On the Excitation of Zonal Flows by Wave Particle Resonances

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Abstract

Excitation of zonal flows by ion temperature gradient driven modes have been studi using both fluid and kinetic models. Previous fluid derivations have shown that a strong excitation of zonal flows occurs through the nonlinearity in the energy equation and is enhanced by the fluid magnetic drift resonance. Thus a new derivation has been made by nonlinear kinetic theory, confirming that a strong excitation occurs through the kinetic magnetic drift resonance. A new fluid derivation is also presented. Fluid and kinetic results are compared.

Introduction

The excitation of zonal flows, i.e. poloidal flows in tokamaks, has recently attracted strong interest[1-5]. This is mainly because it is a phenomenon of considerable theoretical interest which has also turned out to be relevant for turbulent transport in tokamaks. The discovery of the direct tokamak relevance was made in the Cyclone project[1] where fluid transport models were tested against nonlinear gyrokinetic codes. In plasma turbulence contexts, zonal flows are generally supposed to be generated nonlinearly by the turbulence itself. Zonal flows can be regarded as convective cells which are very elongated in the poloidal direction. Thus mathematical tools developed for the study of convective cells can be used. For driftwave turbulence, zonal flows play two different roles. First they can provide the damping of long wavelength eddies in quasi 2d systems with dual cascade. Second they can give a nonlinear upshift (Dimits shift) in the critical temperature gradient needed for a steady turbulent transport. A majority of studies have been focused on effects of the Reynolds stress[2]. However, recent work[3-5] has shown that the nonlinear upshift actually is caused by the convective nonlinearity in the energy equation. This is also in agreement with the nonlinear gyrokinetic simulations in the Cyclone work. The reason for this conclusion is that the nonlinear upshift regime turned out to be particularly sensitive to convergence with regard to number of particles. This convergence has later been shown to be critical in regions of wave particle resonances in velocity space[6]. Because the fluid closure enters in the energy equation this means that fluid results will be sensitive to the fluid closure. In the present paper we present an analytical kinetic derivation which confirms that the excitation of zonal flows is enhanced in the wave particle resonance region. The fact that the zonal flow gives an upshift in the critical gradient is also related to the fact that the linear threshold occurs exactly at the fluid resonance in the fluid description. The zonal flows will be mainly driven by toroidal drift waves which are mainly excited on the outside of a tokamak. Thus it is natural that the flow gets a slow poloidal variation due to this, Such modes are usually called Geodesic Acoustic Modes (GAM's).

Formulation

We start by considering fluid theory. Using the closure with the diamagnetic heat flow we have:

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + v\cdot\nabla\right) T + P\ \nabla\cdot v = -\nabla\cdot q \qquad (1a)$$

$$\mathbf{q} = \mathbf{q}_{*} = \frac{5}{2}\frac{P}{m\Omega}(\mathbf{e}_{çç}\ x\ \tilde{N}T) \qquad (1b)$$

This leads to the temperature perturbation:

$$\frac{dT}{T} = \frac{w}{w - \frac{5}{3}w_D}\left[\frac{2}{3}\frac{dn}{n} + \frac{w_{*e}}{w}(h - \frac{2}{3})\frac{ef}{T_e}\right] \qquad (2a)$$

With Boltzmann electrons we can rewrite this as

$$\frac{dT_i}{T_i} = \frac{w}{w - \frac{5}{3}w_{Di}}\left[\frac{2}{3} + d(w)\right]\frac{ef}{Te} \qquad (2b)$$

$$d(w) = \frac{w_{*e}(h_i - h_{ith})}{w - \frac{5}{3}w_{Di}}$$

$$h_{ith} = \frac{2}{3} + \frac{10}{9t}e_n \qquad (3a)$$

with real frequency at threshold given by:

$$w_r = \frac{5}{3}w_{Di} \qquad (3b)$$

This means that *we have a resonance in the temperature perturbation at marginal linear stability*. This leads to a large nonlinear convective term in the energy equation.
The linear dispersion relation can be written:

$$\Omega(\Omega + \mathrm{a} + \mathrm{i}a(1 + \frac{5}{3}e_{\mathrm{n}})) + (\mathrm{b} + \mathrm{i}a)(h_{\mathrm{i}} - h_{\mathrm{ith}}) = 0 \qquad (4a)$$

$$\mathrm{a} = \frac{1}{1 + \mathrm{k}^2 r^2}\left[e_{\mathrm{n}} - 1 - \mathrm{k}^2 r^2(\frac{5}{3t}e_{\mathrm{n}} - \frac{1 + h_{\mathrm{i}}}{t})\right];\quad \mathrm{b} = \frac{1}{1 + \mathrm{k}^2 r^2}\frac{e_{\mathrm{n}}}{t} \qquad (4b)$$

$$a = \frac{e_{\mathrm{n}}}{4}\frac{|\mathrm{s}|}{\mathrm{q}} \qquad (4c)$$

The convective nonlinearity in the energy equation is:

$$\mathbf{v_E}\cdot\nabla d\mathrm{T_i} = \frac{1}{\mathrm{B}}(\mathbf{e}_{\|} \times \nabla f)\cdot\nabla d\mathrm{T_i} \qquad (5)$$

Substituting (2b) into (5) and considering the generation of a slow mode with small poloidal Modenumber, the two contributions almost cancel and we have to expand $d(w)$. This leads to A coupling factor of the form:

$$D = \frac{h_i - h_{ith}}{(\bar{w} + \frac{5}{3i} e_n)^2} \frac{\partial \hat{w}}{\partial k_q} \;; \quad v = \frac{w}{w_{*e}} \tag{6}$$

$$\frac{\partial v}{\partial k} = -\frac{\Omega \frac{\partial a}{\partial k} + (h_i - h_{ith}) \frac{\partial b}{\partial k}}{2\Omega + a} \;; \quad \Omega = \hat{w} + \frac{5}{3t} e_n \tag{7}$$

$$2\Omega + a = -ia(1+\frac{5}{3t}e_n) + \sqrt{-4\frac{e_n}{t}(h_i - h_{ithl}) - a^2(1+\frac{5}{3t})^2 + ia(a+h_{ith}-h_i)} \tag{8}$$

$$h_{ithl} = h_{ith} + \frac{t}{4e_n}(1-e_n)\left[1-e_n-k^2r^2(1-e_n+\frac{2}{t}(1+h_i-\frac{5}{3}e_n))\right] \tag{9}$$

Here h_{ithl} is the linear threshold in the pure toroidal ITG (local limit).

Due to cancellations $2\Omega + a$ in the denominator of (7) is the only resonance that finally remains. For the Cyclone base case we have a = 0.125 and the local and nonlocal thresholds are close so there is a resonance. We can also see from (11) that there is a strong sensitivity to the FLR parameter when e_n is close to 1 (0.9 in Cyclone case). It actually gives an enhanced resonance for larger FLR for the Cyclone basecase. The sensitivity to magnetic shear was already pointed out.[3,4]

Resonant ordering

We now follow Ref 5 and apply the resonant ordering from: K. Nozaki, T. Taniuti and K. Watanabe, J. Phys. Soc. Japan **46**, 983 (1979). We expand the fields as:

$$f = \sum_n \sum_l e^{1+(2/3)n} f_l^{1+(2/3)n}(x,x,t) e^{il(k_\| z + k_y y - wt)} + c.c + \sum_n e^{4/3+(2/3)n} f_0^{4/3+(2/3)n}(x,x,t) \tag{10}$$

Here x=x, $x = e^{2/3}(y - lt)$, $t = e^{4/3}t$, l is a velocity of the envelope and e is the small parameter of order ef/T which is considered to be of order 10^{-2} in the core. The parameter l is the harmonic number. It is 1 for the linear drift waves and 0 for the flow. We assume a standing wave in the radial (x) direction. This expansion leads to:

Order $e^{5/3}$

$$lD_l f_l^{5/3} + i\Omega_l \frac{\partial}{\partial x} f_l^1 = 0$$

Here the dispersion relation is $D_l = 0$ so only the second term remains. It can be fulfilled only if $l = \partial w / \partial k_q$ thus l is the group velocity.

Order e^2

$$U\frac{\partial}{\partial V}f_0^{\,4/3}=0$$

Here U is a quantity which vanishes for a particular group velocity $l=l_0$. This velocity is a reference velocity for zonal flows.

To orders $e^{7/3}$ and $e^{8/3}$ we now obtain the coupled equations for drift waves and flow as:

$$e^{7/3} \qquad \frac{\partial f_1}{\partial t}+iC\frac{\partial^2 f_1}{\partial x^2}=C_{nl}f_0f_1 \qquad (11a)$$

$$e^{8/3} \qquad \frac{\partial f_0}{\partial t}+D\frac{\partial f_0}{\partial x}=D_{nl}\frac{\partial}{\partial x}\left|f_1\right|^2 \qquad (11b)$$

These equations are similar to the Zakharov equations for Langmuir turbulence. However, here only first order derivatives occur for the flow. A nonlinear Schrödinger equation is obtained ff we neglect the time derivative of the flow, integrate the flow equation in space and substitute its amplitude into the equation for the driftwave amplitude. We have studied the coupling factor of the flow, D_{nl} for the Cyclone base case parameters as shown in Fig's (1a) and (1b).

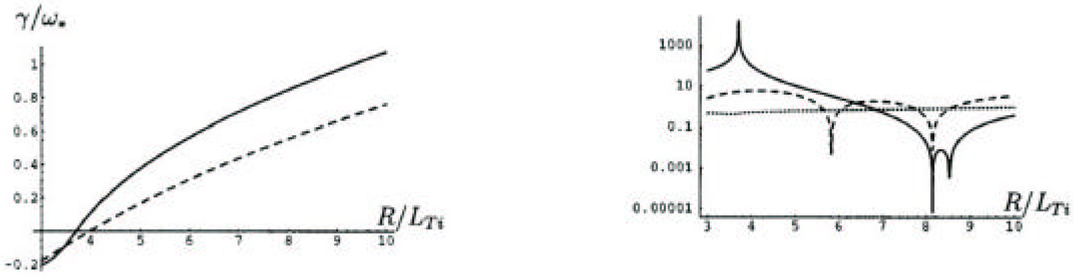

Fig 1a The normalized linear growth-rate as a function of temperature gradient for Cyclone base case parameters. The full line is for the Weiland model and the dotted is with a simple Gyrofluid resonance included in Eq (8).

Fig 1b The absolute value of the coupling factor for zonal flow as a function of temperature gradient. The solid line represents the Weiland model, the dashed line the model including the Gyro-fluid resonance and the dotted line the coefficient of the Reynolds stress

In Fig 1a we note that the model with the Gyro-fluid resonance has the same linear threshold as the kinetic models and the IFS-PPPL model for the Cyclone base case[172] and that the Gyro-fluid resonance reduces the growth rate as expected.

From Fig 1b we conclude that the energy equation nonlinearity dominates in the nonlinear upshift region. A nonlinear upshift extending up to about $R/L_t\approx 6$ as for the Dimit's shift also seems quite reasonable for the reactive model. We note that the resonance has disappeared completely at $R/L_t\approx 6.8$ since there the excitation from the energy equation equals that from the Reynolds stress. At the Dimits upshift limit $R/L_t\approx 6$ the reactive model shows a strength of the resonance comparable to that of the Gyro-fluid model at $R/L_t\approx 5$, the upshift obtained by the IFS-PPPL model in the Cyclone simulations. Note that these are only rough estimates to show that the results are reasonable. The result for the reactive model is also in good agreement with nonlinear simulations[204]. For comparison we note that the IFS-PPPL model, which had zonal flows included in the Cyclone comparison, recovered only half of the Dimits upshift.

Gyrokinetic calculation

We now consider the full kinetic resonance using a nonlinear gyrokinetic equation[7,8] We write the velocity distribution function as:

$$f(\vec{r},\vec{v},t) = f_0(\vec{r},\vec{v}) + f^{(1)}(\vec{r},\vec{v},t) + f^{(2)}(\vec{r},\vec{v},t) \qquad (12)$$

where f_0 is the unperturbed distribution function which depends slowly on space, $f^{(1)}$ is the linear perturbation and $f^{(2)}$ is the nonlinear perturbation. These parts fulfil:

$$f_0 >> f^{(1)} >> f^{(2)}$$

We now express the perturbations in $(w,\vec{k})$ space. The linear gyrokinetic equation obtained from Ref 8 is then:

$$(w - w_D(v_{\|}{}^2, v_{\perp}{}^2) - k_{\|} v_{\|})(f^{(1)}{}_{k,w} + \frac{qf_{k,w}}{T} f_0) = (w - w_{\bullet})\frac{q}{T} f_{k,w} e^{iL_k} J_0(x_k) f_0 \qquad (13)$$

where $L_k = (\vec{v}x\hat{e}_{\|}) \cdot \vec{k}/\Omega_c$ and $x_k = \dfrac{k_{\perp}v_{\perp}}{\Omega_c}$.

The nonlinear gyrokinetic equation then gives $f^{(2)}{}_{k,w}$ as:

$$f^{(2)}{}_{k,w} = \frac{q^2}{T} f_0 \frac{i}{mv\Omega_c} \sum (\vec{k}' \times \vec{k}'') \cdot \hat{e}_{\|} J_0(x')e^{iL_k} f_{k',w'} f_{k'',w''} \frac{w_{\bullet}'' - w''}{v''} J_0(x'') \qquad (14)$$

where $v = w - w_D(v_{\|}{}^2, v_{\perp}{}^2) - k_{\|} v_{\|}$ and $\vec{k} = \vec{k}' + \vec{k}''$

Now, integrating over velocity we obtain for three modes:

$$dn^{(2)}{}_{k,w} = i(\frac{e}{T_i})^2 \frac{T_i}{m_i\Omega_{ci}} \int \frac{1}{v}(\vec{k}'\times\vec{k}'') \cdot \hat{e}_{\|} J_0{}^2(x') \frac{w_{\bullet i}'' - w''}{v''} J_0{}^2(x'') f_0 d^3V f_{k',w'} f_{k'',w''} \qquad (15)$$

$$+ k' \Leftrightarrow k''$$

Now using quasineutrality and Boltzmann electrons we have $dn_{iw,k} = \dfrac{ef}{T_e} n_0$

The dispersion relation is then:

$$1 + \frac{1}{t} = \frac{1}{n_0} \int \frac{w - w_{*i}}{v} J^2{}_0(x_k) f_0 d^3V \qquad (16)$$

We can now separate (5) into an FLR and a non-FLR part by subtracting and adding 1 to $J_0(x)^2$.

$$dn^{(2)}{}_{k,w} = i(\frac{e}{T_i})^2 \frac{T_i}{m_i\Omega_{ci}} \int \frac{1}{v}(\vec{k}\times\vec{k}'') \cdot \hat{e}_{\|} (J_0{}^2(x') - 1) \frac{w_{\bullet i}'' - w''}{v''} J_0{}^2(x'') f_0 d^3V f_{k',w'} f_{k'',w''} \qquad (17)$$

$$i(\frac{e}{T_i})^2 \frac{T_i}{m_i \Omega_{ci}} \int \frac{1}{v} (\vec{k}' \times \vec{k}'') \cdot \hat{e}_{\|} \; \frac{w_{\bullet i}'' - w''}{v''} J_0{}^2(x'') \, f_0 d^3V f_{k',w'} f_{k'',w''} \quad +(k'\text{-> } k'')$$

The first part of Eq (17) is in the form used for deriving the Hasegawa-Mima equation where the nonlinearity is due to FLR effects. However, here we have kept the resonances in the denominator. The second part would vanish due to Eq (7) upon summation over (k'-> k") in the absence of the resonance in $1/v$ as in the derivation of the Hasegawa-Mima equation.
We will now consider the driven mode to have low frequency and to be slowly varying in q. The matching of wave vectors is written:

$$\vec{k} = \vec{k}' + \vec{k}''$$

When $\vec{k}$ is small, $\vec{k}'$ and $\vec{k}''$ are almost equal in magnitude with opposite directions. We can then regard the mode $\vec{k}$ to be driven by self interaction of a mode $\vec{k}_1$ with a finite width. We can then write: $\vec{k}' = \vec{k}_1 + d\vec{k}$, $\vec{k}'' = \vec{k}_1 - d\vec{k}$. Then $\vec{k}' \times \vec{k}'' \approx 2\vec{k} \times d\vec{k}$ and the contributions from the two contributions obtained by interchanging $\vec{k}'$ and $\vec{k}''$ will almost cancel. We then expand the integrand as:

$$\frac{w_*'' - w''}{v''} \approx \frac{w_{1*} - w_1}{v_1} + \frac{\partial}{\partial k_q} (\frac{w_* - w}{v}) dk_q \tag{18}$$

We note that the differentiation will make the denominator squared, i. e.. it will lead to an enhancement of the resonance. After also expanding the Besselfunctions we then obtain:

$$dn^{(2)}{}_{k,w} = -i r_s{}^2 \int \frac{1}{v} (\vec{k}_1 \times d\vec{k}) \cdot \hat{e}_{\|} \; 2\vec{k}_1 \cdot d\vec{k} \, \frac{w_{\bullet i}'' - w''}{v''} \, f_0 d^3V \left| \hat{f}_{k,w} \right|^2 \; + \\ + i r_s{}^2 \int (\vec{k}_1 \times d\vec{k}) \cdot \hat{e}_{\|} \; \frac{2}{v} \left[\frac{\partial}{\partial k_q} (\frac{w_\bullet - w}{v}) \right]_{k_1} dk_q \left| \hat{f}_{k,w} \right|^2 d^3V \tag{19}$$

The first part of (10) is clearly of FLR order. It represents the effect of Reynolds stress. However, the second part will also be of FLR order because the differentiation of the factor $\frac{w - w_*}{v}$ depends on dispersion. Since dispersion is only caused by FLR terms this derivative becomes of FLR order. We may compare these terms by differentiating (7) with respect to k_q.
We can not use (13) directly in (12) because of the factors $1/v$. However, we can conclude that the two parts of (12) are comparable. This is because the nonlinear FLR term is here included in the Reynolds stress. However, The present results have been obtained by substituting the linear perturbation of the distribution function into the nonlinear terms. If we, however, use a resonant ordering, i.e. assume that $v \ll w$ the resonance becomes stronger and the energy equation nonlinearity becomes dominant.
We can go to higher nonlinear order by including a nonlinear frequency shift:

$$dn^{(2)}{}_{k,w} = i r_s{}^2 \int \frac{1}{v} (\vec{k}_1 \times d\vec{k}) \cdot \hat{e}_{\|} \; 2\vec{k}_1 \cdot d\vec{k} \left[\frac{w_{\bullet i}'' - w''}{v''} + \frac{\partial}{\partial w} \left(\frac{w_{\bullet i}'' - w''}{v''} \right) dw_{NL} \right] f_0 d^3V \left| \hat{f}_{k,w} \right|^2 \; + \\ + i r_s{}^2 \int (\vec{k}_1 \times d\vec{k}) \cdot \hat{e}_{\|} \; \frac{1}{v} \left[\frac{\partial}{\partial k} \frac{w_\bullet - w}{v} dk_q + \Delta\{ \frac{\partial}{\partial w} \left(\frac{w_{\bullet i} - w}{v} \right) dw_{NL} \} \right]_{k_1} \left| \hat{f}_{k,w} \right|^2 d^3V \tag{20}$$

after expanding the nonlinear terms in w and introducing the nonlinear frequency shift dw_{NL}. The symbol $\Delta\{\}$ indicates difference between the values at k' and k''. Here $\hat{f} = ef/T_e$. The usual Reynolds stress originates from the ion polarization drift. It is now clear that the resonance is important since it enters in both parts. Since, at the resonance, the linear temperature perturbation becomes large, we have to be careful when applying an expansion in the nonlinearity. This calls for employing a systematic scheme like the reductive perturbation method. However, we have here expanded the nonlinear terms in w, adding the effect of the nonlinear frequency shift dw. Now, entering the linear dispersion relation in the nonlinear terms will not necessarily make them comparable. Expanding in w actually has the same effect as taking a resonant ordering since a small denominator means large nonlinear terms and we have to go to higher order in the nonlinearity. Thus the resonant ordering above should be applied also to the kineic equations.

Summary

We have shown that excitation of zonal flows through the wave particle resonance can be important. Since this resonance coincides with marginal stability of ITG modes, this is the dominant mechanism for the Dimits upshift[1]. The resonance comes through the resonant denominator in a gyrokinetic description but is stronger if wave-particle resonances modify the distribution function in such a way that a reactive closure is valid. Promising results have been obtained with our reactive model both in turbulence simulations[4] and in the excitation strength of zonal flows[5] shown here.

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