Long Time Scale Plasma Dynamics Driven by the Double Tearing Mode in Reversed Shear Plasmas

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Abstract. The new nonlinear destabilization process is found in the nonlinear phase of the double tearing mode(DTM) by using the reduced MHD equations in a helical symmetry. The nonlinear destabilization causes the abrupt growth of DTM and subsequent collapse after long time scale evolution in the Rutherford-type regime. The nonlinear growth of the DTM is suddenly triggered, when the triangular deformation of magnetic islands with sharp current point at the x-point around the outer rational surface exceeds a certain value. Such structure deformation is accelerated during the nonlinear growth phase. Decreasing the resistivity increases the sharpness of the triangularity and the spontaneous growth rate in the abrupt growth phase is almost independent on the resistivity. Current point formation is also confirmed in the multi-helicity simulation, where the magnetic fields become stochastic between two rational surfaces.

1 Introduction

The formation of the non-monotonic safety factor (q-) profile, or the reversed shear profile, is considered to be one of the attractive methods to attain high performance steady state operation of a tokamak. The MHD stability for this profile is one of important issues to be theoretically clarified for the development of a steady state tokamak. The plasma with this q profile can be linearly unstable against resistive modes even in a low-beta state. In high β region, the resistive interchange mode becomes unstable around the inner rational surface [1]. In low β region, however, there is a possibility that the double tearing mode (DTM) becomes unstable even when the Mercier criterion for the resistive interchange mode is broken. Figure 1 shows the eigen mode structure, $V_r(m/n = 3/1)$, obtained by the resistive MHD analysis for the reversed shear equilibrium based on the JT-60U reversed shear discharge [2, 3]. The eigen mode extends between two q=3 rational surfaces and has the odd parity around each rational one, which means this mode is the double tearing one. In some situations, DTM shows the large growth rate of the resistive internal mode and can drive the plasma to termination almost exponentially in time with the linear growth rate. The nonlinear behaviors of the double tearing mode have been intensively studied by some authors through MHD simulations [4, 5]. Recently, we found the new phenomena of the double tearing mode in the nonlinear phase; that is, when two resonance surfaces are apart from each other, the mode gently grows magnetic

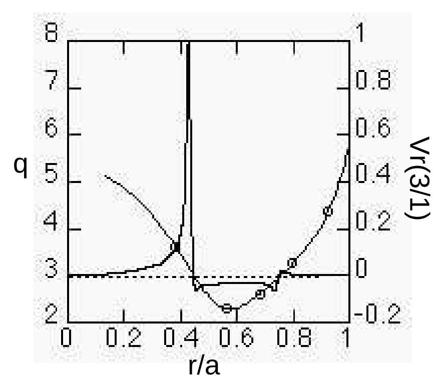

FIG. 1. q-profile (marked solid line) and the eigen mode structure of $V_r(3/1)$ (solid line) obtained by the resistive MHD analysis

islands at each resonance surface like in Rutherford regime of the conventional tearing mode [6, 7] but it suddenly shows the rapid growth after both magnetic islands grow enough to interact with each other [8]. The remarkable feature of this new phenomenon is the weak dependence of the mode growth rate on the resistivity η in the explosive growth phase [9]. It must be noted that this process is observed in a plasma with helical symmetry, where all harmonics have the resonance surfaces at the same radius, so that the newly observed phenomena seems to be very different from any theories proposed so far like the nonlinear coupling among different helicities and also the destabilization through the renormalized turbulence transport process, which have been observed in MHD simulations of the major disruption [10, 11]. This process is very important because, even if the plasma safely passes the regime unstable against the conventional double tearing mode with assistance of the magnetic well or the detail current profile control, the slowly growing tearing-like modes can be suddenly destabilized by the nonlinear process and lead to the reconstruction of the current profile to the monotonic q profile. The purpose of this paper is to show the details of this nonlinear destabilization of double tearing mode.

2 Model and Simulation Results

We employ the reduced set of resistive MHD equations in cylindrical plasma with helical symmetry and solve them by the finite difference in the radial direction and Fourier expansion in the angular directions [12].

$$\frac{\partial u}{\partial t} = \frac{1}{r}[u, \phi] + \frac{1}{r}[\psi, j] + \frac{B_0}{R_0}\frac{\partial j}{\partial \varphi} + \nu \nabla_\perp^2 u \tag{1}$$

$$\frac{\partial \psi}{\partial t} = \frac{1}{r}[\psi, \phi] + \frac{B_0}{R_0}\frac{\partial \phi}{\partial \varphi} + \eta j - E \tag{2}$$

$$j = \frac{\partial^2}{\partial r^2}\psi + \frac{1}{r}\frac{\partial}{\partial r}\psi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\psi \tag{3}$$

$$u = \frac{\partial^2}{\partial r^2}\phi + \frac{1}{r}\frac{\partial}{\partial r}\phi + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\phi \tag{4}$$

$$[a, b] = \frac{\partial a}{\partial r}\frac{\partial b}{\partial \theta} - \frac{\partial b}{\partial r}\frac{\partial a}{\partial \theta}. \tag{5}$$

The safety factor profile used in the following is

$$q(r) = q_c\{1 + (\frac{r}{r_0})^{2\lambda}\}^{\frac{1}{\lambda}}[1 + A\exp\{-(\frac{r - r_\delta}{\delta})^2\}], \tag{6}$$

$$\psi(r) = -\frac{B_0}{R_0}\int_0^r \frac{rdr}{q(r)}.$$

Here, ψ is the poloidal flux function, ϕ is the stream function, η is the resistivity, ν is the viscosity, j is the toroidal current density, u is the vorticity, E is the electric field at the wall, B_0 is the toroidal magnetic field, R_0 is the major radius and $\perp$ means the derivative perpendicular to the magnetic field. In these equations, uniform plasma density is assumed and the time is normalized to the poloidal Alfvén transit time $\tau_{pa} = \sqrt{\rho}a/B_\theta(a)$ (ρ is the plasma mass density, a is the plasma minor radius and $B_\theta(a)$ is the poloidal magnetic field at the plasma surface). The resistivity η is normalized such that $\eta = \tau_{pa}/\tau_\eta$, where τ_η is the plasma skin time. The rationalized MKS unit is used. The magnetic field and the velocity field are related to the poloidal flux ψ and the stream function ϕ by $\vec{B} = B_0\vec{e_\varphi} + \nabla\psi \times \vec{e_\varphi}$

and $\vec{V} = \nabla\phi \times \vec{e_\varphi}$, where $\vec{e_\varphi}$ is the unit vector in the toroidal direction. In the following 3 sections, we consider only the MHD activity with helical symmetry of $f(r,\theta,\varphi) = f(r,\zeta = \theta - (n/m)\varphi)$, where m and n are poloidal and toroidal mode numbers of the MHD mode, respectively, and the helical flux function is defined as $\psi^*(r,\zeta) \equiv \psi(r,\zeta) - (r^2/2)(n/m)$. We fix the parameters $\lambda = 1$, $r_0 = 0.412$, $\delta = 0.273$, $r_\delta = 0$ and $A = 3$ through this paper and change only q_c which changes the distance of two resonance surfaces, Δr. Also, we fix the poloidal/toroidal mode numbers (m/n) to 3/1, respectively. The maximum number of the Fourier components of the mode is taken to be 100 and the maximum number of equally spaced radial grid is 1600, in order to reproduce fine structures.

The linear stability analysis against the resistive mode in this q-profile shows that, as increasing Δr, the exponent factor α of resistivity η with respect to the growth rate $\gamma(\propto \eta^\alpha)$ changes from $\alpha = 1/3$ of the resistive internal mode in the limit of $\Delta r = 0$ to $\alpha = 3/5$ of the conventional tearing mode in the limit of $\Delta r = \infty$ [13]. The dependence of α on Δr is shown in Fig.(2) for the q profile of Eq.(6). Corresponding to this change of the linear stability, the nonlinear behavior of the mode also changes. That is, in the small Δr region, the mode evolves exponentially with the linear growth rate, and the fundamental mode is essential in this behavior, while, in the large Δr region, the mode enters the Rutherford regime and the magnetic islands saturate at each resonance surface. The nonlinear behavior has been considered smoothly to transit from the exponential growth to the island saturation so far. The new type of the nonlinear instability is found in this midway of these nonlinear behaviors of DTM, as shown in the shaded area ($0.22 < \Delta < 0.31$) in Fig.2.

The typical examples of the temporal evolution of magnetic and kinetic energies of this new phenomena for the cases (B), $\Delta r = 0.285$, and (C), $\Delta r = 0.310$, are shown in Figs.3. After the exponential growth in the linear regime, the mode reduces its spontaneous growth rate and tends to enter the Rutherford-type regime. In this phase, the kinetic energy almost saturates, while the magnetic energy continues to increase with reduced temporal rate and magnetic islands grow in the resistive time scale. Then, after magnetic islands growing around each resonance surface to contact with each other, the mode shows the abrupt growth. In this phase, the inner islands are expelled outside the outer ones and squeeze. The outer islands cover the almost whole region between two rational surfaces and the averaged q-profile becomes flat including the magnetic axis for this case. This may be the plasma collapse or disruption for low beta negative shear plasmas. For the case (C), the simulation results for $\eta = 5 \times 10^{-6}$ are also plotted in Fig.3(c) and (d). It is clearly shown that the kinetic energy quasi-saturation regime for $\eta = 5 \times 10^{-6}$ is about two time longer than that for $\eta = 1 \times 10^{-5}$. This is the same as the Rutherford regime in the conventional tearing mode.

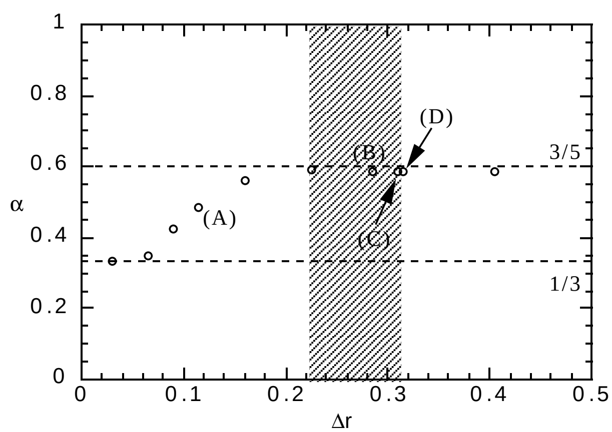

FIG. 2. The dependence of α, which is the exponent of the resistivity, $\gamma \propto \eta^\alpha$, on the distance between two rational surface, Δr.

3 Nonlinear Mode Coupling Effects

In order to study the origin of this abrupt growth of DTM during the nonlinear phase, we have performed several simulations. One of the possible candidates for this destabilization is the quasi-linear modification of the q profile and the acceleration of the linear instability. The simulation, where the perturbations set to zero on the way of the abrupt growth and the small one is set on the main($m/n = 3/1$) harmonics, shows that the mode returns back to the linear growth phase and again enters the Rutherford-type regime. This means that the modified q-profile does not destabilize the mode in the linear stability sense, including any other higher harmonics. In this way, the quasi-linear modification of the q-profile cannot reproduce the abrupt growth of the mode. This was also confirmed by the comparison of simulations with reducing the maximum number l_{max} of Fourier mode. In Figs.3(a) and (b), time traces for different numbers of l_{max} are also plotted. Figures 3(a) and (b) show that the temporal evolution of the mode before the abrupt growth is not sensitive so much on l_{max} ,while the behavior of the abrupt growth strongly depends on l_{max}; that is, reducing l_{max} from some critical number, l_c, the growth becomes more gentle. On the other hand, the simulations with l_{max} greater than the critical number l_c give almost the same result. The critical number l_c depends on the distance between resonance surfaces, Δr, and it is $l_{max} = 20$ for the typical example in Figs. 3(a) and (b). Beyond the upper boundary of the shaded region in Fig.2, the mode does not show any nonlinear destabilization, even if the number of Fourier harmonics is increased. At the boundary of $\Delta r = 0.315$, the outer separatrix of the inner islands and the inner separatrix of the outer ones reach the same radial position. The nonlinear destabilization of DTM, however, does not occur, which means that the interaction of the inner and outer islands is not the sufficient condition of this phenomenon. These simulations clearly show that the abrupt growth of DTM after the Rutherford-type phase is induced by the nonlinear coupling among the higher harmonics, although the harmonics higher than some critical number do not play an essential role in this process. In the case (C), the nonlinear destabilization is triggered for $l_{max} = 7$, but not for $l_{max} = 6$. This corresponds to the fact that the degree of the island deformation, or the formation of the sharp triangular edge, is important for triggering the nonlinear destabilization, as shown in detail later.

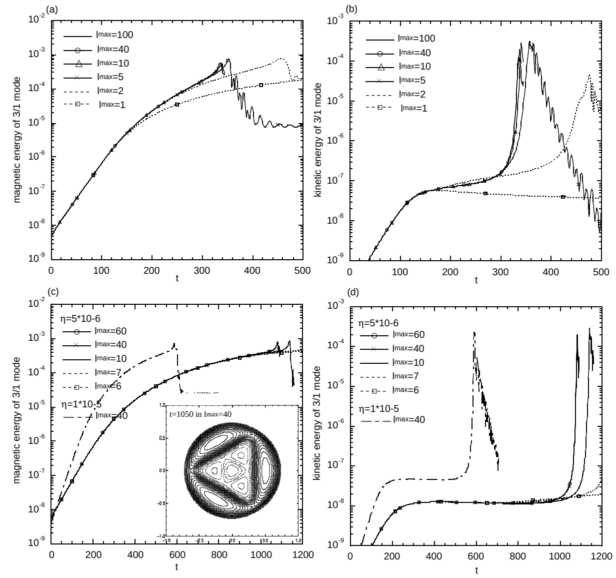

FIG. 3. Time evolutions of (a) magnetic and (b) kinetic energies of 3/1-mode with the different harmonics number, l_{max}, for the case (B). Time evolutions of (c) magnetic and (d) kinetic energies of 3/1-mode with the different harmonics number, l_{max}, for the case (C).

Next we move to the details of the nonlinear destabilization. It is interesting to know whether the magnetic perturbations $\psi_{l\geq1}$ or the kinetic ones $\phi_{l\geq1}$ are the key factor of the nonlinear destabilization. For this purpose, we reset magnetic or kinetic perturbations to zero on the way of the abrupt growth and investigated the subsequent phenomena. Results are shown in Figs.4. In simulations resetting the kinetic perturbations to zero, the kinetic perturbations recovered to the same level as the original ones in a very short time and shows the abrupt growth(Fig.4(a)), while in the case of resetting the magnetic perturbations to zero, the abrupt growth is not reproduced(Fig.4(b)). For the case retaining fundamental magnetic perturbation(i.e.$\psi_{l\geq 2} = 0$), the mode resumes the abrupt growth after the higher harmonics of magnetic perturbations grow up to sufficient amplitudes through the mode coupling. This comparison confirms that the nonlinear destabilization originates from the coupling among magnetic perturbations through $J \times B$, not from the driven reconnection type instability. By considering the quasi-linear and the nonlinear mode coupling effects, it was shown that the higher harmonics are important for the abrupt growth of DTM, and they are produced from the magnetic harmonics, ψ_l.

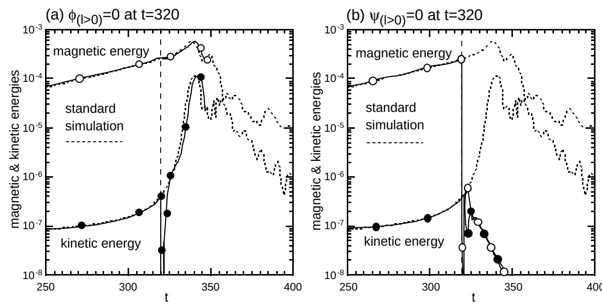

FIG. 4. Time evolutions of magnetic and kinetic energies of 3/1-mode for the standard and restarted simulations: (a) perturbations of $\phi(l > 0)$ are set to zero, (b) perturbations of $\psi(l > 0)$ are set to zero.

4 Current Point Formation

The growth of the mode pushes the inner magnetic islands toward the separatrix of the outer magnetic islands and generates the skin current along the separatrix surface. This skin current prevents the further growth of the mode and leads to the saturation of the mode. This is the nonlinear behavior of the standard DTM with small $\Delta r (< 0.22$ in Fig.2). Contrary to this, in the case of the nonlinearly destabilized DTM, the further growth of the magnetic island increases the triangular deformation of the island shape and forms the skin current highly concentrated to the X-points of the outer magnetic islands. This difference of the nonlinear behaviors between the standard DTM and the nonlinearly destabilized DTM is shown in Figures 5, where the contours of the helical flux surfaces, ψ^*, the flow potential, ϕ, and the toroidal current excluding the fundamental harmonics, $j_{l>0}$, in the nonlinear phase are plotted.
Figure 5(a) and (b) shows the case of the standard DTM. The mode grows exponentially with the linear growth rate and the convective force to push the magnetic flux is larger than the magnetic reconnection rate. Then, the separatrix of the inner island is uniformly pushed toward the outer islands. This changes the reconnection region from the X-point type to the Y-type layer with skin current flowing along the finite distance as shown in Fig.5(b) [14]. In contrary to this, in the case of the nonlinear destabilization of DTM, there are quadruple vortexes and the magnetic flux sustains the X-point structure as shown in Fig.5(c). In this case, the mode enters the Rutherford-type regime growing slowly proportionately to the resistivity, η. During this phase of weak convection, the mode coupling generates the higher harmonics and deforms the magnetic surface to the sharp triangularity. As the result, the plasma current concentrates in the small region and forms the current point, which is a current sheet with very short width, as shown in Fig.5(d). In this way, the increase of the triangular deformation of magnetic islands and the resultant localization of the skin current to the x-point is the key factor of this new

process. From this feature, this can be said the structure driven mode. A remarkable feature of this structure driven mode is the dependency of the spontaneous growth rate, γ_{temp}, on the resistivity, η; that is, the dependence of γ_{temp} on η in the explosive growth phase is very weak, $\gamma_{temp} \sim \eta^{\alpha}$, $\alpha \simeq 0$, as shown in Fig.6. This interesting feature of the nonlinear process was confirmed by simulations showing that the result does not change by the increasing the numbers of radial grid and Fourier harmonics. It is also noted that, by reducing the resistivity, the current peak becomes sharp and high in almost inversely proportional to the resistivity. That is, the local quantities do change sensitive to the resistivity, while the evolution rates of the energies, or the integration quantities, do not change.

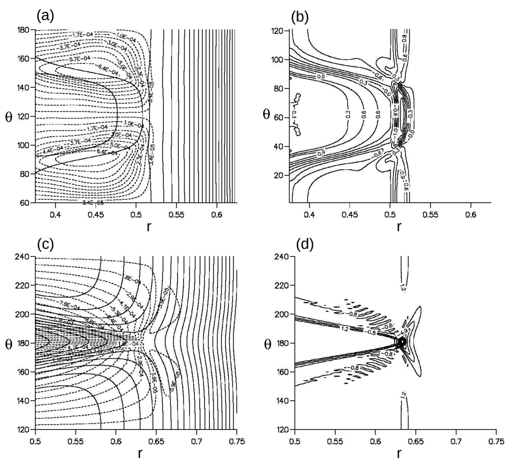

FIG. 5. Contours of the helical flux function, ψ^*, solid curves in (a) and (c), the flow potential, ϕ, dotted curves in (a) and (c), and the current, $j_{l>0}$, solid curves in (c) and (d): (a) contours of ψ^* and ϕ and, (b) contours of $j_{l>0}$ at t=130 for the standard DTM($\Delta r = 0.115$). (c) contours of ψ^* and ϕ and, (d) contours of $j_{l>0}$ at t=330 for the nonlinearly destabilized DTM($\Delta r = 0.285$)

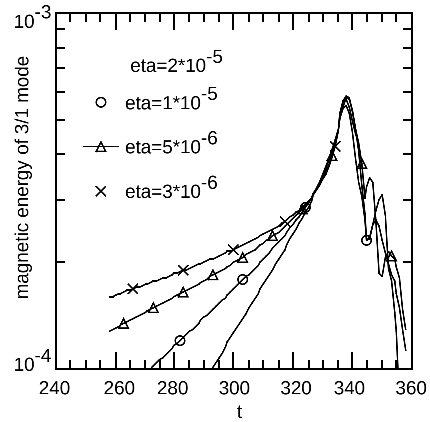

FIG. 6. Time evolutions of the magnetic energies of 3/1-harmonics at the nonlinear destabilization phase of DTM for the different resistivity.

5 Multi Helicity Simulation

In the above sections, the simulations have been carried out for the helical symmetry assumption with single helicity m/n=3, and have shown that the formation of the current point is essential for the destabilization. In the toroidal geometry, however, the different helicity harmonics are coupled with each other through the toroidal coupling. In some case, the mode with different helicity can be unstable simultaneously. These unstable or toroidally coupled modes form the magnetic islands and make the stochastic magnetic field through the island overlapping. This loss of the coherence may affect the formation of the current point and the nonlinear destabilization. In order to investigate this effect, we have done the nonlinear reduced MHD calculations in toroidal geometry, include the pressure effects. The pressure value is set as $\beta_p = 1.0 \times 10^{-6}$ on the magnetic axis, which is low enough so that the most unstable mode is not the pressure driven one but the double tearing one. Figure 7 shows the time evolution of the energies of the fundamental harmonics, m/n=3/1. As shown in Fig.7, the essential feature of the nonlinear destabilization of DTM does not changed in the toroidal multi-helicity case. Figures 8(a) and (b) show the poincare plots of the magnetic field B and the plasma current at $t = 320$. As expected, as the magnetic energy increases, the stochasitization of the magnetic field line rapidly expands between the inner and outer $q = 3$ rational surfaces and finally covers its whole region . The current point is, however, formed at the reconnection region for the $m/n = 3/1$ islands. This means that even under the stochastic magnetic fields, the current point is formed and causes the nonlinear destabilization of DTM in the toroidal geometry. The more detail analysis shows that the different helicity mode ($m/n = 8/3$) is also linearly unstable for this typical case and the coupling with this mode slightly enhances the destabilization of the fundamental mode and the timing of the abrupt increase of the energies becomes faster.

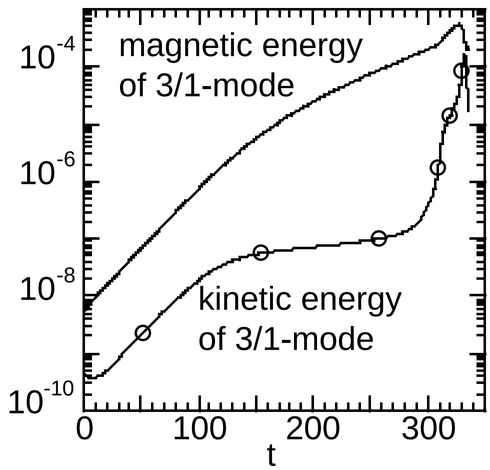

FIG. 7. Time evolutions of the magnetic and kinetic energies of 3/1-harmonics in the toroidal geometry.

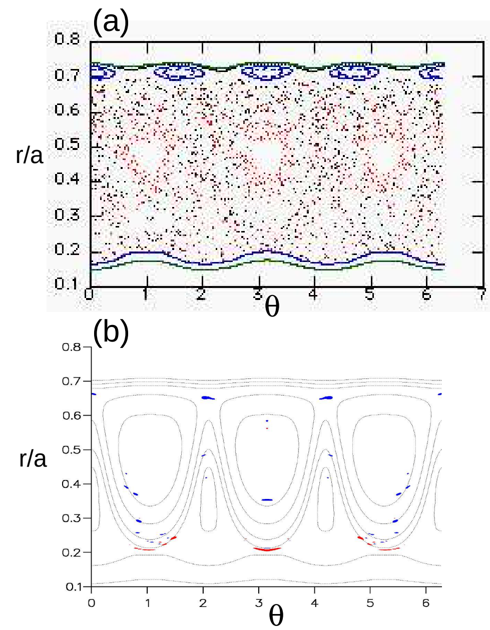

FIG. 8. (a) poincare plot of the magnetic fields and (b)the local maximum and minimum of plasma current and the poloidal flux of m/n=3/1-mode (dotted line) in toroidal simulation at t=320.

6 Summary and Discussions

In summary, we have shown the new process of the nonlinear destabilization of DTM which can be caused in the reversed shear profile in a tokamak. It was found that the slowly growing DTM can be nonlinearly destabilized and changes to the explosively growing DTM. Moreover, the spontaneous growth rate at this explosive phase is almost independent on the resistivity, due to the efficient reconnection of the magnetic field. In this phenomenon, the formation of the current point during the long time scale evolution phase is the key process. We have also shown that the current point can be formed even under the stochastic magnetic fields in a toroidal geometry. In the recent large tokamak plasmas, the resistivity, η, becomes about $\eta \simeq 10^{-8}$. Hence, after long term evolution of

DTM in the Rutherford type regime, the nonlinear destabilization of DTM occurs in the fast time scale.

In the case of the low β disruption in a negative shear plasma, the perturbations growing with a resistive time scale are sometimes observed around each rational surfaces [2]. After the growth in the resistive time scale, the perturbation shows the explosive growth. These feature is roughly consistent with our observation of the nonlinear destabilized DTM, although, at the present stage, the relationship between the precursor with the resistive time scale and the fast time scale phenomenon is not clear in experiments.

Finally, the present paper clarified the new mechanism of the nonlinear destabilization of the MHD mode. The explosive growth of DTM was shown to be originated not from both any type of the quasi-linear destabilization and the turbulence driven instability, where the increase of the transport coefficients driven by the higher harmonics accelerates the growth of the mode. Instead of them, the increase of the triangular deformation of islands plays the key role of these new phenomena, where the skin current concentrates to the X-point, relaxing the excess magnetic energy effectively through dissipation, and the explosion is almost independent on the resistivity.

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