Anomalous particle transport and self-consistent wave evolution of magnetized electrons interacting with electron-cyclotron waves

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Abstract. The nonlinear interaction of relativistic electrons with electron-cyclotron waves in a constant magnetic field is studied. The electron diffusion across the magnetic field is analyzed over and near the local threshold to chaos for ionosphere plasma parameters assuming that the ampitude of the wave is constant. The diffusion is found to obey simple power law in time and the scaling exponent is indicant of sub-diffusion. The anomalous difusion is caused by the effect of the resonant phase-space islands in the particle motion. The self-consistent treatment uses a closed set of nonlinear equations consisting from the equations of motion under the electromagnetic field as well as the wave equation for the evolution of the vector potential. The electron motions drive the evolution of the wave amplitude and frequency mismatch through the current. We use the above model to study the relativistic electron-cyclotron absorption in the ionosphere and in fusion plasma.

1. Introduction

The nonlinear interaction of electrons with electron-cyclotron waves is of great importance for the laboratory and astrophysical plasmas. This problem has been investigated in detail for the last forty years [1][2][3]. Under conditions of resonance between the electron's cyclotron motion and the Doppler-shifted wave frequency, the wave-particle interaction is characterized by a significant energy exchange and an electron acceleration [3]. This effect has been considered in the study of electron-cyclotron instabilities [4] as well as for the interpretation of radiation observations in the ionosphere, and it is widely applied in fusion experiments for plasma heating and current drive. The linear theory for the wave absorption and the quasilinear theory for the electron distribution function are currently the main tools for the study of wave-particle interactions. However, in cases where nonlinear effects are important, the validity of these theories becomes questionable. In a recent work, electron-cyclotron heating simulations were performed using a nonlinear treatment, in contrast with the linear and quasilinear theories [5]. The results show that the deviation can be strong for present day fusion experiments. Furthermore, in numerous publications it is shown that the quasilinear theory breaks down due to the presence of resonant islands in the system phase-space [6][7]. These formations cause large time-space scaling of the particle kinetics, and thus non-Gaussian diffusion. In this report, we focus on the interaction of magnetized relativistic electrons with electron-cyclotron waves. We present the results of a recent analysis on the anomalous diffusion of electrons in the presence of a monochromatic electron-cyclotron wave of constant amplitude, without using a quasilinear approximation for the phase space [8]. The complex formation of the phase-space is underlined, which is strongly connected to the anomalous particle diffusion. We also perform a self-consistent analysis of the wave-electron interaction. A set of nonlinear and relativistic equations is derived, which account for the effects of electron motions on the temporal evolution of the wave. As an application, the problem of electron-cyclotron absorption for the cases of ionosphere and fusion plasma is studied.

2. Anomalous electron diffusion under a constant-amplitude wave

We briefly discuss the results of Ref. [8] on the interaction of magnetized relativistic electrons with an electron-cyclotron wave (ω,$\mathbf{k}$), which propagates at an angle θ with respect to a uniform magnetic field $\mathbf{B}=B_0\mathbf{z}$. We assume that the wave does not have a temporal evolution other than the phase-term $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$.

The knowledge obtained from this simplified approach are a guide towards a more complete, self-consistent treatment. The wave field is described by the normalized vector potential

$$\mathbf{A}=A_0\left(\cos\theta\sin\varphi\hat{\mathbf{x}}+\cos\varphi\hat{\mathbf{y}}-\sin\theta\sin\varphi\hat{\mathbf{z}}\right) \tag{2.1}$$

where $\varphi=\mathbf{k}\cdot\mathbf{r}-\omega t$ is the wave phase. In (2.1), the amplitude A_0 is normalized with m_ec^2/e, where c is the speed of light and e, m_e are the electron charge and rest mass. A value A_0 corresponds to a power flux $S\approx 30N_0^2\omega^2A_0^2$ (W/cm^2). The spatial coordinates are normalized with c/ω_e, the time with ω_e^{-1}, the frequencies with ω_e and the wave-vectors with ω_e/c, where $\omega_e=eB_0/m_ec$ is the cyclotron frequency. The wave-particle interaction can be described by the two-dimensional, autonomous Hamiltonian [9]

$$H=\left[1+(p_x+A_0\cos\theta\sin\varphi)^2+(p_y+x+A_0\cos\varphi)^2+(p_z-A_0\sin\theta\sin\varphi)^2\right]^{1/2}-p_z/N_0\cos\theta \tag{2.2}$$

where H is normalized with m_ec^2 and the canonical momenta with m_ec. We study the system for parameters corresponding to radio-wave heating of the night-time ionosphere: the frequency is $\omega/2\pi=3$MHz, the magnetic field is $B_0=3.5\cdot10^{-5}$T, the plasma density is $n_e=10^2$cm^{-3} and the initial electron distribution is monoenergetic with $E_0=1.279$MeV. For this case, the dispersion relation for circularly polarized waves, $N_0^2=1-\omega_p^2/\omega(\omega-\omega_e)$, where N_0 is the refraction index and $\omega_p=(4\pi n_ee^2/m_e)^{1/2}$ the plasma frequency, is a good approximation. An extensive study on the dynamics of the Hamiltonian (2.2) has been performed in Ref. [9]. It is found that significant chaos exists only for amplitudes larger than a critical value A_{0cr}, which depends on the other wave parameters ω, θ. A local estimate of A_{0cr} can be found by utilizing the fact that acceleration comes together with chaos. The complexity of the phase space is visualized in Fig. 1(a), where we present a Poincarè surface-of-section for $\theta=40^0$ and $A_0=0.1$. Clearly, the phase space is a highly-complex, inhomogeneous mixture of periodic and stochastic behavior. The time-scaling of the diffusion is determined by the exponent α of the power-law $<(\mathbf{r}-\mathbf{r_0})^2>\sim t^\alpha$. In Fig. 1(b) we show the exponent α as a function of A_0 for $\theta=40^0$. We observe that for all $A_0>A_{0cr}\approx0.03$ it is $\alpha<1$, which corresponds to sub-diffusion. This is connected to the resonant islands of the phase-space seen in Fig. 1(a). These formations cause particle trapping, which suppresses the diffusive behavior. Obviously, this is a case where the quasilinear theory breaks down because chaos is not complete. Also, the wave slows down the radial transport of the electrons, acting as a barrier, and this may have important consequences for the overall particle transport.

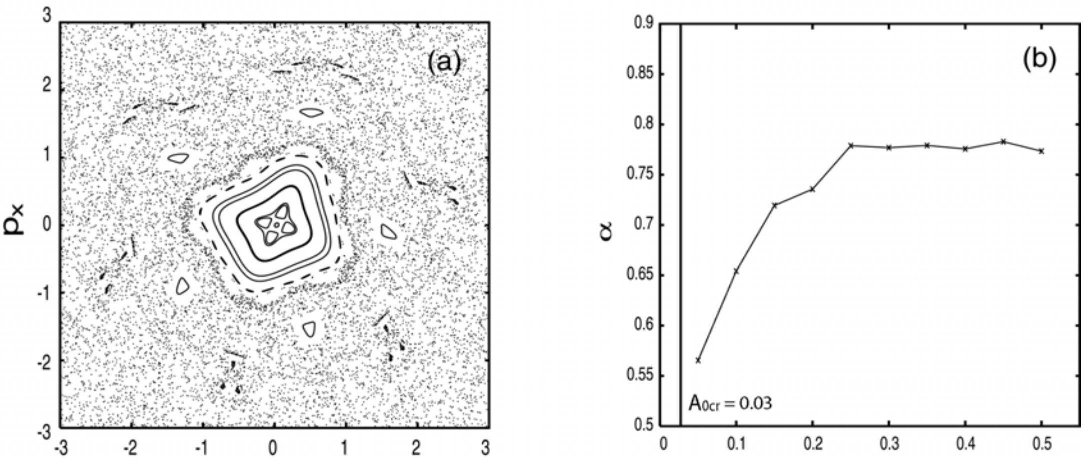

FIG. 1. (a) Poincarè surface-of-section (x,p_x), (b) Scaling exponent α as a function of amplitude A_0.

3. Self-consistent model for wave-particle interaction

In this section, a self-consistent treatment for the nonlinear interaction of electron-cyclotron waves with magnetized relativistic electrons is presented. The model relies on the coupling of the relativistic equations of motion under the electromagnetic field with the wave equation. The vector potential is given again by (2.1), but in the self-consistent model the amplitude A_0 and frequency ω have a time dependence. The electron motions drive the temporal evolution of the wave amplitude and frequency through the current density. The normalized equations of motion are

$$\dot{\mathbf{p}}=-\dot{\mathbf{A}}+\hat{\mathbf{z}}\times\mathbf{p}/\gamma+\mathbf{p}\times(\nabla\times\mathbf{A})/\gamma\ ,\qquad \dot{\gamma}=-\mathbf{p}\dot{\mathbf{A}}/\gamma \tag{3.1}$$

where $\mathbf{p}$ is the relativistic mechanical momentum and γ the Lorentz factor. The normalizations applied in (3.1) are the same as in Sec. 2. Using (2.1) for the vector potential, the equations of motion become

$$\begin{aligned}\dot{p}_\perp=&-\dot{A}(\cos\theta\cos\psi\sin\varphi+\sin\psi\cos\varphi)-A\omega(\cos\theta\cos\psi\cos\varphi-\sin\psi\sin\varphi)\\&-Akp_{\parallel}(\cos\theta\sin\psi\sin\varphi-\cos\psi\cos\varphi)/\gamma\end{aligned} \tag{3.2.a}$$

$$\dot{p}_{\parallel}=-\dot{A}\sin\theta\sin\varphi+A\omega\sin\theta\cos\varphi-Akp_\perp(\cos\psi\cos\varphi-\sin\psi\sin\varphi)/\gamma \tag{3.2.b}$$

$$\begin{aligned}\dot{\psi}=&1/p_\perp\gamma+\dot{A}(\cos\psi\cos\varphi-\cos\theta\sin\psi\sin\varphi)/p_\perp+A\omega(\cos\psi\sin\varphi+\cos\theta\sin\psi\cos\varphi)/p_\perp\\&-Ak\sin\theta\sin\varphi/\gamma-Akp_{\parallel}(\cos\theta\cos\psi\sin\varphi+\sin\psi\cos\varphi)/p_\perp\gamma\end{aligned} \tag{3.2.c}$$

$$\dot{\varphi}=kp_\perp\sin\theta\cos\psi/\gamma+kp_{\parallel}\cos\theta/\gamma-\omega \tag{3.2.d}$$

$$\begin{aligned}\dot{\gamma}=&\dot{A}p_\perp(\cos\theta\cos\psi\sin\varphi+\sin\psi\cos\varphi)/\gamma-A\omega p_\perp(\cos\theta\cos\psi\cos\varphi-\sin\psi\sin\varphi)/\gamma\\&-\dot{A}p_{\parallel}\sin\theta\sin\varphi/\gamma+A\omega p_{\parallel}\sin\theta\cos\varphi/\gamma\end{aligned} \tag{3.2.e}$$

In (3.2), $p_{\parallel}$, $p_\perp$ are the parallel and perpendicular momenta with respect to the magnetic field and ψ is the phase of the perpendicular momentum, $\psi=\tan^{-1}(p_y/p_x)$, which depends on time. The normalized wave equation for the evolutiuon of the vector potential reads

$$\nabla^2\mathbf{A}-\ddot{\mathbf{A}}=-\omega_p^2\mathbf{j} \tag{3.3}$$

where $\mathbf{j}$ is the current density, normalized with en_ec. Using again the representation (2.1) for $\mathbf{A}$, we obtain equations for the temporal evolution of the amplitude A_0 and the frequency ω

$$\ddot{A}_0+(k^2-\omega^2)A_0=-\omega_p^2\left[j_\perp(\cos\theta\cos\psi\sin\varphi+\sin\psi\cos\varphi)-j_{\parallel}\sin\theta\sin\varphi\right] \tag{3.4.a}$$

$$A_0\dot{\omega}+2\omega\dot{A}_0=-\omega_p^2\left[j_\perp(\sin\psi\sin\varphi-\cos\theta\cos\psi\cos\varphi)+j_{\parallel}\sin\theta\cos\varphi\right] \tag{3.4.b}$$

where $j_{\parallel}$, $j_\perp$ are the parallel and perpendicular current densities. Assuming an initial electron distribution function $f_0(\mathbf{r_0},\mathbf{p_0})$, the normalized current density is given by $\mathbf{j}=-\iint d^3\mathbf{r}_0 d^3\mathbf{p}_0 f_0\delta(\mathbf{r}-\mathbf{r}_0)\mathbf{p}$. The right-hand sides in the equations (3.4), which represent the effect of the particle motions on the wave, are spatially-dependent through the wave phase and the current densities. This dependence is periodic in space, because the wave-number $\mathbf{k}$ is constant, and since we are interested in the temporal evolution of A_0,ω, we may average (3.4) over space in order to obtain equations only over time. Taking into account the form of the current density, the result after the averaging is

$$\ddot{A}_0+(k^2-\omega^2)A_0=\omega_p^2\int p_{0\perp}dp_{0\perp}\int dp_{0\parallel}f_0\left\langle\left[p_\perp(\cos\theta\cos\psi\sin\varphi+\sin\psi\cos\varphi)-p_{\parallel}\sin\theta\sin\varphi\right]/\gamma\right\rangle \tag{3.5.a}$$

$$A_0\dot{\omega}+2\omega\dot{A}_0=-\omega_p^2\int p_{0\perp}dp_{0\perp}\int dp_{0\parallel}f_0\left\langle\left[p_\perp(\sin\psi\sin\varphi-\cos\theta\cos\psi\cos\varphi)-p_{\parallel}\sin\theta\cos\varphi\right]/\gamma\right\rangle \tag{3.5.b}$$

where the mean values are taken over the plasma electrons. Equations (3.2), (3.5) form a closed self-consistent set of equations describing the wave-particle system, where A_0, ω in (3.2) are calculated as solutions of (3.5), while the integrals involved in (3.5) are determined from the electron orbits, which are solutions of (3.2). We apply this model to the case of electron-cyclotron absorption in the ionosphere and in fusion plasma. We reconsider the case of Sec. 2, in order to underline the connection of the self-consistent system to the simplified model. In Fig. 2(a), the amplitude A_0 is shown as a function of time. The evolution of the amplitude is in accordance with the behaviour presented in Sec. 2. Wave power is absorbed by the plasma particles, which gain significant amounts of energy, until the amplitude reaches the threshold to chaos. After this point, the absorption procedure saturates due to electron trapping in regions of phase-space where the motion is regular. For small amplitudes, the phase-space is dominated by the islands and the absorption of the electromagnetic radiation is not possible. In Fig. 2(b) the evolution of the average electron energy is shown. The energy gain of the plasma electrons due to absorption of the electron-cyclotron wave is obvious. The results presented in Fig. 2 imply a consistency with the energy conservation law. This consistency can be verified quantitatevily as follows: the energy conservation theorem in the plasma volume occupied by the test particles reads $\omega_p^2\Delta\langle\gamma\rangle+\Delta(A_0^2\omega^2)=0$, where the first term stands for the total particle energy and the second for the wave energy. During the test-particle simulations, we numerically followed the validity of this relation, and the resulting accuracy was of the order 10^{-4}-10^{-5}.

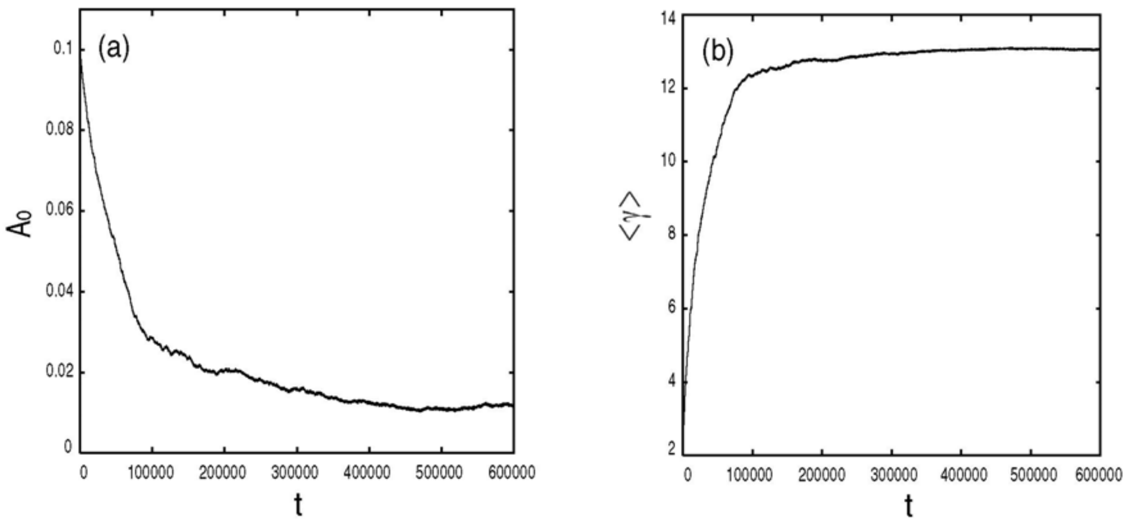

FIG. 2. (a) Amplitude A_0 and (b) mean electron energy $\langle\gamma\rangle$ vs time for absorption in the ionosphere.

We further consider a simple model for the absorption region in a fusion plasma. In our simulation, we consider the absorption of the 2nd harmonic X-mode injected in the infinite slab at an angle θ=600, which corresponds to a toroidal angle θ′=300. We follow the wave-particle interaction for $t\approx1500\Omega_e^{-1}$, which is approximately the time needed by the wave, moving with velocity c/N0, to cover the minor diameter dT=1m of the tokamak. The magnetic field is B0=2.5T, the plasma density is ne=1012cm-3 and the initial wave-power is Pwav=400KW. We assume that the initial distribution is Maxwellian with temperature≈2.55KeV. The refraction index is calculated using the cold plasma dispersion relation for the X-mode [10]. In Fig. 3(a), the vector potential amplitude A0 is plotted as a function of time, together with the result predicted by the linear theory of absorption. The nonrelativistic linear absorption coefficient reads [5]

$$\Gamma_L=\sqrt{\pi/8}\,\omega_p^2k^2\sin^2\theta v_{th}\left(\sin^2\theta N_0^4+B_1N_0^2+C_1\right)\Big/\omega\cos\theta\left(B_2N_0^2+C_2\right) \qquad (3.6)$$

where v_{th} is the thermal velocity and the coefficients B_1, B_2, C_1, C_2 are functions of the wave and plasma parameters θ, ω, ω_p and ω_e. The relation (3.6) is a good approximation for our case, where the Doppler effect is dominant over the relativistic corrections. This is verified by the fact that the relevant condition $N_0\cos\theta>v_{th}/c$ [10] is valid during the entire simulation. The disagreement of the nonlinear result with the prediction of the linear theory is evident. The nonlinear calculations show a significant reduction of the absorption. This is in accordance with recent results on the importance of nonlinear effects during electron-cyclotron heating [5]. In fig 3(b) the evolution of the wave frequency is shown. The frquency, despite its nonlinear variation, remains confined near the initial second-harmonic value.

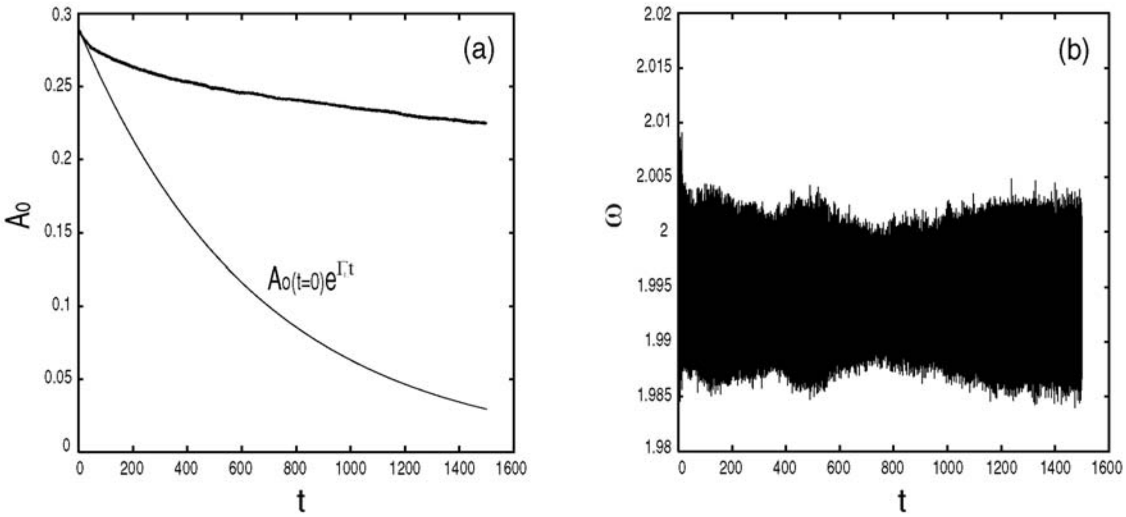

FIG. 3. (a) A_0 ,together with the linear prediction, and (b) ω vs t for absorption in a fusion plasma.

4. Conclusions

In this report, we studied the nonlinear interaction of magnetized relativistic electrons with electron-cyclotron waves. Using a simple model where the wave has constant amplitude, we demonstrated that the quasilinear theory breaks down because chaos is not complete and the phase space is an inhomogeneous mixture of periodic and stochastic orbits. The wave slows down the radial transport of the electrons, which may be of importance for the overall particle transport. The self-consistent analysis showed that the main characteristics of the constant wave amplitude particle dynamics are preserved, leading the absorption of the electromagnetic wave to a minimum value in a relatively short time. For the case of a fusion plasma, the disagreement with the linear theory is significant. We feel that there is a need to reconsider the importance of nonlinear effects during electron-cyclotron heating. The configuration used in this report is relatively simple and our current work focuses on more realistic tokamak slab geometries.

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REFERENCES

[1] ROBERTS, C.S., BUCHSBAUM, S.J., Phys. Rev. **135** (1964), A381.
[2] KARNEY, C.F.F., BERS, A., Phys. Rev. Lett. **39** (1977), 550.
[3] MENYUK, C.R., et al., Phys. Fluids **31** (1988), 3768.
[4] SPRANGLE, P., VLAHOS, L., Phys. Rev. A **33** (1986), 1261.
[5] KAMENDJE, R., "On Nonlinear Effects during Electron-Cyclotron Resonance Heating in Fusion Plasmas", PhD Thesis (2001).
[6] SHLESINGER, M.F., et al., Nature **363** (1993), 31.
[7] ZASLAVSKY, G.M., Chaos **4** (1994), 25.
[8] TSIRONIS, C., VLAHOS, L., Plasma Phys. Control. Fusion **47** (2005), 131.
[9] POLYMILIS, C., HIZANIDIS, K., Phys. Rev. E **47** (1993), 4381.
[10] BORNATICI, M., Plasma Phys. **24** (1982), 629.