + \frac{2n}{mR}g\nabla_g^2\tilde{u} - 4g\left(\frac{n}{mR}\right)^2\tilde{V}_h\right] \qquad (8)$$

with $\nabla_g^2 \equiv \frac{\partial^2}{\partial r^2} + (2g-1)\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{gr^2}\frac{\partial^2}{\partial\theta^2}$, $X_h \equiv (\mathbf{X}\cdot\mathbf{b})g$ and $[p,q] = \frac{1}{r}\left(\frac{\partial p}{\partial r}\frac{\partial q}{\partial\theta} - \frac{\partial p}{\partial\theta}\frac{\partial q}{\partial r}\right)$. In the numerical simulations the magnetic field is normalised as $B \rightarrow B'B_{0z}$ and a normalised minor radius $x = r/a$ and toroidal alfvén velocity $V_A = B_{0z}/\sqrt{\mu_0\rho_0}$ time scale are used. The magnetic Reynolds and Prandtl numbers are defined as $S = \tau_R/\tau_A$ and $Ga = \tau_R/\tau_V$, with the resistive and viscous times given by $\tau_R = a^2\mu_0/\eta_0$ and $\tau_V = \rho_0 a^2/\nu$ respectively. Vacuum boundary conditions at the plasma edge are assumed. A parabolic q(x) profile ($q_0 + (q_a - q_0)x^2$) with q_0=1.33 and q_a=3.7 and rotation profile $V_{x=0} - (V_{x=0} - V_{x=1})x^3$ are assumed throughout the text.

3. Shear flow effect on mode growth

While an uniform equilibrium plasma flow merely provides a propagating frequency to the tearing mode without affecting its' growth rate, a non uniform toroidal plasma flow can have a significant impact on the general characteristics of the mode. Recent results have shown that non uniform toroidal plasma rotation, mediated by the anomalous perpendicular plasma viscosity, can lead to a significant poloidal deformation of the magnetic islands associated to tearing modes [10].

In this paper, the joint effect of plasma rotation and viscosity on the linear growth rate of an unstable m=2,n=1 tearing mode ($\Delta' = 25$ where Δ' is the jump in the logarithmic derivative of the perturbed helical flux over the rational surface [11]) was investigated with a linear stability code solving the eigenvalue problem (Eqs. (5)-(8)). As discussed in Ref. 10, in static plasmas, parallel components of the perturbations start to give a non negligible contribution to the growth rate magnitude when the aspect ratio A (R_0/a) drops below A~10. In fact, retaining parallel perturbations leads to a larger growth rate and, for very compact geometries (A<3), to a growth rate scaling with the magnetic Reynolds number S closer to that of an ideal instability. To isolate the effect of sheared rotation on the linear growth rate, we have therefore done calculations both at small (3.4) and large (100) aspect radius scenarios. We have also used for comparison both the complete set of single helicity MHD equations (Eqs. (5)-(8)) and a reduced model version (where $\widetilde{B}_h$ and $\widetilde{V}_h$ are set to zero). Moreover, the magnitude of the equilibrium plasma velocity remains constant in both scenarios (the angular rotation frequency in the large aspect ratio scenario is 100/3.4 times smaller).



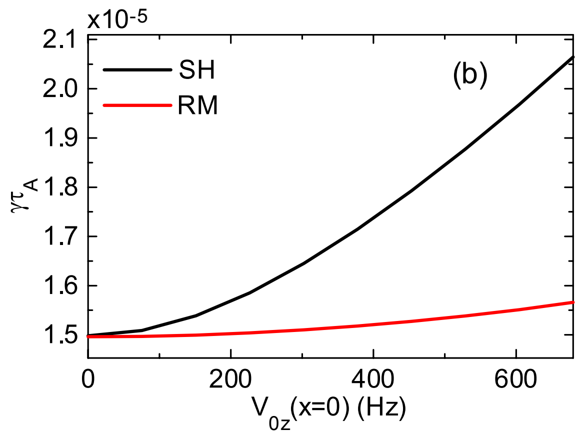


Figure 1 – Growth rate scaling with plasma rotation for the m=2 tearing mode in a low viscous plasma (Ga=0.01) with S=5x10^7, using the single helicity (in black) and the reduced (in red) MHD models for two different values of the aspect radius (A=3.4 in case (a) and A=100 in case (b)).

In Figure 1 we show the scaling of the growth rate with plasma rotation for the low viscosity regime (plasma rotation effect on the growth rate dominantly an inertial effect). At small aspect ratio (see Figure 1 (a)), the different order of magnitude of the growth rate is a consequence of the rather significant parallel component of the perturbations both in static and rotating plasmas. In addition, the variation of the growth rate caused by the sheared plasma rotation is larger for the single helicity model (black curve). One therefore might conjecture that toroidal flow (dominantly parallel to $\mathbf{b}$, in contrast with purely poloidal flow) enhances parallel perturbations and consequently the effect on mode stability is stronger than that arising when the reduced MHD model is considered (where perturbations are purely transverse to $\mathbf{b}$). This is confirmed in Figure 1 (b), where the scaling of the growth rate with plasma rotation is shown for the large aspect ratio (A=100) scenario. As expected, for a static plasma both MHD models yield the same growth rate since parallel perturbations are negligible. However, for a rotating plasma (with a non uniform radial profile), the destabilising effect due to the spatial gradient in the plasma rotation is much more noticeable in the single helicity model.

For a viscous dominated plasma (Ga=100), sheared plasma rotation plays a stabilising role (see Figure 2). In addition, as for the low viscosity scenario, parallel perturbations ($\tilde{B}_h$ and $\tilde{V}_h$) clearly give a substantial contribution to the growth rate magnitude. For this particular set of plasma equilibrium, the tearing unstable mode can be stabilised with a toroidal plasma rotation profile that on axis exceeds ~13kHz.

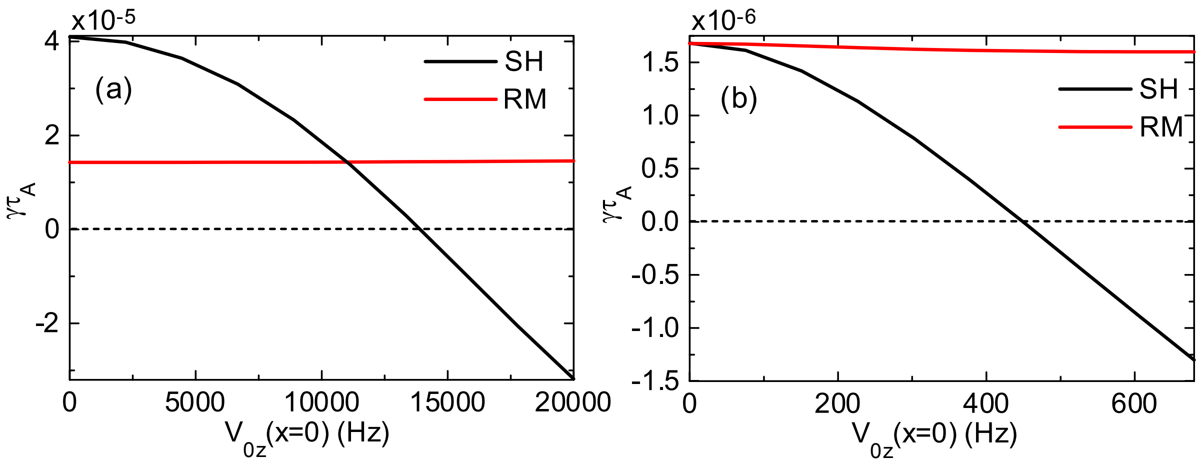


Figure 2 – Growth rate scaling with plasma rotation for the m=2 tearing mode in a viscous plasma (Ga=100) with S=5x10^7, using the single helicity (in black) and the reduced (in red) MHD models for two different values of the aspect radius (A=3.4 in case (a) and A=100 in case (b)).

A scan in the magnetic Reynolds number (S) also reveals that the threshold in plasma rotation for mode stabilisation, with the Prandtl number (Ga) fixed, decreases with increasing S. For instance, at S=5x10^8, the threshold drops to ~11kHz. We have also performed a scan in the Prandtl number (Ga) to determine the threshold in Ga (for different values of the Reynolds number but for the same toroidal rotation profile) above which the mode becomes stable (see Figure 3). The magnitude of plasma viscosity necessary for mode stabilisation, as anticipated, drops with increasing S. It is worth while stressing that mode stabilisation is a joint effect of both plasma viscosity and plasma rotation. For a static plasma and for the same plasma parameters as those of Figure 3, the mode is still unstable (plasma viscosity is stabilising but doesn't suppress by itself the mode).

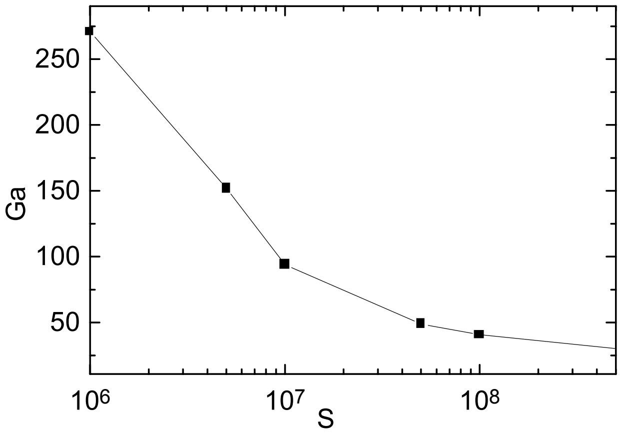


Figure 3 – Threshold in Ga for mode stabilisation as a function of the magnetic Reynolds number S for a rotating plasma with $V_{z0}(x=0)=20kHz$ and aspect ratio A=3.4.

4. Conclusions

The role of sheared toroidal plasma rotation on the stability of the tearing mode in a cylindrical geometry has been investigated. In order to correctly capture the essential features of the interaction of the plasma flow (mediated by plasma viscosity) with the mode, a single helicity MHD model was used. The need of such a model was evidenced by the linear growth rate results at a large plasma aspect ratio (A= 100). It is shown that with finite plasma rotation, retaining parallel (to the helical vector $\mathbf{b} = \mathrm{nr}/\mathrm{mR}_0\boldsymbol{\theta} + \mathbf{z}$) perturbations enhances considerably the effect of plasma rotation and viscosity on the growth rate. Mode stabilisation is shown to occur in viscous dominated plasmas for moderate plasma rotation, with the threshold in rotation decreasing with both the Reynolds (S) and Prandtl (Ga) numbers.

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