Study of nonlinear phenomena in a tokamak plasma using a novel Hilbert transform technique

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Abstract. A new technique based on Hilbert transform is used to analyze edge turbulence data from tokamak ADITYA. In this technique, empirical mode decomposition has been introduced to extract finite number of intrinsic mode functions (IMFs) from the data. Hilbert transform of these IMFs help to determine the instantaneous frequency and energy which gives a clear representation of localized time-frequency spectrum, referred as Hilbert-Huang spectrum. In this paper, Langmuir probe data from the turbulent edge plasma show signatures of intermittency in the form of sporadic bursts of mode energy. The Hilbert-Huang spectrum also allows the evaluation of the degree of non-stationarity. It is observed that only high frequency signals (exceeding 20 kHz) are non-stationary.

1. Introduction

Study of tokamak transients during start-up phase, disruptive phase and flattop phase events such as, ELMs or onset of NTM, sawtoot crashes or nonlinear interaction of modes data is important to understand many associated physics. However, most of the tokamak diagnostics data related to transients have one or more of the following problems: (i) short data segment; (ii) non-stationary data; and (iii) the data represents nonlinear processes. The Fourier spectral analysis of such data may not give any physical sense because for such analysis, the data must be linear; and periodic or stationary. A data segment shorter than the longest time scale of a stationary process can even appear to be non-stationary. The requirement of stationary data is a general condition for most of the data analysis methods. Several Fast Fourier Transform (FFT) based techniques and their variants such as Short-time FFT, Wavelet Transform, Wigner-Ville Distribution, Bi-orthogonal Decomposition etc., have been used to process such data due to lack of alternatives. In short-time FFT method, a suitable time window is selected such that the data remain stationary. However, it is difficult to guarantee that the window size adopted will always coincides with the stationary time scales. Another practical problem arises when the window width is made narrower to localize an event in time, as this conflicts the requirement of frequency resolution for which the requirement is a longer window size. Therefore, this technique has a limited use. The most popular method is Wavelet Transform which provides a uniform resolution for all the scales. However, this method can only give physically meaningful interpretation to linear phenomena. The difficulty with the Wigner-Ville method is the cross terms as indicated by the existence of the negative power for some frequencies. Although this problem can be eliminated but the results will be that of the windowed FFT and would suffer all the limitations of Fourier analysis. Bi-orthogonal decomposition gives a distribution of the variance in the modes but this distribution by itself does not suggest scales or frequency components. In summary, these methods can have only limited use. Results of such analysis may generally give misleading energy-time-frequency spectrum as both the non-linearity and non-stationarity may induce spurious harmonic components that cause energy spreading.

A new method based on the direct extraction of the energy associated with various intrinsic time scales, has been developed, referred as empirical mode decomposition (EMD) for studying such kind of data [1]. In this method signal is decomposed into a finite number of intrinsic mode functions (IMF) which have well behaved Hilbert transforms, from which the instantaneous frequencies can be calculated. This helps us to localize any event on the time as well as the frequency axis in the spectrum. This decomposition is based on the physical time scales that characterizes the oscillations of the phenomena under consideration. The local energy and the instantaneous frequency derived from the IMFs through the Hilbert transform can give a full energy-frequency-time distribution, represented as the Hilbert spectrum of the data.

This paper is organized as follows: Hilbert Transform technique is discussed in section-2; application of this technique on edge turbulence data of ADITYA tokamak is presented in section-3. Conclusions and future scope of work are described in section-4.

2. Brief description of Hilbert transform technique

Evolution of a signal $X(t)$ between two consecutive extrema (say, two minima occurring at time t_1 and t_2), can define a (local) high-frequency part $D(t)$, or local *detail*, which corresponds to the oscillation terminating at the two minima and passing through the maximum which necessarily exists in between them. In addition, it is easy to identify the corresponding (local) low-frequency part $M(t)$, or local *trend*, so that $X(t)= M(t) + D(t)$ for $t_1 < t < t_2$. Extraction of various local details and trend from a signal $X(t)$,can be obtained by the following steps [2]:

(1) identify all extrema of $X(t)$
(2) interpolate between minima (and maxima) ending up with some envelope $E_{min}(t)$ (and $E_{max}(t)$)
(3) compute the mean of these envelopes, $M(t)$
(4) extract the detail, $D(t)=X(t)-M(t)$
(5) iterate on the residual

The above procedure is done by a *sifting* process [1] which is stopped by some criteria (discussed in the next paragraph). Once this is achieved, the detail is referred to as an Intrinsic Mode Function (IMF). By construction, the number of extrema is decreased when going from one residual to the next, and the whole decomposition is ended up with a finite number of modes.

As described in step-2 in above algorithm,the upper and lower envelopes are computed by interpolated curves between the extrema. The interpolation scheme is therefore important. Cubic spline interpolation is the right choice as linear or polynomial interpolation schemes tend to increase the *sifting* iterations. Since this algorithm operates on discrete data, care must be taken for identifying correct extrema.The extraction of a mode is considered as satisfactory when the *sifting* process is terminated. Two conditions are to be fulfilled [1]: the first one is that the number of extrema and number of zero-crossings must differ at most by 1; the second one is that the mean between the upper and lower envelopes must be close according to some criteria. Imposing a too low threshold for terminating the iteration process can lead to over- or under-decomposition. Therefore, an improved criteria is proposed in [2] by introducing two thresholds δ_1 and δ_2 to guarantee globally small fluctuations in the mean while taking large excursions. Following conditions for continuing iteration of the *sifting* are made on an evaluation function ,$S(t) = |M(t)/A(t)|$ (here, $A(t) = (E_{max}(t)- E_{min}(t)) /2$ being the amplitude of the corresponding mode) for these thresholds: $S(t)< \delta_1$ for some prescribed fraction ($1-\alpha$) of the total duration, while $S(t)< \delta_2$ for the remaining fraction. For this paper, $\alpha = .05$, $\delta_1 = .05$ and $\delta_2 = 10\ \delta_1$ are set.

Once the IMF components are extracted, the Hilbert transform of each IMF can give information of instantaneous frequency. A localized frequency and energy spectrum, referred to as Hilbert-Huang spectrum, can be obtained. More detailed description can be found in [1].

3. Application to the edge turbulence data in tokamak ADITYA

ADITYA tokamak is a medium sized tokamak with major radius of 0.75m and minor radius of 0.25m. The basic operating parameters are: density ~ $10^{19}m^{-3}$, temperature~500eV and B_T~0.8T. The edge turbulence is measured by Langmuir probes arranged in a 10×10 matrix form [3]. Intermittency has been reported for edge fluctuation data [3] in tokamak ADITYA and a new look at the intermittent behaviour of the fluctuation can be further strudied by using this novel Hilbert spectral technique. By obtaining localized frequency and energy spectrum as well as the degree of non-stationarity [1] can give new insights on intermittency physics .

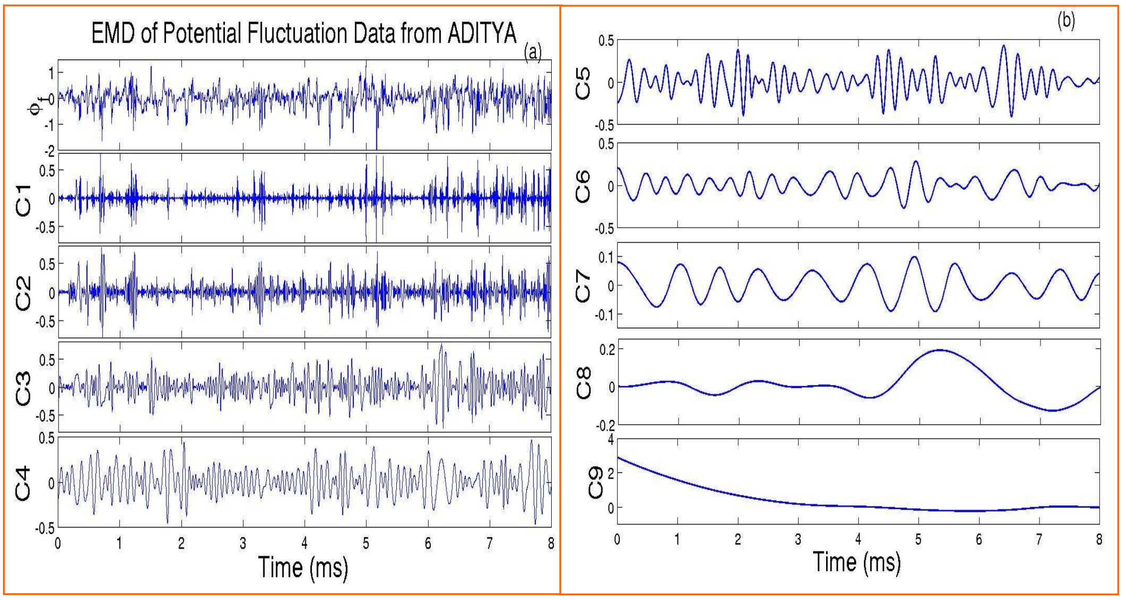

FIG. 1. Edge fluctuation data and its empirical mode decomposition. EMD analysis decomposes into eight IMF components and a trend(the ninth component).

Various IMF components decomposed by EMD analysis are shown in Fig.1. It is observed that the ninth component is just a trend and not and an IMF. When the IMF components are hilbert transformed, the instantaneous frequencies and energy densities can be calculated as described in [1].

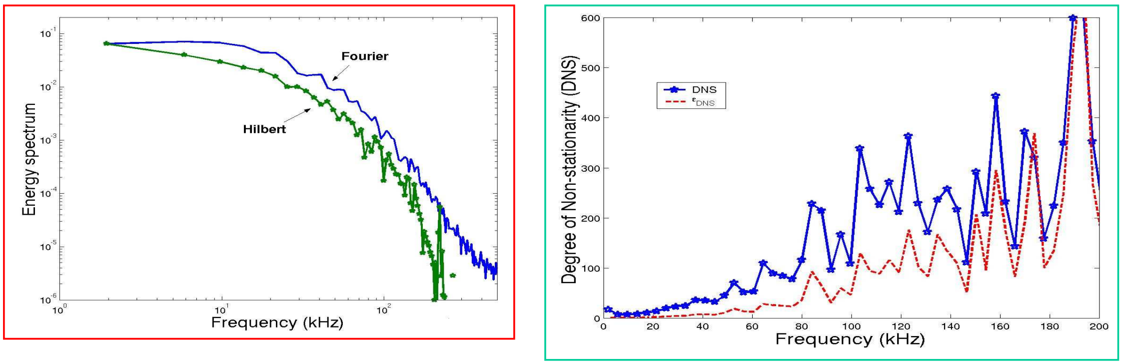

FIG. 2. (a) Comparison of Hilbert spectrum and Fourier spectrum. Power at high frequencies fall sharply as compared to the Fourier spectrum (b) from this analysis it is observed that frequecies exceeding 20kHz are non-stationary. Also, error in the calculation for degree of non-stationarity is high for frequencies above 80kHz.

The energy spectrum thus obtained by this technique can be compared by the Fourier spectrum (see Fig.2) which clearly shows the presence of high frequency modes at the tail end of Fourier spectrum may be due to the non-stationary nature of the signal. This is furture confirmed by the computation of degree of non-stationarity which shows that frequencies exceeding 20kHz are non-stationary. However, the calculation error is high for frequecies above 80kHz. In Fig.3, the instantaneous energy gives a clear indication of intermittent behaviour of edge potential fluctuation. In this figure, correlation of the fluctuating signal with the sporadic bursts of mode energy is clearly seen.

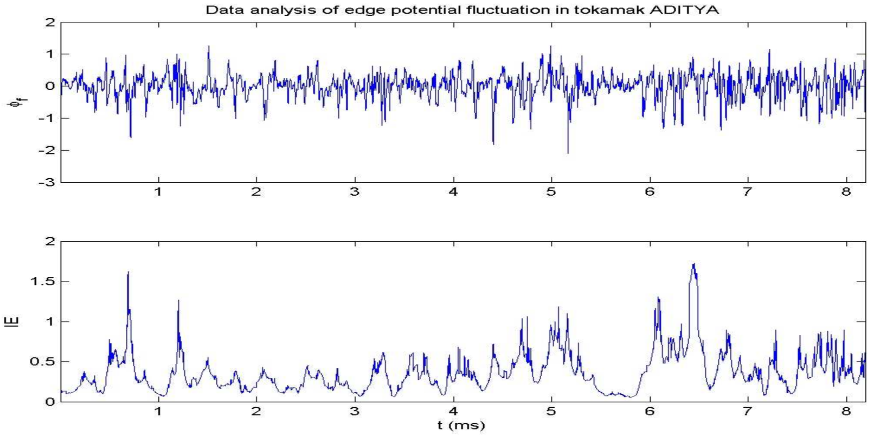

FIG. 3. The instantaneous energy obtained from Hilbert spectrum technique shows the intermittent behaviour of fluctuation more clearly.

With this localized informations of instantaneous frequency and energy, it is easy to show a time-frequency-energy spectrum, referred to as Hilbert-Huang spectrum. For a comparison, it is shown along with the spectrum obtained by Wavelet transform. It is clearly seen that Hilbert-Huang spectrum gives more localized informations as compared to the one obtained by Wavelet transform.

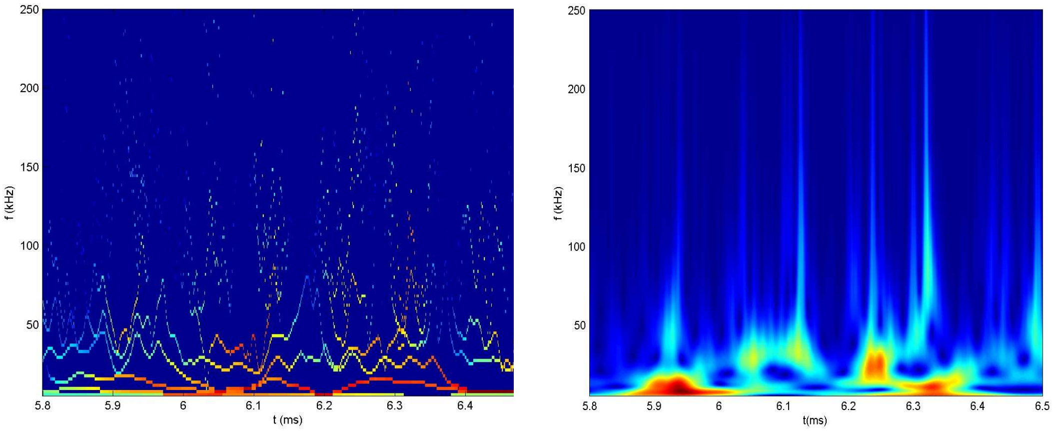

FIG. 4. A localized frequency and energy information can be obtained by Hilbert-Huang spectrum as compared to the Wavelet transform spectrum.

4. Conclusion and future scope of work

A newly emerging Hilbert Transform technique has been applied to extract a finite number of 'intrinsic mode functions (IMFs)' from the edge fluctuation data of tokamak ADITYA. Instantaneous frequencies calculated from these IMFs are used to localize any event on a Hilbert-Huang spectrum. Advantage of this technique is that it handles signals having small number of data points with non-linearity and/or non-stationarity. The edge potential fluctuation data show signatures of intermittency in the form of sporadic bursts of mode energy. By using Hilbert spectrum, the degree of non-stationarity is evaluated which shows that only the high frequency components (exceeding 20 kHz) are non-stationary.

Results obtained by Hilbert transform technique are quite encouraging and motivates to apply it to many diagnostics data of fusion plasmas particularly to transient events. For such events the diagnostics data could be non-stationary, nonlinear and of a short span. Therefore, this technique may be useful and give appropriate informations. In addition, application of this technique may also give more physical insight to several other plasma phenomena such as, triggering of sawtooth events, crashing of sawtooth, triggering of NTMs, modes and its structures during ELMs etc.

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