Investigations of the Role of Nonlinear Couplings in Structure Formation and Transport Regulation: Experiment, Simulation, and Theory

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Abstract. Understanding the physics of shear flow and structure formation in plasmas is a central problem for the advancement of magnetic fusion because of the roles such flows are believed to play in regulating turbulence and transport levels. In this paper, we report on integrated experimental, computational, and theoretical studies of sheared zonal flows and radially extended convective cells, with the aim of assessing the results of theory-experiment and theory-simulation comparisons. In particular, simulations are used as test beds for verifying analytical predictions and demonstrating the suitability of techniques such as bispectral analysis for isolating nonlinear couplings in data. Based on intriguing initial results suggesting increased levels of nonlinear coupling occur during L-H transitions, we have undertaken a comprehensive study of bispectral quantities in fluid and gyrokinetic simulations, and compared these results with theoretical expectations. Topics of study include locality and directionality of energy transfer, amplitude scaling, and parameter dependences. Techniques for inferring nonlinear coupling coefficients from data are discussed, and initial results from experimental data are presented. Future experimental studies are motivated. We also present work investigating the role of structures in transport. Analysis of simulation data indicates that the turbulent heat flux can be represented as an ensemble of "heat pulses" of varying sizes, with a power law distribution. The slope of the power law is shown to determine global transport scaling (i.e. Bohm or gyro-Bohm). Theoretical work studying the dynamics of the largest cells (termed "streamers") is presented, as well as results from ongoing analysis studying connections between heat pulse distribution and bispectral quantities.

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[*] This research was supported by Department of Energy Grant. No. FG-03-88ER-53275.

[†] C. Holland performed this research under an appointment to the Fusion Energy Sciences Fellowship Program, administered by Oak Ridge Institute for Science and Education under a contract between the U.S. Department of Energy and the Oak Ridge Associated Universities.

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1. Introduction

A central challenge of plasma turbulence research in the last decade has been to develop a cohesive picture for understanding how zonal flows [1,2,3] are nonlinearly generated and regulate turbulence. It is also known the turbulent flux is highly intermittent or "bursty". The largest bursts have been suggested to be due to with large, radially extended convective cells (often termed "streamers" [2,4]) which are generated by nonlinear couplings and secondary instability processes as well. A detailed understanding of such couplings and structures is clearly needed. The aim of the report is to build upon previous investigations via the use of more detailed and advanced methods of quantifying the results of simulations and physical experiments. In particular, three issues are focused upon:

1) The impact of shear flows on nonlinear energy transfer in different models of drift-wave turbulence, such as changes in the directionality, locality, and isotropy.
2) The utility of the bicoherence as an experimentally realizable tool for studying the nonlinear generation and dynamics of shear flows and their impact on turbulence.
3) Probability distribution function (PDF) based representations of turbulent fluxes.

For each issue, we seek to compare results from simulation and experiment with expectations from analytic theory. By doing this, we hope to create points of contact between all three approaches to investigations of turbulence upon which future studies can build.

2. Shear Flows and Energy Transfer

The first step of this study was to analyze the effects of shear flows on energy transfer in simulations of two variants of a basic fluid model for electrostatic curvature-driven drift-wave turbulence. The model equations used are simplified versions of the equations in Horton et. al. [5], with the change that $\delta n_k / n_0 = \delta(k_y) |e| \phi_k / T_e$. The quantity $\delta(k_y)$ is a simple attempt to include the response of density perturbations to the flux-surface averaged, which is known to have a strong impact on zonal flow formation. For all finite values of k_y, $\delta(k_y) = 1$. If $\delta(0) = 0$, the equations describe ITG-like turbulence (which exhibits strong zonal flows), while if $\delta(0) = 1$, they describe ETG-like turbulence (which exhibits weak or absent zonal flows). A mockup of Rosenbluth-Hinton [6] collisional damping is also used. For these models, one can define two energy transfer functions, $T_k^{\phi}(k') = \left(\vec{k} \times \vec{k'}\right) \cdot \hat{z} \left|k'\right|^2 \mathrm{Re}\left\langle \phi_k^* \phi_{k-k'} \phi_{k'} \right\rangle$ and $T_k^{p}(k') = \left(\vec{k} \times \vec{k'}\right) \cdot \hat{z}\, \mathrm{Re}\left\langle p_k^* \phi_{k-k'} p_{k'} \right\rangle$.

It is useful to interpret the structure of the ETG transfer functions as "base" cases, or controls, in that they reflect the structure of the couplings in the absence of significant shear flows. For the ETG model, one finds that both $T_k^{\phi}(k')$ and $T_k^{p}(k')$ are best described as having interactions regions correlated with the largest amplitude modes (as indicated by the energy spectrum), but no other clear structure or net directionality. The primary change in the ITG model is the appearance of significant transfer or "scattering" of energy between modes with wavevectors $\vec{k} = \left(0, k_y^{DW}\right)$ and $\vec{k'} = \left(\pm \delta k_x, k_y^{DW}\right)$ which is the expected signature of strong shear flows (via momentum conservation / triad wavevector sum rules), although there are also lower magnitude isotropic couplings similar to the ETG case. The k_x' scattering does not exhibit a clear directionality (the $k_x = 0$ drift-waves are both *giving and receiving energy to / from the finite k_x modes*). In addition, modes near the peak of the energy spectrum also *show significant energy transfer into and out of the zonal flow ($k_y = 0$) modes*. The energy transfer functions for the zonal potential and pressure have also been investigated. Examination of

$T_k^\phi(k')$ and $T_k^p(k')$ indicates that the majority of energy transfer is characterized by an interaction region that correlates with the core of the energy spectrum. Particularly interesting is that $T_k^\phi(k')$ indicates that *there is significant transfer both into and out of the zonal flows.* Having examined the effects of shear flow on energy transfer in the simple fluid models, this analysis is extended to include the results of a gyrokinetic simulation by Candy et. al. [7] of ITG turbulence. For the bulk of the drift-waves, there is a clear scattering of energy in k_x' at fixed k_y, and an *equally clear transfer of energy from the zonal flows into the drift waves, comparable in magnitude to the scattering in* k_x'. For all of the zonal flows, there is a clear transfer of energy from drift-waves with $k_x > k_x^{ZF}$ to drift-waves with $k_x < k_x^{ZF}$. The key change from the fluid ITG case is the much clearer directionality of the interactions between the drift-waves and zonal flows; this difference may be a result of much stronger zonal flows in the gyrokinetic case relative to the fluid models.

Both the scattering of energy in k_x at fixed k_y, and transfer of kinetic energy from drift-waves to zonal flows are expected by theory, but the transfer of energy from the zonal flows to the drift-waves is not necessarily expected. One particularly interesting question is whether the energy transfer from zonal flows to drift-waves is a signature of some instability of the zonal flows. If one were to suppose this hypothesis to be the case, these results would suggest that energy is exchanged between drift-waves and zonal flows in a "loop": finite k_x, k_y drift-waves transfer energy to zonal flows with lower k_x via modulational instability, the zonal flows give energy to even lower k_x drift-waves via an undetermined mechanism, and energy is scattered from low k_x drift-waves to larger k_x at the same k_y by the zonal flows. It should be noted that the zonal flow instabilities might not manifest themselves as strong Kelvin-Helmholtz type (K-H) instabilities which destroy the zonal flow, but rather as non-destructive "flow defects" [8], such as *spatially local* "kinks" or "bumps" in the flow profile generated by the nonlinear noise in the zonal flow evolution equation. Rapid mixing and shearing of such defects could manifest itself as energy transfer from zonal flows back to the drift-waves. It should also be noted that the K-H or flow defect feedback on drift-waves could induce spatial spreading of drift-wave intensity, causing "fast transport" or "non-locality" phenomena.

3. Bicoherence Studies and Connecting To Experiment

In this section, we present the results of a simple "thought experiment" which attempts to connect computational/analytic and experimental investigations of nonlinear dynamics via studying the bicoherence [9] in a heuristic computational model of published experimental results by Moyer et. al. [10]. The experimental results to be modeled report on the temporal dynamics of the bicoherence near the separatrix during a spontaneous low-high (L-H) confinement transition in the DIII-D machine. A numerical experiment was undertaken, in which zonal flows are suppressed, and the turbulence is allowed to evolve into a saturated state; zonal flow generation was then "turned on", to mimic the transition from a highly turbulent regime to a shear-flow dominated one. The computational data is divided into uniform time segments, and for each segment, the k_x-integrated values of the electrostatic potential, $\bar{\phi}(k_y, t)$ are calculated, which are then used to calculate the (squared) bicoherence. The idea here being that one might hope that k_y could be a suitable proxy for frequency, based on ideas such as the "frozen-flow" hypothesis. The kinetic energy of the zonal flows and total bicoherence are plotted in Fig. 1, similar to Fig. 4 of Moyer et. al. Zonal flow evolution clearly impacts the dynamics of total bicoherence, but quantifying the exact connection is a harder question. Examination of wavenumber-resolved bicoherences indicate the presence of

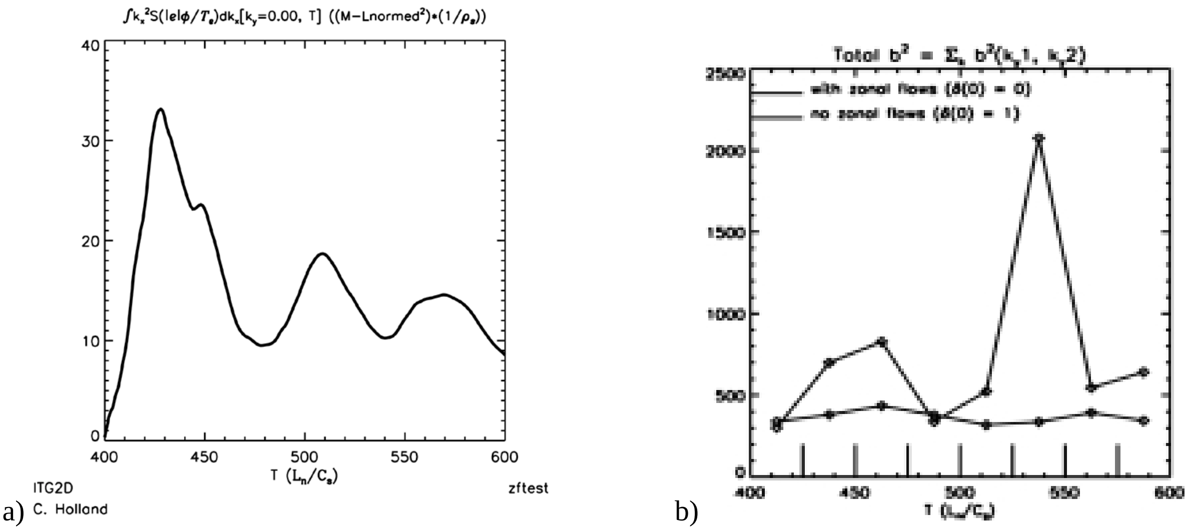

FIG. 1. (a) Zonal flow energy. (b) Total bicoherence for cases with and without strong shear flow.

banding structures similar to those seen in Moyer et al. Thus, this simple experiment seems able to qualitatively capture many of the results from the experimental work.

4. Studies of Heat Flux Probability Distribution Functions

There is now considerable evidence that experimental turbulent transport is quite intermittent or "bursty" (see, e.g., Ref. 2 or 4, and references therein). One method of characterizing the intermittency of a field is via the PDF of that field. Such an investigation has been carried out in the previously described (Sec. 2) fluid models of plasma turbulence. PDFs of the local and flux-surface averaged turbulent fluxes have been calculated, with a striking contrast found between the local and averaged fluxes for all models. The local flux PDFs are strongly non-Gaussian, with extended "wings" and high kurtosis (but little skew), while the average flux PDFs are strongly asymmetric (skewed). The key result is that the PDF profiles change drastically upon averaging over flux surfaces. Since flux-surface averaging is equivalent to removing information relating to poloidal localization, these results suggest that polodially localized flux structures could be important components of the turbulent heat flux.

An alternative method of characterizing intermittency due to coherent structures is to analyze the flux with methods which inherently account for such structures. We present here such an approach, termed "heat pulse analysis." In this analysis, the flux-surface averaged turbulent heat flux is decomposed into a distribution of "pulses" of different characteristic scales. The size of the i^{th} pulse is defined as $h_i \equiv \int drdt\, q_i(r,t)$. The PDF of pulse sizes *f(h)* then replaces the single-point PDF of the flux. Examination of *f(h)* in gyrokinetic simulations reveals that it is roughly flat for $h < h_c$, and falls of as a power law $f(h) \propto h^{-\alpha}$ for $h >> h_c$. If the falloff of *f(h)* at large h is slower than h^{-2} ($\alpha < 2$), then the heat flux is dominated by the largest heat pulses (i.e. is set by box size) and a Bohm-like scaling is obtained, while $\alpha > 2$ gives a gyroBohm-like scaling set by h_c. This analysis has been undertaken for results from a pair of gyrokinetic simulations of plasma turbulence, with varying strength shear flows. When the shear flow is weak or absent ("L-mode"-like), $\alpha = -1.524 \pm 0.1295$ (implying a Bohm-like scaling) while $\alpha = -2.279 \pm 0.1276$ (gyroBohm-like scaling) for the case of strong shear flow ("H-mode"-like). These results demonstrate a clear connection between the presence of large, radially extended heat flux events (manifested as $\alpha < 2$) and global scalings of transport.

5. Conclusions

In this article, results from integrated experimental, computational, and analytic investigations of the role of nonlinear couplings in structure formation and turbulent transport regulation have been reported. The principal results are:

1. Zonal flows have been explicitly shown to scatter turbulent energy in k_x in both fluid and gyrokinetic simulations of drift-wave turbulence, in accordance with theory.
2. Significant energy transfer from drift-waves to zonal flows *and zonal flows to drift-waves* has been observed in both fluid and gyrokinetic simulations, possibly related instabilities of the zonal flows. It is hypothesized that these instabilities could take the form of "flow defects" in addition to K-H type instabilities.
3. A simple numerical experiment which attempts to heuristically model an experimental study of an L – H transition in DIII-D is shown to qualitatively reproduce much of the experimentally measured bicoherence dynamics and structure.
4. Significant differences in the PDFs of local and flux-surface averaged heat fluxes for fluid models of turbulent transport have been found, highlighting the possible importance of polodially localized structures in the flux.
5. A new method for investigating spatio-temporally resolved heat fluxes which incorporates the presence of coherent flux structures demonstrates a clear connection between the presence of radially extended heat flux structures predicted by theory and simulation with experimentally relevant transport scalings.

These findings motivate several future directions of inquiry. Examination of energy transfer in other fluid and gyrokinetic simulations for evidence of the hypothesized energy transfer "loop" should be undertaken (particularly as a function of deviation from marginality). Also, the heat pulse analysis technique should be refined and extended.

6. Acknowledgements

We thank C. Hidalgo, R. Moyer, M. Burin, Z. Lin, T. S. Hahm, and R. Waltz for collaboration and useful discussions. We gratefully thank Z. Lin and A. Dimits for generous use of the gyrokinetic simulation results.

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