Large scale flows and coherent structure phenomena in flute turbulence

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Abstract

The properties of zonal and streamer flows in the flute mode turbulence are investigated. The stability criteria and the frequency of these flows are determined in terms of the spectra of turbulent fluctuations. Furthermore, it is shown that zonal flows can undergo a further non–linear evolution leading to the formation of long–lived coherent structures which consist of self–bound wave packets supporting stationary shear layers, and thus can be characterized as regions with a reduced level of anomalous transport.

1. Flute Equations

Flute (or interchange) modes $k_{\|} = 0$ are low–frequency, $\omega \ll \omega_{ci}$, electrostatic oscillations of a non–uniform magnetoplasma which become unstable due to the combined effects of the density inhomogeneity and the curvature of the magnetic field. To describe flute modes we use the two-fluid equations [1], for an inhomogeneous magnetized plasma with characteristic inhomogeneity scale length L_n along the radial axis x. For the slab geometry, we model the curved magnetic field by $B(x) = B_0\,(1 - x/R)$ and $\mathbf{b} = \hat{z} - (z/R)\hat{x}$. The magnetic–curvature–driven flute modes are described by the following set of dimensionless coupled equations for the perturbed electrostatic potential Φ and density n [2] :

$$(\partial_t - v_{ni}\partial_y)\,\nabla_\perp^2\Phi + v_g\partial_y n = \tau_i \text{div}\left\{\nabla_\perp\Phi, n\right\} + \left\{\nabla_\perp^2\Phi, \Phi\right\} \quad (1)$$

$$(\partial_t + v_{ge}\partial_y)\,n + (v_{ne} - v_{ge})\partial_y\Phi = \{n, \Phi\}\,, \quad (2)$$

where $\{f, g\} = \hat{z} \times \nabla f \cdot \nabla g$ denotes the Poisson bracket. The electrostatic potential has been normalized by T_e/e, the time by ω_{ci}, the length scales by $\rho = c_s/\omega_{ci}$ where $c_s^2 = T_e/m_i$, and the perturbed density by n_0. Furthermore, $v_g = v_{ge} + v_{gi}$. These equations generalizes previous descriptions of the magnetic–curvature driven flute instability [3] as include rigorously the diamagnetic drift, $v_{nj} = T_i/(eB_0L_n)$, the magnetic curvature drift, $v_{gj} = 2T_j/(eRB_0)$ of both electron and ion fluids ($j = i, e$), and the finite ion Larmor radius effect described by the term proportional to τ_i $(= T_i/T_e)$. The linear dispersion relation of the flute modes gives;

$$\omega_k = -\frac{k_y(v_{ni} - v_{ge})}{2}\left(1 \pm \epsilon\sqrt{1 - \frac{k_{cr}^2}{k_\perp^2}}\right), \quad (3)$$

where $\epsilon \equiv (v_{ni} + v_{ge})/(v_{ni} - v_{ge})$ and $k_{cr}^2 \equiv 4v_g(v_{ne} - v_{ge})/(v_{ni} + v_{ge})^2$. Modes of finite poloidal wavenumber with $k_\perp \leq k_{cr}$ are linearly unstable.

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2. Coupled Dynamics of flute mode turbulence and Large Scale Flows

For the description of the dynamics of large-scale plasma flows that vary on a longer time scale compared to the small-scale fluctuations, a multiple scale expansion is usually employed assuming that there is a sufficient spectral gap separating the large and the small scale motions. In what follows, $[\tilde{\Phi}(\mathbf{r},t), \tilde{n}(\mathbf{r},t)]$ denote the small scale fluctuations and $[\overline{n}(\mathbf{r},t), \overline{\Phi}(\mathbf{r},t)]$ the large scale ones. By averaging equations (1, 2), we get

$$(\partial_t - v_{ni}\partial_y)\,\nabla_\perp^2\overline{\Phi} + v_g\partial_y\overline{n} = -\overline{R^\Phi} - \overline{R^n} \tag{4}$$

$$(\partial_t + v_{ge}\partial_y)\,\overline{n} + (v_{ne} - v_{ge})\partial_y\overline{\Phi} = \overline{\left\{\tilde{n}, \tilde{\Phi}\right\}}, \tag{5}$$

where $R^\Phi = \left\{\tilde{\Phi}, \nabla^2\tilde{\Phi}\right\}$ is the standard Reynolds force due to the polarization drift non–linearity and $R^n = \tau_i\left\{\tilde{n}, \nabla_\perp^2\tilde{\Phi}\right\} + \tau_i\left\{\nabla_\perp\tilde{n}, \nabla_\perp\tilde{\Phi}\right\}$ is the diamagnetic Reynolds force due to the fluctuating ion pressure and it is a finite ion Larmor radius effect. The equations above describe the formation of large scale structures by the flute turbulence. This is ensured by the inverse cascade properties of the polarization drift non–linearity [4]. However, the diamagnetic component of the polarization drift non–linearity leads to direct energy cascade towards short scales [5].

Equations (1, 2) conserve the energy integral

$$I_1 = \int\left\{\overline{n}^2 + \tilde{n}^2 - \frac{v_{ne} - v_{ge}}{v_g}\left[(\nabla\overline{\Phi})^2 + (\nabla\tilde{\Phi})^2\right]\right\}dxdy = const.,$$

which shows that the modulation of flows and turbulence are coupled and cannot be addressed in isolation.

The propagation of the flute modes in weakly inhomogeneous media can be described by employing the wave kinetic equation for the wave-action density $N_k(r,t)$ in the $\mathbf{r}$-$\mathbf{k}$ space. The source of the slow spatial and temporal variations are the large scale flows induced by the velocity and the density perturbations. The wave kinetic equation for the generalized wave action allows us to determine the modulations of $N_k(r,t)$ due the mean flow. The method of constructing the adiabatic invariant has been previously discussed in [6].

The generalized action density is found to be

$$N_k \equiv |\Psi_k|^2 = k_\perp^4\left(\frac{v_{ni} + v_{ge}}{v_g}\right)^2\left|\frac{k_{cr}^2}{k_\perp^2} - 1\right||\Phi_k|^2. \tag{6}$$

The WKB-type wave kinetic equation which describes the evolution of the generalized action invariant $N_k(\mathbf{r},t)$ in the flute mode turbulence due to the interaction between the mean flow and the small fluctuations, is given by [3]

$$\frac{\partial N_k}{\partial t} + \frac{\partial N_k}{\partial \mathbf{r}}\cdot\frac{\partial \omega_k^{NL}}{\partial \mathbf{k}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}}\cdot\frac{\partial N_k}{\partial \mathbf{k}} = \gamma_k N_k - \Delta\omega_k N_k^2. \tag{7}$$

The non–linear frequency is defined through $\omega_k^{NL} = \omega_k + \mathbf{k}\cdot\mathbf{V_0}$, where the non–linear shift is due to the presence of the large scale flows and it is given by $\mathbf{V}_0 = \mathbf{V}_\Phi + \mathbf{V}_n$, where

$$\mathbf{V}_\Phi = -\frac{1}{2}\left(\nabla\bar{\Phi}\times z\right) \quad \text{and} \quad \mathbf{V}_n = -\frac{\tau_i}{4}\left(\nabla\bar{n}\times z\right). \tag{8}$$

The non–linear frequency shift $\Delta\omega_k$ in the rhs of Eq. (7) represents the part of the non–linear interactions among the flute modes which balance the linear growth rate. Considering small deviations of the spectrum function from the equilibrium, $N_k = N_k^0 + \tilde{N}_k$, the perturbed density of the "quasiparticles" $\tilde{N}_k$ can be calculated by the linearized wave kinetic equation for a uniform equilibrium $\partial N_k^0/\partial \mathbf{r} = 0$:

$$\frac{\partial \tilde{N}_k}{\partial t} + \frac{\partial}{\partial \mathbf{k}}(\omega_k + \mathbf{k}\cdot\mathbf{V_0})\frac{\partial \tilde{N}_k}{\partial \mathbf{r}} - \frac{\partial}{\partial \mathbf{r}}(\omega_k + \mathbf{k}\cdot\mathbf{V_0})\frac{\partial N_k^0}{\partial \mathbf{k}} = -\gamma_k \tilde{N}_k. \tag{9}$$

In the local approximation, i.e. $\partial\omega_k/\partial\mathbf{r} = 0$, equation (9) can be solved by assuming that the large scale variation of the action density is of the form $N_k \sim \exp[i\mathbf{qr} - i\Omega t]$. This yields the resonant part of the distribution:

$$\tilde{N}_k^{res} = \frac{\partial}{\partial \mathbf{r}}\left(\mathbf{k}\cdot\mathbf{V_0}\right)\frac{\partial N_k^0}{\partial \mathbf{k}} R(\Omega, q, \delta\omega_k). \tag{10}$$

The response function here is defined by $R(\Omega, \mathbf{q}, \delta\omega_k) = i/(\Omega - \mathbf{q}\cdot\mathbf{V_g} + i\delta\omega_k)$, $\delta\omega_k$ is the total decorrelation frequency which includes the linear growth rate and a nonlinear shift, and $\mathbf{V_g} = \partial\omega_k/\partial\mathbf{k}$ is the group velocity. In a weakly nonlinear regime it is $R(\Omega, q, \delta\omega_k) \to \pi\delta(\Omega - \mathbf{q}\cdot\mathbf{V_g})$, while for a wide fluctuating spectrum $R(\Omega, \mathbf{q}, \delta\omega_k) \to 1/\delta\omega_k$. The broad spectrum of large scale structures regulates the flute turbulence by the process of random shearing and governs the self–regulative and saturation mechanism of the flute mode turbulence [3].

3. Long Term Dynamics of Zonal Flows

Calculating the averaged Raynold stress forces in Eqs. (4, 5), we obtain the equations describing the evolution of the zonal flow $q(q_x, q_y) = q(q_x, 0)$

$$\frac{\partial \overline{\Phi}_{q_x}}{\partial t} = \int k_x k_y \left(1 - \frac{v_{ni}}{2v_g}k_\perp^2\right)\left|\tilde{\Phi}_k\right|^2 \, d^2k, \tag{11}$$

$$\frac{\partial \overline{n}_{q_x}}{\partial t} = 0. \tag{12}$$

The second term in the right hand side of Eq. (11) is attributed to the ion diamagnetic drift and to the finite ion Larmor radius and may lead to the suppression of the zonal flow generation. Adding Eqs. (11,12) and using Eqs. (6,8), we get a relation which connects the zonal flow velocity with the spectra of the short scale fluctuations,

$$\frac{\partial V_{0y}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial x}\int k_x k_y \zeta(k_\perp)|\Psi_k|^2 d^2k. \tag{13}$$

Here $\zeta(k_\perp)$ is defined by

$$\zeta(k_\perp) = \frac{1}{2k_\perp^2}\frac{v_g v_{ni}}{(v_{ni} + v_{ge})^2}\left(\frac{2}{k_\perp^2}\frac{v_g}{v_{ni}} - 1\right)\left|\frac{k_{cr}^2}{k_\perp^2} - 1\right|^{-1}.$$

Closure conditions for the N_k modulations in terms of the mean flow are provided by Eq. (7). Inserting Eq. (10) into (13) and assuming the zonal flow variation to be of the form $V_{0y} \sim \exp[iq_x x]$, we obtain

$$\frac{\partial V_{0y}}{\partial t} = q_x^2 D_{xx} V_{0y}$$

This equation determines the stability of the zonal flow since $\gamma_{zf} = q_x^2 D_{xx}$. The coefficient D_{xx} is given by

$$D_{xx} = -\frac{1}{2}\int k_x k_y^2 \frac{\partial N_k}{\partial k_x}\zeta(k_\perp)R(\Omega, q_x, \delta\omega_k)d^2k. \tag{14}$$

The zonal flow gets unstable when $D_{xx} > 0$. This instability can be interpreted as a result of the resonant interaction between zonal flow and the small scale modulations of the turbulence. In the flute turbulence, where it is usually $k_x(\partial N_k^0/\partial k_x) < 0$, the zonal flow may become unstable due to the contribution of the modes with $k_\perp^2 < 2v_g/v_{ni}$. Part of these modes is linearly unstable. However, when $v_{gj}/v_{nj} < \sqrt{3\tau_i^2 + 2\tau_i} - 2\tau_i$ (for $\tau_i < 2$), it turns that $k_{cr}^2 > 2v_g/v_{ni}$ and subsequently, the modes responsible for the instability of the zonal flow may contribute significantly to the value of the integral (14). When $k_x(\partial N_k^0/\partial k_x) > 0$, it is more likely that the zonal flow is stable similar to the drift wave turbulence [7].

For perturbations with $\Omega \ll q_x V_{gx}$, we can take into account the non–resonant response $\tilde{N}_k^{(1)}$ of the turbulent spectra over the perturbations of the induced zonal flow. In this case the solution of the linearized wave kinetic equation (7) yields

$$\tilde{N}_k^{(1)} = k_y V_{0y}\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}.$$

Substituting the later expression into Eq. (13), we obtain the oscillation frequency of the zonal flow, $\Omega_{zf} \simeq -u_x q_x$, where

$$u_x = \frac{1}{2}\int k_x k_y^2\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\zeta(k_\perp)d^2k.$$

As the amplitude of the zonal flow grows, non–linear effects become significant. Using the derived expression of $\tilde{N}_k^{(1)}$, we determine iteratively from Eq. (9), the next order non–linear response $\tilde{N}_k^{(2)}$ for the non–resonant interactions,

$$\tilde{N}_k^{(2)} = \frac{1}{2}\left(k_y V_{0y}\right)^2\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial}{\partial k_x}\left[\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\right].$$

Including the total response, $\tilde{N}_k = \tilde{N}_k^{res} + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$ into Eq. (13), we obtain, similar to the case of the drift wave–zonal flow turbulence [7], a non–linear equation which describes the evolution of the zonal flow;

$$u_x\frac{\partial^2}{\partial x^2}V_{0y} + b_x\frac{\partial^2}{\partial x^2}V_{0y}^2 - D_{xx}\frac{\partial^3}{\partial x^3}V_{0y} = \frac{\partial}{\partial t}\frac{\partial}{\partial x}V_{0y}. \tag{15}$$

The coefficient b_x in the non–linear term is given by

$$b_x = \frac{1}{4}\int k_x k_y^3\zeta(k_\perp)\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial}{\partial k_x}\left[\left(\frac{\partial\omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\right]d^2k.$$

Equation (15) admits localized solutions. The simplest solution is of the kink type and is given by

$$V_{0y} = \frac{1}{2}\left\{V_{1y} + V_{2y} + (V_{1y} - V_{2y})\tanh\left[x\, b(V_{1y} - V_{2y})/2D_{xx}\right]\right\}$$

where $V_{2y} = -V_{1y} - (u_{0x} + u_x)/b_x$. This solution describes the transient region between two different values of the flow. So, the cooperative effects of the wave motion, steeping and instability gives the possibility to the formation of stationary or moving kink solitons. The values of the parameters which determine the characteristic lengths of these structures are determined by the value of the group velocity and by the spectral density of the background fluctuations. The above simple analysis demonstrates the self–organization properties of the flute modes–zonal flow coupled system.

4. Summary

The properties of the large scale flows, developed and interacting in an electrostatic turbulent environment of the flute type were investigated and determined by using a kinetic wave equation coupled with averaged fluid equations which describe the flute turbulence. The resonant interaction between the variations of the mean flow and the turbulent spectra may lead to the stabilization of the large scale flows. The non-linear evolution of the large flows can lead to the formation of stationary coherent structures in the transition layer between surfaces of different flow velocities, modifying significantly the transport properties of the turbulent plasma.

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