EFFECTIVE ELECTROSTATIC PLASMA LENS FOR FOCUSSING OF HIGH-CURRENT ION BEAMS

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The optimum plasma lens, intended for the focussing of high-current ion beams, has been investigated.

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INTRODUCTION

Optimum plasma lens (PL) realized in experiments [1] at weak magnetic fields has been theoretically researched. The optimum PL is lens, in which the perturbations are not excited and particle density is uniform. Three possible reasons of perturbation damping in PL have been researched: namely, finite time of ion movement through PL, finite time of electron renovating in it, proximity of PL parameters to optimum ones. It has been shown, that the vortices are not excited in PL, if the overbalance of ions by electrons is close to limiting one.

INFLUENCE OF PLASMA VORTICES ON ION BEAM FOCUSSING IN PLASMA LENS

In short cylindrical electrostatic PL for ion beam focusing the vortices can be excited in wide on radius, $R-r_t<r<R$, near wall region. Two kinds of vortices are excited. Namely, quick vortices are excited due to development of the resonant hydrodynamic instability of interaction of electron vortices of PL with ions. Also slow vortices are excited, which phase velocities are much smaller then the electron drift velocity in PL crossed fields. The excitation of the vortices leads to anomalous electron radial transport, therefore, to decrease of electron density and to PL focusing quality deterioration. The numerical simulation shows (see Fig. 1) that if the overbalance of ions by electrons decreases monotonically due to vortices excitation in wide near wall layer, which width equals 4/7 of PL radius, then ion beam can be focused six times.

At increase of the PL magnetic field, H_o, electron confining properties of the magnetic field increase and, though vortices are excited, PL focusing quality is improved. The latter dependence has been observed in [1].

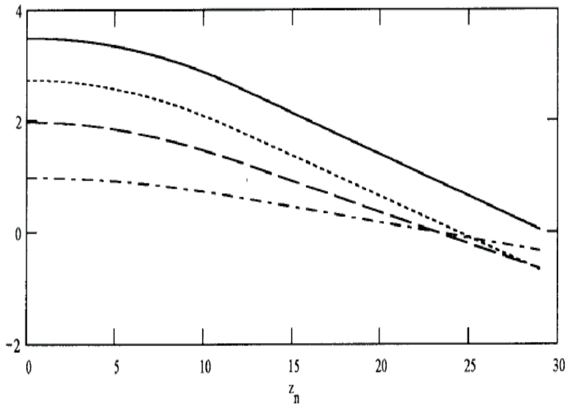

Fig. 1. Focusing of ion beam by plasma lens if n_e-n_i decreases in its circumferential region, R-r_t<r<R, monotonically on radius to 0 due to excitation of vortical perturbations

Vortices are not excited in PL, if the overbalance of ions by electrons $\Delta n \equiv n_{eo}-n_{io}$ is closed to limit Δn_{th}. Let us determine Δn_{th} from the condition of balance upset of the radial forces, confining electrons in the region of finite radius: $\omega_{He}V_\theta m_e = m_e V_\theta^2/r - eE_r$. From this expression one can obtain that two last forces exceed first one if overbalance of ions by electrons, $\Delta n \equiv n_{eo}-n_{io}$, is not smaller then Δn_{th}, determined by $\Delta n_{th} = H_o^2/8\pi m_e c^2$. If Δn could be formed such that the following inequality is executed $\Delta n > \Delta n_{th}$, then the electron cloud could propagate freely transversally to magnetic field. Then the vortices can not be excited in PL in the limiting case $\Delta n = \Delta n_{th}$. Let's explain it. The instability development of the vortical perturbation excitation leads to the electron bunch formation. But at Δn, closed to Δn_{th}, the electron bunches can not be formed due to the vortices excitation, because any electron bunches formation leads at once to their destruction by centrifugal and electric scattering forces.

The following expression $\Delta n_{th} = H_o^2/8\pi m_e c^2$ shows observed in [1] quadratic relation of optimum electron density n_{eo} on optimum H_o.

The expression $\Delta n_{th} = H_o^2/8\pi m_e c^2$ also shows fulfillment of criterion that the radius, $r_{He} = eE_r/m_e\omega^2_{He}$, of the radial electron oscillations in crossed radial electric, E_r, and longitudinal magnetic, H_o, fields should be approximately equal to the PL radius, R.

SUPRESSION OF EXCITATION OF VORTICES IN OPTIMUM PLASMA LENS

Let us derive the dispersion relation and show that the oscillation excitation in the cylindrical PL is damped at its optimum parameters. We take into account that the beam ions pass through the plasma lens of length L during time $\tau_i = L/V_{bi} \equiv 1/\nu_i$. The electrons are renovated in PL also during finite time, $\tau_e \equiv 1/\nu_e$.

We use the electron, ion hydrodynamic eq.s

$$\partial_t\mathbf{V}+\nu_e(\mathbf{V}-\mathbf{V}_{\theta o})+(\mathbf{V}\nabla)\mathbf{V}=(e/m_e)\nabla\varphi+[\boldsymbol{\omega}_{\mathbf{He}},\mathbf{V}]-(V^2_{th}/n_e)\nabla n_e,\ \partial_t n_e+(n_e-n_{eo})/\tau_e+\nabla(n_e\mathbf{V})=0 \quad (1)$$

$$\partial_t\mathbf{V_i}+\nu_i(\mathbf{V}-\mathbf{V}_{bi})+(\mathbf{V_i}\nabla)\mathbf{V_i}=-(q_i/m_i)\nabla\varphi,\ \partial_t n_i+(n_i-n_{io})/\tau_i+\nabla(n_i\mathbf{V}_i)=0 \quad (2)$$

and Poisson eq. for the electrical potential, φ ,

$$\Delta\varphi=4\pi(en_e-q_in_i) \quad (3)$$

Here $\mathbf{V}$, n_e are the velocity and density of electrons; V_{th} is the electron thermal velocity; $\mathbf{V}_{\theta o}$ is the electron azimuth drift velocity in crossed fields of PL; $\mathbf{V_i}$, n_i, q_i, m_i are the velocity, density , charge and mass of ions.

As the sizes of vortices are much greater than the electron Debye radius, $r_{de}\equiv V_{th}/\omega_{pe}$, then the last term in (1) can be neglected. Here $\omega_{pe}\equiv(4\pi n_{oe}e^2/m_e)^{1/2}$, n_{oe} is the unperturbed electron density.

From eq.s (1) one can derive non-linear eq.s

$$d_t[(\alpha-\omega_{He})/n_e]=[(\alpha-\omega_{He})/n_e]\partial_z V_z-\alpha\nu_e/n_e,\ d_tV_z+\nu_eV_z=(e/m_e)\partial_z\varphi \quad (4)$$

describing both transversal and longitudinal electron dynamics. Here

$$d_t=\partial_t+(\mathbf{V}_\perp\nabla_\perp)\ ,\ \partial_t\equiv\partial/\partial t\ ,\ \partial_z\equiv\partial/\partial z \quad (5)$$

$\mathbf{V}_\perp$, V_z are the transversal and longitudinal electron velocities, $\alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}$ is the vorticity.

Taking into account higher linear terms, from (1) one can obtain

$$\mathbf{V}_\perp\approx(e/m\omega_{He})[\mathbf{e}_z,\nabla_\perp\varphi]+(e/m\omega^2_{He})(\partial_t+\nu_e)\nabla_\perp\varphi \quad (6)$$

From (6) we derive

$$\alpha\approx-2eE_{ro}/rm\omega_{He}-(eE_{ro}/m)\partial_r(1/\omega_{He})+(e/m\omega_{He})\Delta_\perp\phi+(e/m)(\partial_r\phi)\partial_r(1/\omega_{He})+$$
$$+(e/m)(\partial_t+\nu_e)\mathbf{e}_z[\nabla_\perp,\omega^{-2}_{He}\nabla_\perp\phi]\equiv\mu\omega_{He}\ ,\ \nabla\varphi\equiv\nabla\phi-\mathbf{E}_{or} \quad (7)$$

Here E_{or} is the radial focusing electric field, ϕ is the electric potential of the vortices; $-2eE_{ro}/rm\omega_{He}=(\omega^2_{pe}/\omega_{He})(\Delta n/n_{oe})\equiv\eta\omega_{He}$, $\Delta n\equiv n_{oe}-q_in_{oi}/e$.

From (3), (7) $\alpha\approx(\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo}$ approximately follows. Thus the vortical motion begins, as soon as the electron density perturbation, δn_e, appears.

We use that, as it will be shown below, the characteristic frequencies of perturbations approximately equal to ion plasma frequency, ω_{pi}.

As beam ions have large mass and propagate through PL with fast velocity V_{ib}, we will describe their dynamics in linear approximation. We derive ion density perturbation from eq.s (2)

$$\delta n_i=-n_{io}(q_i/m_i)\Delta\varphi/(\omega-k_zV_{ib}+i\nu_i)^2 \quad (8)$$

Here k, ω are wave number and frequency of perturbation, V_{ib} is the ion beam velocity. Substituting (8) in Poisson eq. (3), one can obtain

$$\beta\Delta\varphi/4\pi e=\delta n_e,\ \beta=1-\omega_{pi}^2/(\omega-k_zV_{ib}+i\nu_i)^2,\ n_e=n_{oe}+\delta n_e \quad (9)$$

Let us consider instability development in linear approximation. Then we search the dependence of the perturbation on z, θ in the form $\delta n_e\propto\exp(ik_zz+il_\theta\theta)$. Then from (4) we derive

$$d_t(\omega_{He}/n_e)(1-\mu)=\alpha\nu_e/n_e-(e\omega_{He}/m_e n_{eo})ik_z^2\varphi(1-\mu)/(\omega-l_\theta\omega_{\theta o}+i\nu_e),\ \omega_{\theta o}\equiv V_{\theta o}/r. \quad (10)$$

From (5), (6), (9), (10) we obtain, using the radial gradient of the short coil magnetic field, the following linear dispersion relation, describing the instability development

$$1-(1-\eta)\omega^2_{pe}(l_\theta/r)\partial_r(1/\omega_{He})/k^2(\omega+i/\tau_e-l_\theta\omega_{\theta o})-\omega^2_{pi}/(\omega+i/\tau_i-k_zV_{bi})^2-(1-\eta)\omega^2_{pe}k_z^2/k^2(\omega+i/\tau_e-l_\theta\omega_{\theta o})^2=0. \quad (11)$$

It is necessary to note, that at optimum PL parameters $\eta=1$.

Let us mean the quick vortices those, which phase velocities $V_{ph}\approx V_{\theta o}$. For them from (11) we derive in approximation $k_z=0$, $\omega=\omega^{(o)}+\delta\omega$, $|\delta\omega|<<\omega^{(o)}$ and neglecting τ_e, τ_i

$$\omega^{(o)}=\omega_{pi}=l_\theta\omega_{\theta o},\ \omega_{\theta o}=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe}),\ \Delta n\equiv n_{oe}-q_in_{oi}/e \quad (12)$$

$$\delta\omega=i\gamma_q,\ \ \gamma_q=(\omega_{pe}/k)[(1-\eta)(\omega_{pi}/2)(l_\theta/r)|\partial_r(1/\omega_{He})|]^{1/2}$$

Here n_{oi} is the unperturbed ion density. At obtaining (12) we used a validity of an inequality

$$(1-\eta)(\Delta n/n_{oe})(r/\omega_{He})\omega^2_{pe}|\partial_r(1/\omega_{He})|<<m_e/m_i \quad (13)$$

It is fulfilled at a weak overbalance of beam ion space charge by electrons of PL, $\Delta n/n_{oe}<<1$, at small radial non-uniformity of ω_{He}, small plasma density, $\omega_{pe}/\omega_{He}<<1$ and for PL, closed to optimum one. From (12) one can see that in PL, which parameters are closed to optimum ones, the excitation of the quick vortices is damped.

From (12) it follows $l_\theta=(m_e/m_i)^{1/2}(\omega_{He}/\omega_{pe})(n_{oe}/\Delta n)$, that for typical parameters of experiments, $\Delta n/n_{oe}\approx0.1$, the perturbations with $l_\theta>1$ are excited at a large magnetic field and at small electron density.

As γ_q grows with r, taking into account τ_e, τ_i can lead to that perturbations with r smaller then critical value can not be excited.

Let us introduce slow vortices as those ones, whose phase velocity is small, $V_{ph}<<V_{\theta o}$. We derive for them from (11) in approximation $k_z=0$ and neglecting τ_e, τ_i the following expressions

$$\gamma_s=(\sqrt{3}/2^{4/3})[\omega^2_{pi}l_\theta(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})]^{1/3},\ k^2=-(1-\eta)(1/V_{\theta o})\omega^2_{pe}\partial_r(1/\omega_{He}),\ \ \mathrm{Re}\,\omega_s=\gamma_s/\sqrt{3} \quad (14)$$

Here γ_s is the growth rate of excitation of slow vortices of small amplitudes, $\mathrm{Re}\,\omega_s$ is the real part of the frequency. As γ_s grows with r, then taking into account τ_e, τ_i should lead to that perturbations with r smaller than some value are not excited. In other words, the vortices are excited far from the PL axis.

(14) is obtained in approximation of a validity of a following inequality $l_\theta>>(n_{oe}/\Delta n)2\omega_{pi}\omega_{He}/\omega^2_{pe}$. From (14) one can show that not large l_θ are excited. From (21) it also follows that in optimum PL the slow vortices are not excited.

So, the quick vortices are excited at relatively small plasma density, and at parameters of optimum PL the quick vortices are not excited. The slow vortices are excited at large plasma density, and the growth rate equals zero for optimum PL parameters.

SIMULATION OF HIGH-CURRENT ION BEAM FOCUSSING IN PLASMA LENS

Let us numerically simulate the ion beam focusing in high-current PL of different structures. Namely, the spatial structure of magnetic field lines of short cylindrical permanent magnet or short coil such that the electron cloud of PL can be solid cylinder of length L and radius R with hollow cones of radius R and with cones lengths Z_1 and Z_2 on the ends. In Fig. 2 the simulation results of ion beam focusing in such PL are presented.

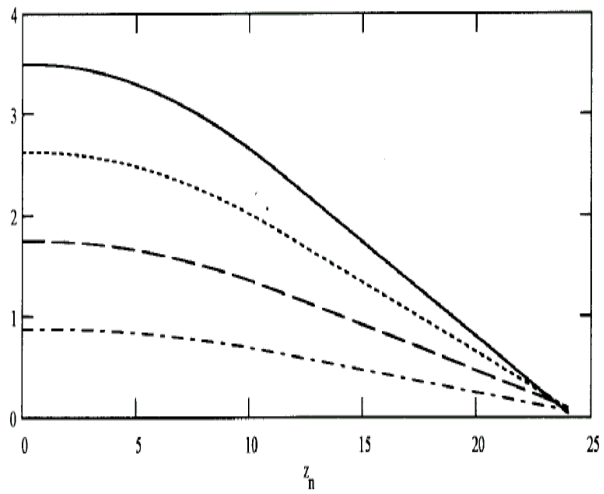

Fig. 2. Focusing of ion beam by plasma lens if electron column has hole cones on its ends

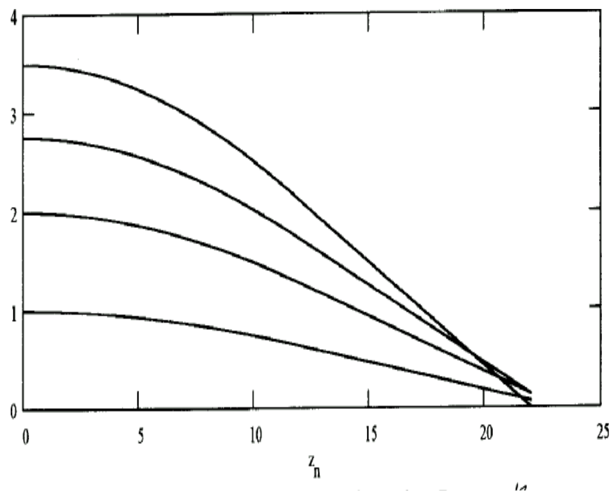

Fig. 3. Focusing of ion beam by plasma lens if electron density grows in region, R-r_t<r<R, on 1/3

One can see that all trajectories do not converge in one point, however if the altitudes of hollow cones are small, the aberrations are small.

In Fig. 1 the results of the ion beam focusing in PL, in which in wide near wall region, R-r_t<r<R, the strong vortices are excited, are shown. These perturbations lead in the region of their excitation to anomalous radial transport of electrons, and certainly to strong decrease of electron density in this region.

The excitation of the oscillated fields in the near wall region of PL can be damped by providing of the positive radial gradient of the electron density. Certainly, this gradient of density could lead to aberrations of focused beam. The numerical simulation, presented in Fig. 3, shows that aberrations are not large.

REFERENCES

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