Neoclassical tearing Modes in the presence of sheared flows

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Abstract. The influence of equilibrium shear flows on the evolution of neoclassical tearing modes is an important issue for future long pulse experiments on tokamaks and for reactor grade machines like ITER. Sheared flows can be generated in a tokamak plasma due to a variety of reasons, such as due to neutral beam injection, ion cyclotron heating and self-consistent drift turbulence. In this paper we study certain aspects of this problem through numerical solutions of a set of generalized reduced MHD equations that includes viscous force effects based on neoclassical closures. Our principal findings are that differential flow has a strong stabilizing influence leading to lower saturated island widths and mode energy for the neoclassical tearing modes. Velocity shear on the other hand is seen to make a destabilizing contribution. An analytic model calculation, consisting of a Rutherford island evolution equation is also presented.

1. Introduction

It is now widely recognized that the β limit of advanced tokamaks is determined by the nonlinear instabilities associated with neoclassical tearing modes (NTMs) and not by the linearized theory of ideal MHD instabilities [1–3]. In recent years a great deal of work has been carried out on the Rutherford theory of neoclassical tearing modes and many important results on critical β values and their improvement by the use of RF current drive and heating methods, stabilization by the use of external helical current coils etc. have been obtained [4–6]. There are, nevertheless, a number of issues related to the origin of excitation of the mode, its excitation threshold, its nonlinear behaviour and its interaction with error fields and equilibrium shear flows that have not been satisfactorily resolved and need to be better understood [7]. The influence of shear flows is a particularly important issue since sheared velocity flows are known to be widely prevalent in tokamak devices and can be generated by neutral beams, ion cyclotron heating and self-consistent drift turbulence. A number of past studies have examined the effect of flows on tearing modes, particularly in the linear regime and for simplified geometries [8]. There have also been a few nonlinear studies [9, 10] but the problem is quite complex, particularly in realistic toroidal geometries, and is an important area of present and future study for major numerical initiatives such as NIMROD [11]. In this paper we report on numerical studies that we have begun on this problem with the help of a finite difference code NEAR that solves a set of generalized reduced MHD equations [12] and that includes viscous force effects based on neoclassical closures. While not as comprehensive or sophisticated as the NIMROD initiative, the present numerical model and the code is a lot simpler to implement and has been tested in the past for the effect of flows on linear resistive tearing modes. Our emphasis in this work is to extend the investigation to the nonlinear regime for both classical tearing modes and NTMs. To complement the numerical work and to gain some physical insight we also present an analytic calculation that describes the evolution of a single helicity NTM within the framework of an extended Rutherford model.

2. Model Equations

Our numerical simulations are based on the solutions of a set of reduced MHD equations originally proposed by Kruger *et al* in 1998 [12]. These equations, which are valid at any aspect ratio, are derived using $k_\parallel/k_\perp$ as a small expansion parameter. In the limit of $\beta \sim \delta^{1/2}(\delta \ll 1)$, a simplified set of the evolution equations are as follows,

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1) \cdot \nabla \phi_1 - \boldsymbol{b}_1 \cdot \nabla \phi_0 = \eta \tilde{J}_\parallel - \frac{1}{ne} \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}_e \tag{1}$$

$$\begin{aligned} \nabla \cdot \left(\frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (\boldsymbol{V}_1 \cdot \nabla) \left(\nabla \cdot \left(\frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) &= (\boldsymbol{B}_0 \cdot \nabla) \frac{\tilde{J}_\parallel}{B_0} + (\boldsymbol{B}_1 \cdot \nabla) \frac{J_{T\parallel}}{B_0} \\ + \nabla \cdot \frac{\boldsymbol{B}_0 \times \nabla p_1}{B_0^2} &\qquad + \nabla \cdot \frac{\boldsymbol{B}_0}{B_0^2} \times \nabla \cdot \boldsymbol{\Pi} \end{aligned} \tag{2}$$

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot \boldsymbol{V}_1 = (\Gamma - 1) \left[\eta J_{T\parallel}^2 - \boldsymbol{\Pi} : \nabla \boldsymbol{V} + \boldsymbol{\Pi}_{\boldsymbol{e}} : \nabla \frac{\boldsymbol{J}}{ne} - \nabla \cdot \boldsymbol{q} \right] \tag{3}$$

$$\rho \frac{d\widetilde{V}_\parallel}{dt} + (\boldsymbol{V}_1 \cdot \nabla) V_{\parallel_0} = -\boldsymbol{b}_0 \cdot \nabla p_1 - \boldsymbol{b}_1 \cdot \nabla p_T - \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi} \tag{4}$$

$$\begin{aligned} \nabla \cdot \boldsymbol{q} \;=\; & -\chi_\perp \nabla^2 p_1 - (\chi_\parallel - \chi_\perp)[\boldsymbol{b}_1 \cdot \nabla(\boldsymbol{b}_0 \cdot \nabla p_0) + \boldsymbol{b}_0 \cdot \nabla(\boldsymbol{b}_0 \cdot \nabla p_1 + \boldsymbol{b}_1 \cdot \nabla p_0) \\ & + \boldsymbol{b}_0 \cdot \nabla(\boldsymbol{b}_1 \cdot \nabla p_1) + \boldsymbol{b}_1 \cdot \nabla(\boldsymbol{b}_1 \cdot \nabla p_0) + \boldsymbol{b}_1 \cdot \nabla(\boldsymbol{b}_0 \cdot \nabla p_1)] \end{aligned} \tag{5}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla \;\; ; \;\; \boldsymbol{V} = \Omega(\psi) R^2 \boldsymbol{\nabla} \zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0 \times \nabla \Phi_0}{B_0^2} + V_{0\parallel} \boldsymbol{b}_0 + \frac{\boldsymbol{B}_0 \times \nabla \Phi_1}{B_0^2} + \tilde{V}_\parallel \boldsymbol{b}_T$$

where the notations are standard (for a more detailed discussion see [12]). The equilibrium toroidal velocity which is conveniently expressed in terms of a function $\Omega(\psi)$ is ordered such that $V_0/V_A \sim \epsilon \ll 1$ so that the flows are restricted to the sub-Alfvenic range. The above equations have been programmed into an initial value code, called NEAR, which is a derivative of an older code called FAR. An early benchmarking of this code was carried out in [13], where terms proportional to $\boldsymbol{\Pi}_e$, $\boldsymbol{\Pi}$ and $\nabla \cdot \boldsymbol{q}$ were dropped and the tests were restricted to the linear growth regime of classical tearing modes. Our emphasis in the present work is to explore the effects of shear flow in the nonlinear regime and in particular to examine its influence on the evolution of neoclassical tearing modes.

3. Numerical simulation results

3.1. Neoclassical tearing modes

As is well known, the neoclassical tearing mode is driven unstable by a perturbation of the bootstrap current To study the evolution of neoclassical tearing modes it is therefore necessary to retain the stress tensor terms in the reduced MHD equations in order to

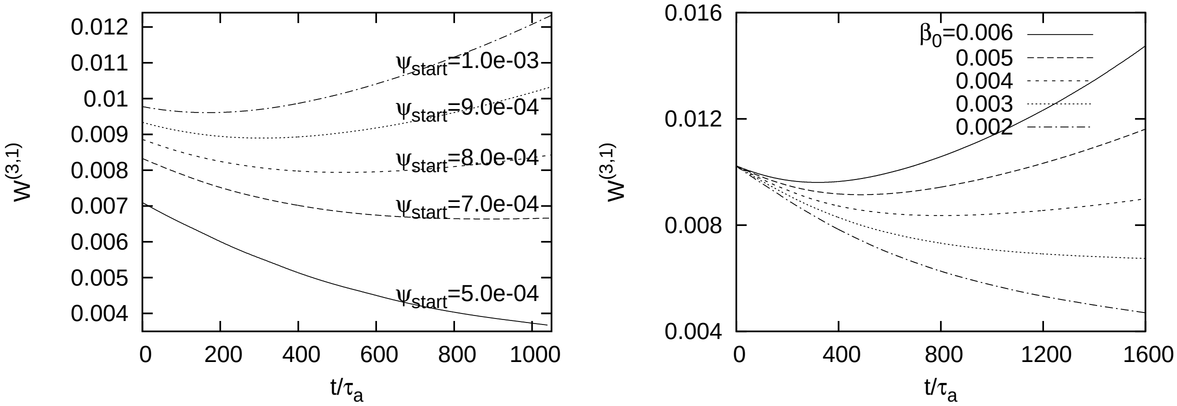

Figure 1. Benchmark results showing the existence of a threshold amplitude and threshold β for the $(3,1)$ NTM.

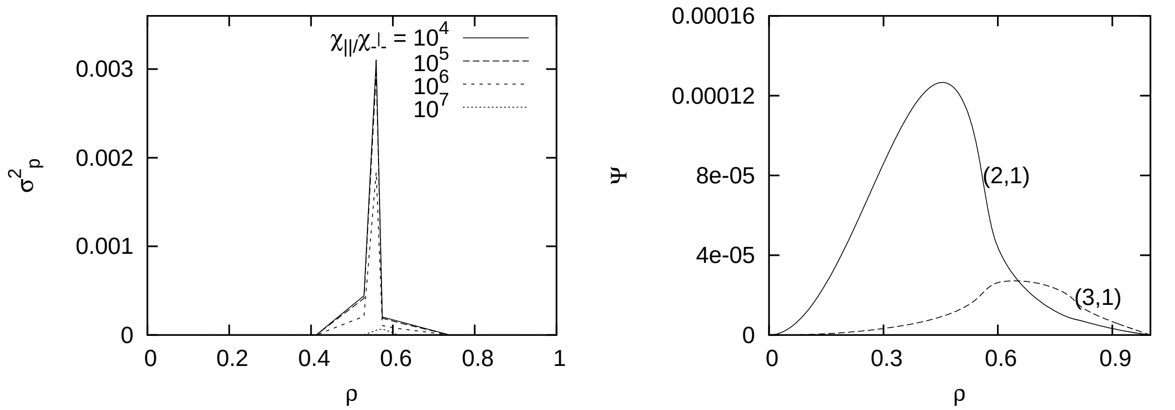

Figure 2. The pressure variance σ_p^2 as a function of $\chi_\parallel/\chi_\perp$ and benchmark result showing eigenfuction for a typical NTM evolution.

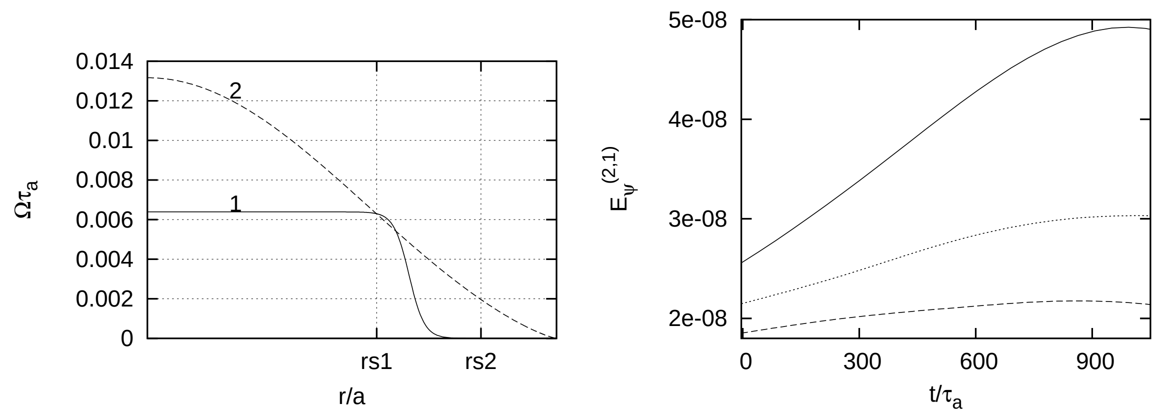

Figure 3. Equilibrium toroidal flow profiles and nonlinear evolution of the $(2,1)$ NTM for those profiles: no flow (solid curve), flow profile-1 (heavy dotted curve), flow profile-2 (dashed curve).

provide the drive term and to keep the heat flow terms in the pressure evolution equation. For the neoclassical viscous stress tensor we have used the following closure ansatz [12],

$$\nabla \cdot \vec{\Pi}_j = \rho_j \mu_j \left\langle B^2 \right\rangle \frac{\boldsymbol{v}_s . \nabla\theta}{(\boldsymbol{B} \cdot \nabla\theta)^2} \nabla\theta, \tag{6}$$

where $j = i, e$ and μ_j is the viscous damping frequency of each species j.
Before investigating the effect of flows on NTMs we have benchmarked the NEAR code by reproducing these characteristic features of the NTMs and paid particular attention to

pressure equilibration [14]. Here we generate an equilibrium profile by numerically solving the Grad-Shafranov equation with the help of an equilibrium code called TOQ [15]. Using this equilibrium in NEAR we study the evolution of the tearing modes from an initial perturbation. The typical ratio of $\chi_{\parallel}/\chi_{\perp}$ in most of our runs has been of the order of 10^6or more and the Reynolds number S has been kept at 10^5 or higher. In Fig. 1, we show the dependence of the mode evolution on the initial amplitude and the existence of a threshold amplitude and threshold β for the destabilization of the mode. Fig. 2 is showing the pressure equilibration which is key for NTM physics and the characteristic eigenfunctions of NTMs obtained from numerical runs on NEAR. In Fig.3 we have shown toroidal flow in the system using equilibrium flow profiles 1 and 2 and the island width evolution for three different cases - the top curve is without any flow, the bottom curve is for flow profile 1 (pure differential flow) and the intermediate curve is for flow profile 2 (differential flow + shear). It has been observed that the differential flow have a stabilizing influence and leads to a lower level of mode saturation as shown by the lowest curve of in Fig. 3. When we use profile 2 we find a decrease in the stabilization effect (the intermediate curve in Fig. 3 indicating that velocity shear has a destabilizing trend.

4. Rutherford model equations in the presence of sheared flows

To gain some analytic understanding of the nature of the sheared flow contributions we have tried to construct a Rutherford model description of the island evolution in the presence of flows. The nonlinear evolution equation of the magnetic island is derived from the matching conditions obtained by integrating Ampere's equation across the nonlinear region, where the longitudinal current can be obtained, following standard procedures by perturbative solutions of the parallel Ohm's law [16]. The polarization drift term, which is proportional to the plasma inertia incorporates the flow effects through the potential Φ Using the lowest order solution of the parallel Ohm's law, $\tilde{\phi}$ can eventually be obtained in the form,

$$\tilde{\phi} = \frac{B_0}{ck_\theta}(\omega - \omega_E)(x - \lambda) - \frac{B_0}{ck_\theta}\frac{\omega'_E}{2}(x^2 - \lambda^2) \tag{7}$$

where $\omega_E = k_\theta c \Phi'_0(r = r_s)/B_0$ is the drift frequency due to the equilibrium electric field and is the flow contribution, and $\omega'_E = k_\theta c \Phi''_0(r = r_s)/B_0$ is the flow shear contribution. The function $\lambda(\psi)$ is an integration constant which is chosen to provide the correct asymptotic behaviour for $\tilde{\phi}$. Our final island evolution equations are as follows,

$$\begin{aligned} 0.41\frac{\partial W}{\partial t} &= D_R^{neo}\left[\frac{\Delta'_c}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2 + W_\chi^2} \right. \\ &\left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega - \omega_E)(\omega - \omega_E - \omega_*)}{W^3} + 0.24\frac{{\omega'_E}^2}{W}\right)\right] \end{aligned} \tag{8}$$

$$0.82\frac{\partial}{\partial t}\left[W(\omega - \omega_E) + \frac{\omega'_E}{2}W^2\right] = -12.6\frac{\mu_e}{W}(\omega - \omega_E) - \frac{1}{4\sqrt{2}}\left(\frac{nsV_A}{R^2 q}\right)^2 W^4 \Delta'_s \tag{9}$$

where, $D_R^{neo} = c^2/4\pi\sigma_{neo}$ is the magnetic diffusion coefficient calculated using the neoclassical resistivity, $\beta_\theta = 8\pi p_e/B_\theta^2$, $L_p = -(dlnp/dr)^{-1}$, $L_q = (dlnq/dr)^{-1}$. In eqn.(8) the second termm on the RHS is the perturbed bootstrap current contribution which drives the mode unstable when it is larger than the $(\Delta'_c < 0)$ term. For evaluating the neoclassical contribution we have adopted the standard procedure outlined in [16] The factor $W^2/(W^2+W_\chi^2)$ in the neoclassical term is the usual effect associated with finite radial thermal diffusion and sets a critical island width $W_\chi = \sqrt{\frac{RqL_q}{m}}\left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4}$, below which radial transport becomes significant and the pressure is no longer flattened across the island. The fifth term of Eqn.(8) is the contribution from velocity shear and is seen to be of a destabilizing nature. Eqn.(9) is the evolution equation for the mode frequency obtained from the second matching condition with the flow shear correction.

5. Summary and Discussion

Our present set of numerical results, using two different profiles of toroidal equilibrium flow, indicate that differential flow has a strong stabilizing influence on the nonlinear evolution of neoclassical tearing modes whereas velocity shear has a destabilizing effect. While a quantitative comparison with any existing analytic model is not possible some qualitative features of the results can be understood on the basis of past theoretical work on shear flows as well as nonlinear evolutionary studies of tearing modes in the absence of flows. The destabilization effect of weak shear flows is consistent with the findings of earlier linear studies as well as the simple Rutherford model derived in the previous section. A major source of the stabilization is due to the influence of differential flow on toroidal mode coupling and equilibrium modifications of the pressure profile caused by the centrifugal effects of flow [17]. Further explorations with different flow profiles and different neoclassical closure models are currently in progress to study their influence on the nonlinear evolution of neoclassical tearing modes.

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