Invited Paper

Electrodynamics and Dispersion Properties of Dusty Plasma with Ferromagnetic Grains

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Abstract. We obtained the analytic expression of the permeability tensor of the dusty plasma consisting of grains with magnetic moments in the external constant magnetic field. In the strong field the magnetic moments are oriented along the force lines and their small vibrations contribute to the transversal magnetic polarization. This motion results in appearance of a new typical frequency in the dusty plasma. It is shown that such a medium may have the negative permittivity and permeability simultaneously in a definite range of frequency.

1. Introduction

Recently the materials are composed from conductive elements and exhibit tensor features of dielectric permittivity and magnetic permeability. Also they have a range at microwave frequencies where these electrodynamics parameters are simultaneously negative. Some experimental verifications of unusual phenomena under wave refraction on an interface between two materials, Doppler's effect, Vavilov-Cherenkov radiation, etc. [1]. Theoretically V.G.Veselago [2] firstly described the electrodynamics properties of the medium with such dielectric permittivity ε and magnetic permeability μ . These media were called the "left handed materials" (LHM). The frequency dispersion of ε and μ of majority of the real materials relates to considerably different frequency ranges. Moreover, even definition of the magnetic permeability becomes meaningless in the optical range of frequencies where a magnitude of μ is close to unit [3]. The known experiments on testing of the corresponding theoretical predictions the LHM physical properties have been performed in the SHF range. In particular, the experiments [4] with the two-dimensional array of repeated unit cells of copper strips and strip ring resonators showed that they could be interpreted with the help of the following negative refraction index $n = \sqrt{(-\varepsilon)(-\mu)} < 0$ within the frequency band around 10 GHz.

In this paper, we consider dispersion properties of the magneto-active dusty plasma with ferromagnetic grains and demonstrate that such a medium may be a good candidate for the LHM. For the sake of simplicity, we suppose that the grains are identical spheres of a radius a carrying the same constant magnetic dipole moment d_m. This system could be easily prepared in laboratory. Plasma as electrodynamics system has negative dielectric permittivity in definite frequency range naturally. Addition of ferromagnetic grains changes its magnetic properties essentially.

We consider the case of "cold magnetic dipoles" embedded in the electron-ion plasma in the strong magnetic fields H_0 when the following inequality holds true

$$d_m H_0 / T >> 1 . \qquad (1)$$

Here T is the grain system temperature. In our case, the low temperatures support this inequality. We do not discuss the process of grain charging. Further, we assume that

$$H_0 >> 4\pi N_g d_m, \quad (2)$$

where $N_g \sim r^{-3}$ is the grain density number (r is a mean distance between grains). The latter inequality means that the magnetization of the dusty plasma, associated with orientation of individual magnetic dipoles, is small in comparison with the external magnetic field. This inequality allows us to ignore both the dipole-dipole interactions between grains and influence of the grain magnetization on the properties of electron-ion plasma.

It is known that the most part of the weakly damping waves in a magneto-active plasma have magnetic components. In the varying magnetic field

$$\vec{H}(\vec{r},t) = \vec{H}\exp[i\vec{k}\vec{r} - i\omega t] \quad (3)$$

the dipoles could slightly change orientation with respect to the H_0 direction that is chosen parallel to the z-axis. Here k and ω are the wave vector and frequency of the wave respectively.

2. Magnetized dusty plasma

We consider the motion of an individual magnetic dipole in the strong constant homogeneous magnetic field H_0 and varying field (3) in the spherical coordinate system. The Lagrange function of the dipole may be written in the form

$$L = \frac{m_g}{2}v^2 + \frac{J}{2}(\dot{\theta}^2 + \dot{\varphi}^2) + d_m H_0 \cos\theta + \vec{d}_m \cdot \vec{H}(\vec{r},t) \quad , \quad (4)$$

where, $\vec{v}$ is the velocity of translation motion of the mass center of magnetic dipole with a mass m_g and a moment of inertia J , φ and θ are the azimuth and polar angles of its orientation, respectively.

In the long wave approximation, when a wavelength of the varying field λ is large in comparison with the typical size of the magnetic dipole a, the force acting on the grain center of mass is of the order of $a/\lambda<<1$. This allows us to ignore the transnational motion of the grain and consider only its rotational degrees of freedom and take into account only the time dependence of the varying magnetic field (3).

It is worth noting that below we study the rotation motion of the grain as a whole in the magnetic field. This motion considerably depends on the grain inertia moment. The motion of magnetization vector associated with the electron subsystem of the magnetic dipole does not depend on its inertia at all and causes the dispersion of the grain magnetic susceptibility as well but relates to the frequency range of the ferromagnetic resonance.

The equations of motion that follow from (4) take the form

$$\ddot{\varphi} = \frac{d_m}{J}\{\sin\theta[-H_x \sin\varphi + H_y \cos\varphi]\}\exp(-i\omega t),$$
$$\ddot{\theta} + \frac{d_m H_0}{J}\sin\theta = \frac{d_m}{J}[\cos\theta\{H_x \cos\varphi + H_y \sin\varphi) - H_z \sin\theta]\exp(-i\omega t). \quad (5)$$

The system of nonlinear equations (5) describe variations of orientation of the grain magnetic moment with time.

Now we consider the case when the amplitudes of the varying magnetic field (3) are of the first order of smallness with respect to the amplitude of the constant field H_x, $H_y << H_0$. Keeping this in mind and taking into account that in this case $\theta<<1$, we solve the system (5) by the method of step by step approximations. In the zero approximation, the system (5) takes the form

$$\ddot{\varphi} = 0,$$
$$\ddot{\theta} + \omega_0^2\theta = 0. \quad (6)$$

where

$$\omega_0 = \sqrt{d_m H_0 / J} \tag{6a}$$

is the frequency of small vibrations of the grain magnetic moment orientation with respect to the z –axis. From the first equation of system (6), we see that the azimuth angle of the magnetic dipole varies with time according to the law

$$\varphi = \Omega t + \varphi_0 , \tag{7}$$

where Ω and φ_0 are integration constants. Obviously, the angular frequency Ω coincides with the thermal frequency $\omega_T = \sqrt{T/J}$. Comparing Ω with the typical frequency of variation of $\theta(t)$ ω_0, we get $\omega_0 / \omega_T = \sqrt{d_m H_0 / T} >> 1$ in accordance with the model of the "cold" magnetic dipoles. This means that in our model, the azimuth rotation of the dipole around the *z* –axis $\varphi(t)$ is slow in comparison with oscillations of its polar angle $\theta(t)$. In another words, while solving the first order equation for $\theta_1(t)$

$$\ddot{\theta}_1 + \omega_0^2 \theta_1 = \frac{d_m}{J} [H_x \cos\varphi + H_y \sin\varphi] \exp(-i\omega t) , \tag{8}$$

we could set φ to be a constant. The partial solution of equation (8) describes the forced oscillations of the magnetic dipole direction and takes the form

$$\theta_1(t) = \frac{d_m}{J} \frac{H_x \cos\varphi + H_y \sin\varphi}{\omega_0^2 - \omega^2} \exp(-i\omega t) . \tag{9}$$

The substitution (9) in the first equation of system (5) with constant φ shows that variations of the azimuth angle with time is the second order of smallness in the parameter $H/H_0 << 1$.

Now it is possible to find the magnetization of the unit volume of the system of magnetic grains by comparing the magnetization

$$M_i = N_g d_{mi} \tag{10}$$

with the phenomenological expression

$$M_i = \chi_{ij} H_j . \tag{10a}$$

In these formulas χ_{ij} is the tensor of magnetic susceptibility, indexes *i,j = x, y, z.* Recalling that in our model

$$d_{mx} = d_m \theta \cos\varphi,\ d_{my} = d_m \theta \sin\varphi,\ d_{mz} \cong d_m , \tag{11}$$

with the help of (9), we get that the tensor of magnetic permeability has the following nonzero components

$$\mu_{xx} = \mu_{yy} \equiv \mu = 1 + \frac{\Omega_m^2}{\omega_0^2 - \omega^2},\ \mu_{zz} = 1;\ \ \Omega_m = \sqrt{\frac{2\pi N_g d_m^2}{J}}. \tag{12}$$

The transversal components of the permeability tensor (12) have a resonance at $\omega = \omega_0$. Thus, the frequency ω_0 is given by relation (6a) may be treated an additional typical frequency of the magneto-active dusty plasma with magnetized grains.

The above reported results relate to the model of spherical grains with built-in magnetic dipoles. Apparently, they will be true for the needle-like "cold" magnetized dusty particles in the strong magnetic fields due to slow azimuth rotation of the magnetic dipoles.

3. Additional waves

Now we consider the additional waves in that appear in the magneto-active dusty plasma with the magnetic grains. The dispersion law for monochromatic waves propagating in a medium with arbitrary permittivity $\hat{\varepsilon}(\vec{k},\omega)$ and permeability $\hat{\mu}(\vec{\kappa},\omega)$ tensors can be obtained from Maxwell's equations. For the electromagnetic fields varying according to the law $\sim\exp(i\vec{\kappa}\cdot\vec{r}-i\omega t)$, we get

$$\begin{aligned}&\vec{\kappa}\times\vec{E}=\frac{\omega}{c}\vec{B},\quad \vec{\kappa}\times\vec{H}=-\frac{\omega}{c}\vec{D},\\ &\vec{k}\cdot\vec{B}=0,\qquad \vec{k}\cdot D=0,\\ &\vec{D}=\hat{\varepsilon}\left(\vec{k},\omega\right)\cdot\vec{E},\ \vec{B}=\vec{H}+4\pi\vec{M}=\hat{\mu}\left(\vec{k},\omega\right)\cdot\vec{H}.\end{aligned} \tag{13}$$

Here E and H are the Fourier transforms of the electromagnetic field. The permeability tensor of our media is given by (12). The permittivity tensor of the cold dusty plasma in the coordinate system with z-axis along the constant magnetic field H_0 could be presented in the following form:

$$\varepsilon_1=\varepsilon_{xx}(\omega)=\varepsilon_{yy}(\omega)=1-\sum_\sigma\frac{\Omega_\sigma^2}{\omega^2-\omega_{H\sigma}^2},\ \varepsilon_3=\varepsilon_{zz}(\omega)=1-\sum_\sigma\frac{\Omega_\sigma^2}{\omega^2}, \tag{14}$$

$$\varepsilon_{xy}(\omega)=\varepsilon^*{}_{yx}(\omega)=-i\varepsilon_2,\ \ \varepsilon_2=\sum_\sigma\frac{\omega_{H\sigma}}{\omega}\frac{\Omega_\sigma^2}{\omega^2-\omega_{H\sigma}^2},$$

$\Omega_\sigma=\sqrt{\dfrac{4\pi e_\sigma^2 N_\sigma}{m_\sigma}}$ is the plasma frequency, $\omega_{H\sigma}=\dfrac{e_\sigma H_0}{m_\sigma c}$ is the cyclotron frequency, σ numerates kinds of charged particles: *e* (electrons), *i* (ions), and *g* (grains).

It will be convenient further to introduce the inverse tensor permeability μ_{ij}^{-1} with the help of the relation $\mu_{ik}^{-1}\mu_{kj}=\delta_{ij}$. It follows from (12) that

$$\mu_{xx}^{-1}=\mu_{yy}^{-1}=\frac{1}{\mu},\ \mu_{zz}^{-1}=1. \tag{15}$$

Eliminating the vector of magnetic induction *B* from the Maxwell equations (13), we obtain the system of linear algebraic equations with respect to the components electric field

$$\begin{aligned}&[\mu\varepsilon_1-\eta^2\cos^2\vartheta]E_x-i\varepsilon_2E_y++\eta^2\sin\vartheta\cos\vartheta\,E_z=0,\\ &i\varepsilon_2E_x+[\mu\varepsilon_1-\eta^2(\cos^2\vartheta+\mu\sin^2\vartheta]E_y=0,\\ &\eta^2\sin\vartheta\cos\vartheta\,E_x+[\mu\varepsilon_3-\eta^2\sin^2\vartheta]E_z=0,\end{aligned} \tag{16}$$

$\eta=kc/\omega$ is the refraction index, ϑ is the angle between the wave vector $\vec{k}$ and the magnetic field $\vec{H}_0$. Equating to zero the determinant of the system (16), we get the equation

$$\mu(\omega)\{a\eta^4 + b\eta^2 + c\} = 0, \qquad (17)$$

$$a = (\varepsilon_1 \sin^2 \vartheta + \varepsilon_3 \cos^2 \vartheta)[1 + (\mu - 1)\sin^2 \vartheta],$$

$$b = \mu(\omega)(\varepsilon_2^2 - \varepsilon_1^2)\sin^2 \vartheta - \varepsilon_1 \varepsilon_3 [1 + \cos^2 \vartheta + (\mu - 1)\sin^2 \vartheta],$$

$$c = \mu^2(\omega)[\varepsilon_1^2 - \varepsilon_2^2]\varepsilon_3 .$$

If the grains have no magnetic moments $\mu(\omega)\to 0$, equation (17) coincides with the corresponding result for the conventional magneto-active plasma [5].

Equation (17) splits into two equations $\mu(\omega)=0$ and $a\eta^4+b\eta^2+c=0$. The first one relates to the collective vibration of magnetic dipoles in the plane perpendicular to the constant magnetic field H_0. We will call them the magnetization waves. The dispersion law of these waves is given by the following expression

$$\omega=\sqrt{\frac{d_m H_0}{J}\left(1+\frac{2\pi N_g d_m}{H_0}\right)}\approx\omega_0. \qquad (18)$$

Here we took into account inequality (2). The wavelength that corresponds to the frequency (18) must be much larger than the average distance between the grains $\lambda\sim c/\omega_0 >> N_g^{-1/3}$ (c is the velocity of light). Substituting in this inequality (18) and $J\sim m_g a^2$, we obtain the following inequality

$$\frac{d_m H_0}{m_g c^2} << (N_g a^3)^{2/3}. \qquad (19)$$

This inequality could be easily satisfied for the typical parameters of dusty plasma and moderate magnetic fields. The second equation describes the dispersion of waves in the dusty plasma with participation of magnetic dipoles. In the case of ferromagnetic grains with anomalous magnitude of magnetic moments their contribution to the dispersion properties of the dusty plasma may be important.

4. **Left handed medium**

Now we would like to note that components of the permeability tensor (12) in the frequency range $\omega > \sqrt{\omega_0^2 + \Omega_m^2}$ are negative. Obviously, that the electromagnetic wave in this range of frequencies may propagate in the medium only provided that the corresponding component of the tensor of dielectric permittivity are negative as well. Here we show that the magneto-active dusty plasma with magnetized grains could be related to these media. We consider the linearly polarized electromagnetic wave that propagates along the y-axis in our media transversally to the constant magnetic field. Let the electric component of this wave E_z be along the z-axis and the magnetic component be along the x-axis. The other components of the wave are equal to zero. The dispersion equation of this wave follows from the third equation of system (16) and takes the form

$$\mu\varepsilon_3 = \frac{k^2 c^2}{\omega^2} \ . \qquad (20)$$

Simultaneous realization of the inequalities ε_3<0 and μ<0 in the dusty plasma with magnetic grains according to (12) and (14) is possible in the following frequency range

$$\sqrt{\omega_0^2 + \Omega_m^2} < \omega < \Omega_e \ . \qquad (21)$$

The left-hand side of this inequality depends on the magnetic parameters of grain and external constant magnetic field and its right hand side depends only on the electron density number. This allows us to claim that the above-described system may be considered as a good candidate of the "left-handed media" with controlled parameters. Here we present some numeric evaluations of parameters of the magnetized dusty plasma when one could expect the realization of ε <0 and μ <0 simultaneously.The ferromagnetic grains may be obtained by grinding of compounds of iron, nickel, and cobalt. To prevent the grain sticking before magnetization a thin film must cover them (for example, polystyrene). After the magnetization up to saturation, the grains are charged in the gas discharge plasma. For the grains of 10^{-6} - 10^{-5} cm, the domain approximation is true. The magnetic moment of a grain is of the order $d_m \approx 10^{-16}$ erg/Gs. The external magnetic of the order of $H_0 \sim 10^4$Gs "freezes" the orientation of magnetic dipoles and they perform only small vibrations around the magnetic field. In particular, at room temperatures, $\frac{d_m H_0}{T} \approx 25 >> 1$. According to () the frequency of these vibrations $\omega_0 = \sqrt{d_m H_0 / J}$ for the spherical grains ($J = \frac{2}{5} m a^2$, $m = \frac{4\pi}{3} \rho a^3$, ρ is the density of a grain and a its radius). Setting $a \approx 10^{-5}$ - 10^{-6} cm and $\rho \approx 8$ g/cm^3, we get $\omega_0 \approx 10^6 - 10^8$ rad /s . Variations of the external magnetic field and the electron plasma density are independent ones. If we chose $\omega \approx \omega_0 + \Omega_m^2 / 6\omega_0$, the magnetic permeability tensor component μ according to (12) is equal to –2. At the same time, the substitution of the chosen ω in the expression of ε_3 according to (14) with account the electron component only gives $\varepsilon_3 \approx 10^2$ at the electron density number $n_e \sim 10^{10}$cm^{-3}. This means that the electromagnetic wave length in the above considered dusty plasma is $\sqrt{\varepsilon_3 \mu}$ ~10 times smaller that in the vacuum. This evaluation simplifies the possible realization of the electrodynamics properties of the left handed materials based on the magneto-active dusty plasma with ferromagnetic grains.

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