Disspative Instability of Overlimiting Electron Beam in No Uniform Cross Section System

Eduard V. Rostomyan

Institute of Radiophysics & Electronics National Ac. Sci. of Armenia
Alikhanyan st. 1, Ashtarack, 378410, ARMENIA,
e-mail: evrostom@freenet.am

Abstract. Paper presents a type of streaming instabilities with new, previously unknown physics. It realizes under very high, overlimiting beam current in no uniform cross section systems with dissipation. In this case two factors that lead to excitation of the beam wave with negative energy superimpose and this superposition results in instability of new type – dissipative instability of overlimiting electron beam. Physics of this instability in no uniform cross section system sharply differs from that in uniform cross section system. The growth rate of presented instability has more critical dependence on dissipation.
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1. Introduction

Propagation of charge particle beams in plasma is accompanied by development of instabilities. As a result the amplitudes of electromagnetic oscillations increase as well as the inner energy of plasma at the expense of beam energy. For vast majority of considered cases it was assumed that dissipation is small and cannot have a pronounced effect on instability evolution. But in many situations dissipation in system (collisions between plasma consistent, heating of metallic surfaces etc) can play an important role in beam-plasma instability development. It can become not only deciding factors in the limiting spatial and temporal growth and determining the magnitude of the fields [1,2]. The dissipation can also significantly influence on the mode structure of instability and reduce the growth rate. However, dissipation never suppresses the instability completely. Dissipation of high level transforms physical nature of the beam instability to that of another type - dissipative instability. This type of instabilities comes to be developing in systems with electron beams where the beam wave with negative energy exists. Actually dissipation is nothing else as a channel of energy withdrawal for excitation of the beam wave with negative energy. This type of beam instabilities has an array of characteristic peculiarities [1-3].

Another type of beam instability, which is caused by excitation of the beam wave with negative energy, is also known in beam-plasma interaction theory. It is instability of overlimiting electron beam. It is known that the beam current which can path through the given elecrtodynamic system is limited by the beam space charge [4]. In order to overcome this restriction and to increase output power of microwave devices on relativistic electron beams the operation space of such a devices must be filled by plasma. Plasma neutralizes the beam space charge and plasma-filled systems can path through beams with current much higher than vacuum systems. However the physical nature of beam-plasma interaction and character of instability development changes as compared with underlimiting electron beams [4-7]. Traditional studies state that instability of electron beam is due to induced radiation of the system proper waves by beam electrons. Physical character of the overlimiting electron beam instability changes and now it is due either to aperiodical modulation of the beam density in medium with negative dielectric constant or to excitation of beam wave with negative energy. The last variety of overlimiting electron beam instability develops in no uniform cross section systems.

Dissipative instability of overlimiting electron beam has been considered for the first type of overlimiting electron beam instability only, i.e. in uniform cross section system [8].

Dissipative instability of overlimiting electron beam is of special interest in no uniform cross-section system. Superposition of the effects both cause excitation of the beam wave with negative energy must lead to many new effects: More critical dependence of the growth rate on the parameter characterizing dissipation, essential influence on mode structure of instability and some other new effects.
Present investigation considers the influence of dissipation on the instability of overlimiting electron beam caused by excitation of the beam wave with negative energy and transition of this instability to that of dissipative type. This mechanism of straight-line electron beam instability reveals itself under high beam density when the influence of the fields of the beam space charge may not be neglected. These additional fields of the beam proper oscillations effect on the process of stimulated radiation of the beam electrons. In physical arrangement radiation become similar to Raman process. This process is most effective in conditions of anomalous Doppler effect. The mechanism of coupling of the beam wave with emitted wave may be arbitrary. The only important condition is the coupling be weak.

2. Statement of the problem

Consider fully magnetized plasma filled waveguide penetrated by overlimiting relativistic electron beam. In such a system physical character of beam-plasma interaction essentially depends on transversal geometry. In the case of inhomogeneous beam and/or plasma the instability is due to excitation of beam wave with negative energy.
Generally say, rigorously treatment of the problem may not be developed based on the perturbation theory with small parameter. But in the case of spatially separated beam and plasma when the integral describing fields overlap is small, the effect may be considered analytically. In this case $p_b(\boldsymbol{r}_\perp)p_p(\boldsymbol{r}_\perp)=0$ (functions $p_{p,b}(\boldsymbol{r}_\perp)$ represent the profiles of the plasma and beam densities in the waveguige cross-section; for homogeneous beam and plasma $p_{p,b}(\boldsymbol{r}_\perp)=1$) for thin annular beam $p_b=\delta(r-r_b)$, where δ is the Dirac function and r_b is the beam radius and in zero approximation the system may be described by solving of following two independent problems

$$\Delta_\perp E_{z\alpha}-\left(k^2-\frac{\omega^2}{c^2}\right)[1-p_\alpha(r_\perp)\delta\varepsilon_\alpha]E_{z\alpha}=0 \qquad (1)$$

with the conditions $E_{z\alpha}=0$ on the metallic surfaces. Here $\alpha = p, b$ for plasma and beam respectively, $\delta\varepsilon_p=\omega_p^2/\omega(\omega+i\nu)$; $\delta\varepsilon_b=\dfrac{\omega_b^2}{\gamma^3(\omega-ku)^2}$, $\Delta_\perp$ is the Laplace operator with respect to transversal coordinates, ω and k are the frequency and wave vector of perturbations E_z is the longitudinal electric field which is represented in a form

$$E_z(\boldsymbol{r}_\perp,z,t)=E_z(\boldsymbol{r}_\perp,\omega,k)\exp(-i\omega t+ikz)$$

where the transversal coordinates $\boldsymbol{r}_\perp$ are separated out, z is the coordinate along the waveguide axis, t is the time, $\omega_{p,b}$ are the Lengmuire frequencies for plasma and beam respectively, γ is the relativistic factor of beam electrons $\gamma=(1-u^2/c^2)^{-1/2}$, u is the velocity of beam electrons, c is the speed of light. The zero order dispersion relations for beam and plasma are

$$D_\alpha(\omega,k) = k_{\perp\alpha}^2 + \left(k^2 - \frac{\omega^2}{c^2}\right)(1-\delta\varepsilon_\alpha) = 0 \tag{2}$$

where $k_{\perp\alpha}$ are determined by the proper function of zero order problems respectively, $\alpha = p, b$. If one applies the perturbation theory to this state he can obtain after cumbersome calculations the dispersion relation, which takes into account the beam-plasma interaction in first order approximation

$$D_p(\omega,k)D_b(\omega,k) = G[(k^2 - \frac{\omega^2}{c^2})^2 \delta\varepsilon_p \delta\varepsilon_b] \tag{3}$$

Here $G > 0$ and it is nothing else as so-called geometrical factor of the space charge. It shows the overlap of the plasma and the beam fields and represents specific properties of considered system.

The factor D_b in the dispersion relation (3) coincides with the dispersion relation describing beam oscillations in magnetized waveguide in the case of full filling. In the simplest one dimensional limit the spectra of beam oscillations are given by well known simplest expression

$$\omega = ku \pm \Omega_b \tag{4}$$

where $\Omega_b = \omega_b \gamma^{-3/2}$. In general case $\Omega_b = \Omega_b(\omega,k)$ and the spectra take complicate form. Obvious expression for the spectra can be obtained if one neglects the biased current as compared with the high frequency beam current i.e. $k << k_\perp \gamma$

$$\omega_\pm = ku \pm \alpha^{1/2} / \gamma \tag{5}$$

where $\alpha = \omega_b^2 / k_{\perp b}^2 u^2 \gamma^3$. Non-potential character of the beam waves is intrinsic for high beam current only, comparable or higher than the limiting vacuum current. Interaction of the beam wave with negative energy in form (5) (lower sign) with plasma will be considered below

3. Growth rates

The dispersion relation (3) determines proper oscillations of the system under consideration with spatially separated beam and plasma. It is known that physical character of beam-plasma interaction as well as the influence of dissipation on it depends on the beam current value. These changes must reveal themselves on the solutions of dispersion relation (3). If one searches the solutions in the form $\omega = ku(1+x)$, the dispersion relation takes following form

$$\left(x+\Delta+i\frac{\nu_0}{ku}f\right)\left(x^2-\frac{\alpha}{\gamma^2}\right)=\frac{G}{2\gamma^4}\frac{\alpha}{1-q} \tag{6}$$

where $\Delta=\dfrac{k^2u^2-\omega_0^2}{2\gamma^2\omega_p^2(1-q)}$; $f=\dfrac{1}{2\gamma^2(1-q)}$, $q=\beta^2\dfrac{k^2u^2}{\omega_p^2}$,

$\omega_0^2=\omega_p^2-k_{\perp p}^2u^2\gamma^2$, ν_0 – is the effective collision frequency in plasma.

It can be easily seen that solutions of (6) essentially depends on the value of parameter α. This parameter actually serves as a parameter that determines the beam current value and the character of beam-plasma interaction. One can easily see that α corresponds (correct to the factor γ^{-2}) to the ratio of the beam current to the limiting current in vacuum waveguide $I_0 = mu^3\gamma/4e$, i.e. $\alpha = I_b/(\gamma^2 I_0)$ (I_b is the beam current). The values $\alpha << \gamma^{-2}$ correspond to underlimiting beam current $I_b << I_0$ and the instability in this case is caused by induced radiation of system proper waves by beam electrons. As this takes place one can obtain (neglecting second and third terms in left hand-side) the growth rate of beam instability in waveguide with separated beam and plasma. In this case the beam instability is due to induced radiation of the systems proper oscillations by the beam electrons. The growth rate attains its maximum at Δ = 0 and is proportional to $\omega_b^{2/3}$ and to the fields' overlap $G^{1/3}$

$$\delta_{sbl}=\frac{\sqrt{3}}{2}\frac{ku}{\gamma}\left(\frac{G}{2\gamma}\frac{\alpha}{1-q}\right)^{1/3} \tag{7}$$

Abovementioned mechanism of beam instability changes to that caused by excitation of beam slow wave with negative energy if: (i) dissipation increases and exceeds the growth rate, (ii) beam current increases and exceeds the limiting vacuum current. In first case we have dissipative instability with growth rate

$$\delta_{\text{sbl}}^{(\nu)}=\frac{1}{\gamma}\sqrt{\frac{k^3u^3}{2\nu_0}G\alpha} \tag{8}$$

The usual dependence of the growth rate on parameter of dissipation as $\sim\nu^{-1/2}$ can be easily seen. Most of previous investigations consider dissipative instability of underlimiting electron beam in unbound system. Presented here cylindrical geometry is more close to real systems for intense microwave generation and charge particle acceleration. Particular case of dissipative beam instability, so called resistive wall instability is an important issue in systems for the last case i.e. for charge particle acceleration. This variety of dissipative beam instability is due to resistance and inductivity of waveguide walls and interaction of the beam with those. This interaction leads to excitation of the beam wave with negative energy.

Increasing of beam current also leads to excitation of the same beam wave with negative energy. If the conditions

$$\gamma^{-2} << \alpha << 1 \tag{9}$$

takes place the beam space charge significantly influence on beam plasma interaction. Instability turns to be due to excitation of the beam wave with negative energy. The growth rate attains its maximum under $\Delta=\sqrt{\alpha}/\gamma$. Taking this into account we can obtain

$$\delta_{\mathrm{sl}} = ku\left\{\frac{G}{4\gamma^3}\frac{\sqrt{\alpha}}{1-q}\right\}^{1/2} \tag{10}$$

One can easily see different (as compared with traditional) dependence on the beam density. Presence of high level dissipation in system transforms physical nature of beam instability. It becomes of dissipative type and also due to excitation of beam wave with negative energy. Corresponding growth rate is equal

$$\delta_{\mathrm{ovl}}^{(\nu)} = \frac{k^2u^2}{\nu_0}\frac{G}{2\gamma^2}\alpha \tag{11}$$

More critical dependence on parameter, characterizing dissipation is obtained. As we think superposition of two effects that lead to excitation of beam wave with negative energy leads to more effective withdrawal of energy and to more critical dependence on ν.

It is known, that conventional beam-plasma instability is due to induced radiation of the system proper waves by beam electrons. As the beam current increases its proper degrees of freedom come into interaction and change the mechanism of beam instability. Interaction becomes of wave-wave type. The beam slow wave with negative energy interacts with plasma waves and such process is known as collective Cherenkov effect [4]. Dissipation coming into interplay intensifies interaction and the growth rate if instability takes more critical dependence on it as compared with low beam currents.

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