Nonlinear Simulation Studies of Tokamaks and ST's

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Abstract. The multilevel physics, massively parallel plasma simulation code, M3D has been used to study ST's and tokamaks. The magnitude of outboard shift of density profiles relative to electron temperature profiles seen in NSTX under strong toroidal flow is explained. IRE's in ST discharges can be classified depending on the crash mechanism, just as in tokamak discharges; a sawtooth crash, disruption due to stochasticity, or high-β disruption. Toroidal shear flow can reduce linear growth of internal kink. It has a strong stabilizing effect nonlinearly and causes mode saturation if its profile is maintained, e.g., through a fast momentum source. Normally however, the flow profile itself flattens during the reconnection process, allowing a complete reconnection to occur. In some cases, the maximum density and pressure spontaneously occur inside the island and cause mode saturation. Gyrokinetic hot particle/MHD hybrid studies of NSTX show the effects of fluid compression on a fast-ion-driven n=1 mode. MHD studies of recent tokamak experiments with a central current hole indicate that the current clamping is due to sawtooth-like crashes, but with n=0.

1. Introduction

The multilevel physics, massively parallel plasma simulation code, M3D [1,2] has been used to study spherical tori (ST) and tokamaks. The effects of strong sheared toroidal flows are included since such flows can be a substantial fraction of the Alfven speed for ST's such as NSTX. Internal reconnection events (IRE) in NSTX are studied and their relation to the sawtooth crash and disruptions in tokamaks are examined. Energetic particle driven instabilities in NSTX are studied using a hybrid model, and n=0 reconnection process is studied to explain current clamping in tokamak experiments with a current hole. M3D currently has physics levels of MHD, two-fluids [3], and a hybrid gyrokinetic-particle/fluid models.[4] It uses unstructured meshes [5], and the parallel processing structure utilizes MPI and the PETSc framework.[6] The equations and methods used in the present studies can be found in Ref. 1.

2. 2D Equilibria with Strong Sheared Toroidal Flow

To explain the measured outboard shift of the plasma density relative to the electron temperature profile in NSTX, time asymptotic numerical 2D steady states are obtained with balanced toroidal momentum, current, density, and pressure sources and sinks. Figure 1 shows steady state density profiles with aspect ratio A=1.3 and peak beta β_0=30%; (a) in MHD, without zeroth order flow, (b) with peak Alfven Mach number M=0.2 in a two-fluid model, (c) with M=0.8 in MHD, and (d) M=0.2 but with mostly trapped hot particles providing about half of the total pressure. Case (b) is relevant to current experimental conditions, and unlike the density, the shift of the magnetic flux surfaces was very small. A two-fluid case is shown, but the MHD model gave a very similar result, and a hybrid model with parallel pressure 20% higher than perpendicular pressure also gave a result similar to within the uncertainty of experimental measurements. Thus, the relative density shift index of the MHD model at the magnetic axis, $(R\partial\rho)/(\rho\partial R) = 2M^2/\beta$

should roughly hold in the experiment. However, the data of Shot 105051 initially gave the right hand side (the centrifugal force term) only half of what was expected. The missing half turned out to be the centrifugal force of the hot particle slowing down distribution.

With 4 times the rotation of case (b), Fig. 1c has a maximum density 5 times larger than the nonrotating case and a small density of only 0.005 over most of the core region, showing that core confinement is basically lost. A similar loss of confinement can occur even at M=0.2, if the pressure is reduced by 16 times. Such a scenario could occur if a thermal quench happens without a corresponding toroidal momentum quench, e.g., in a disruption.

Fig 1d shows that if the hot particles were mostly trapped ($p_\perp \gg p_\parallel$), a much larger outboard shift would occur with a substantial loss of confinement in the core region.

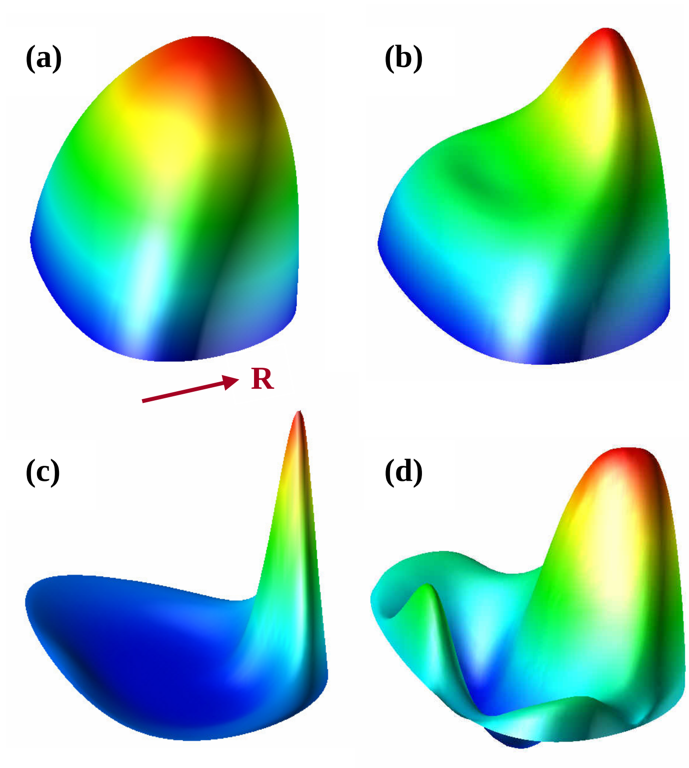

Figure 1. Density profiles of various 2D steady states.

3. Simulation studies of NSTX internal reconnection events

Although 3D global nonlinear simulations with experimental dissipation parameters of large fusion devices are still beyond current computational capabilities, useful insights on experiments can be gained by simulating discharges under less stiff conditions, e.g., more resistive and viscous plasmas. In some cases, e.g., in low-β MHD, the nonlinear evolution goes through similar states as long as the resistive time is long enough compared to the Alfven time, only the rate of change varying with resistivity. With more complex physics, the instability threshold may also depend on dissipation parameters. Even here, the magnetic topology often evolves through similar states when the plasma is unstable to a global mode, because magnetic field line reconnection must satisfy its own constraints. For the effective Lundquist number of a simulation of an experiment to be realistic, the simulation first has to have sufficient resolution of narrow layer structures in all dimensions. For example, global nonlinear simulations with S=10^8 that follow only a few harmonics are equivalent to a much lower effective S, since narrow reconnection layer structures must be resolved in both radial direction and harmonic space. Second, the simulation has to have sufficient 'physical resolution' of the layer structures, e.g., when layer widths become comparable to or smaller than the ion gyroradius, finite ion gyroradius effects should be included in the physics model. The simulations shown in this section are done with S between 10^3 and 10^4, with magnetic Prandtl number of order one. We could have used higher S values if a small number of very long runs were desired, but to maximize physical insights and ensure the reliability of simulation results, many distinct but similar runs have to be compared.

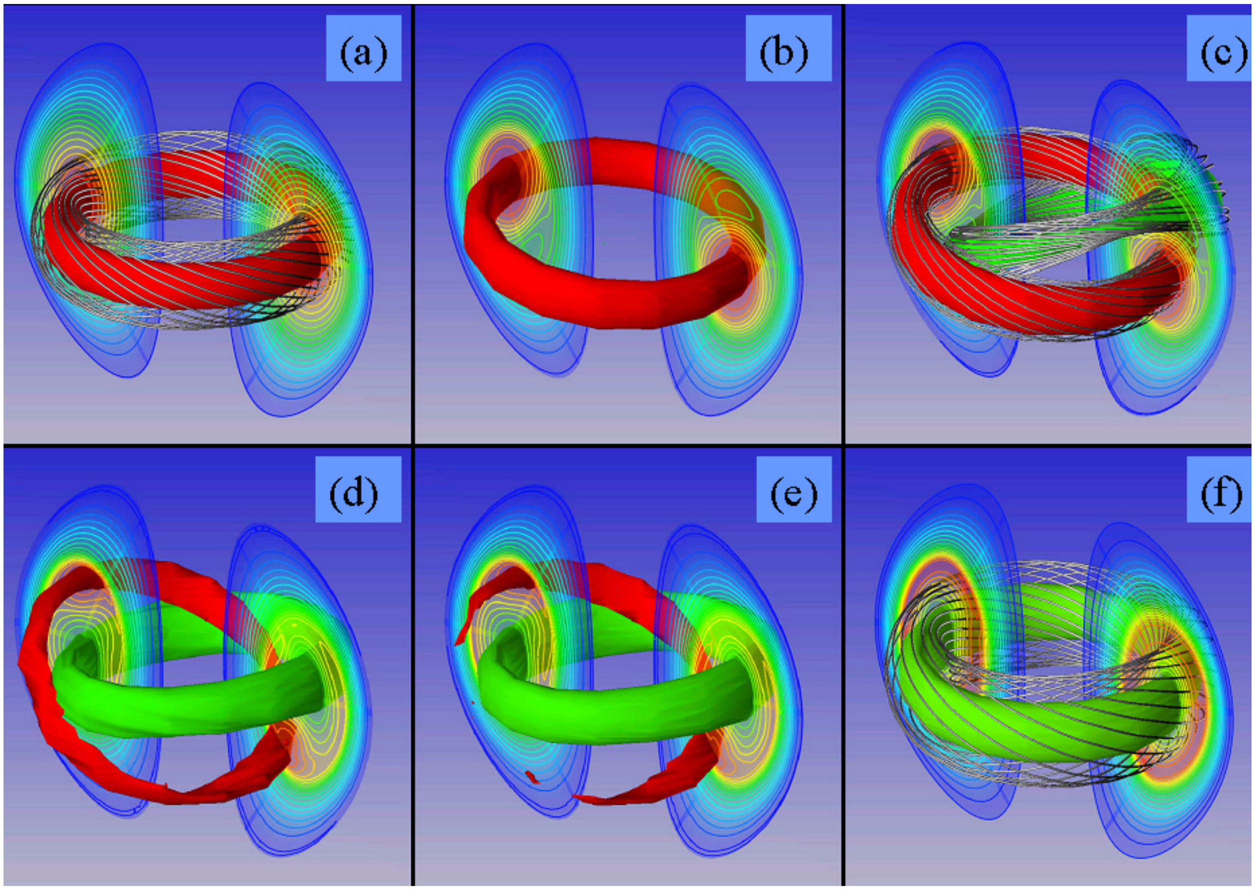

Figure 2. An IRE in a ST simulation which shows a similar behavior as a tokamak sawtooth crash. Temperature iso-surfaces and contours are shown together with selected magnetic field lines. The hot plasma core is replaced by cold peripheral plasma through reconnection of field lines.

3.1. Similarities to Sawtooth Crash and Disruption in Tokamaks

NSTX discharges sometimes end with an internal reconnection event (IRE) with the measured central q value near unity. Numerical simulation of such IRE's yielded three different categories, each with close similarity to a corresponding tokamak phenomenon. The first category, as shown in Fig. 2, is similar to a tokamak sawtooth crash. The hot plasma core is replaced by the colder outer plasma. Such cases are commonly seen in low beta plasmas.

When the inversion radius is large or the plasma pressure is increased, a similar sawtooth crash can result in global stochastic magnetic field lines as shown in Fig. 3. This is a similar phenomenon as a disruption due to stochastic field lines in tokamaks.

When the plasma beta is further increased, the local steepening of pressure driven modes becomes a more common cause of IRE's, for example as shown in the pressure profiles of Fig. 4, both of which have about 20% beta. The one on the right has predominantly m=2 mode, while the one on the left has more mixed m modes. A similar mechanism of disruption, sometimes called the high-β disruption, has been found in tokamaks, both in experiment and theory[7], and has also been found in stellarator simulations when the plasma beta is much higher than the design value.[8] These results indicate that the local steepening of pressure driven modes is a general phenomenon, that can occur in a wide spectrum of plasmas including astrophysical objects. In fact, a high-β disruption mechanism has been used to explain solar flares.[9]

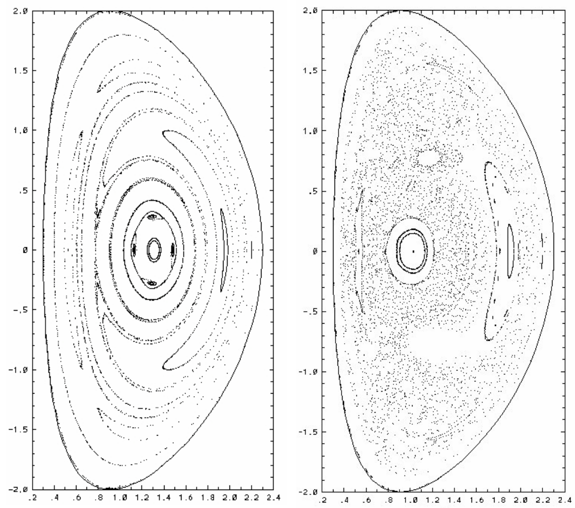

Figure 3. Puncture plots of field lines show that magnetic islands overlap and become stochastic.

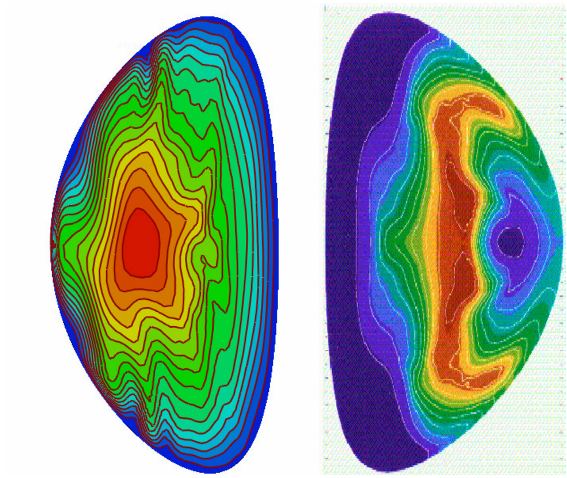

Figure 4. IRE's in high-β ST simulation show similar behavior as tokamak high-β disruptions.

Since IRE's in ST seem to have basically the same mechanism as either the sawtooth crash or disruptions in tokamaks, and scientific knowledge is gained by categorizing depending on the underlying physics, most IRE's in ST's could be categorized as a sawtooth crash, disruption due to stochasticity, or high-β disruption, depending on the crash mechanism. (A disruption process involves a thermal quench sometimes with or sometimes without a current quench) An ST is after all a very small aspect ratio tokamak, but the smallness of the aspect ratio affects the physical processes and the statistics of occurrence of each type of IRE/disruption can be quite different from in a tokamak.

Figure 5 compares the time history of soft X-ray signals (perturbed part), from experiment and simulation, in a case corresponding to a disruption due to stochasticity. The relative position on the short side corresponds to the vertical detector position. Both show similar global structures of a m=1 dominant evolution. However, what the simulation does not explain is the long saturation phase of the mode and the mode locking to the resistive wall prior to the crash. The saturation may be explained by stabilizing effects of strong sheared rotation, the ω* effect, and energetic particle effects. To study mode locking, the resistive wall, vacuum region, and external coils should be included in the simulation. All these physics and hardwares are incorporated in the M3D code, and benchmarks and simulation studies are currently being performed. Concerning IRE's, only the study of plasma rotation effects has been completed and these are reported in the next section.

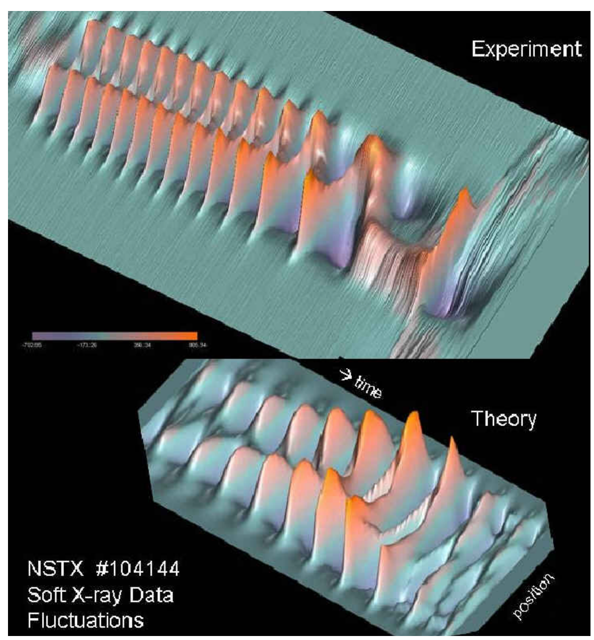

Figure 5. Soft-X ray data are compared between experiment and theory.

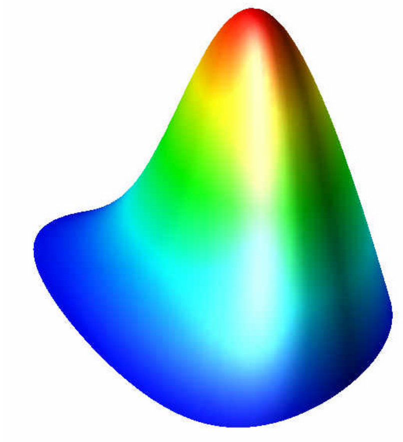

Figure 6. Toroidal velocity profile of a 2D steady state.

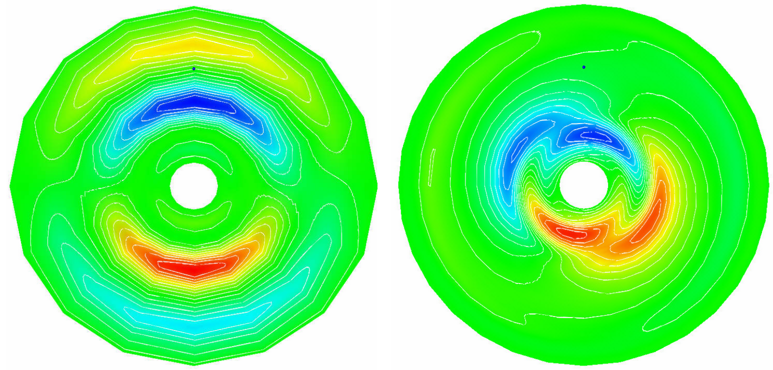

Figure 7. Velocity potential (incompressible part) profiles of n=1 linear eigenmode on the midplane of torus. Cases without flow and with flow of M=0.2 are compared.

3.2. Effects of Sheared Toroidal Flows

Linear and nonlinear studies are performed starting from the 2D steady state shown in Fig. 1b, whose toroidal velocity profile is shown in Fig. 6. Toroidal rotation modifies the linear eigenmodes as shown in Fig. 7, which compares the potential of the incompressible part of velocity in stationary and rotating equilibrium cases. The mode is shifted inboard and the growth rate is reduced from 0.03 to 0.01 (per Alfven time) for this case with the peak Alfven Mach number of M=0.2. The mode rotation frequency is 0.13, which roughly corresponds to the flow rotation frequency of the outboard lobe of the mode.

The nonlinear evolution of this mode shows that the toroidal shear flow is strongly stabilizing nonlinearly. If the momentum source rate were large enough that the toroidal velocity profile of Fig. 6 is roughly maintained, the mode saturates without going through a complete reconnection process. However, with realistic momentum source rate, the toroidal flow profile is flattened and complete reconnection occurs. This is due to the fact that a 'reconnection' process of the toroidal flow itself occurs as shown in Fig. 8, which compares pressure and toroidal flow profiles during the reconnection process. The two profiles are similar, because both evolve under conservation laws, of thermal energy and angular momentum respectively, inside the reconnected flux bundles. The upper regions are flattened due to the existence of a large m=1 magnetic island there. Such flattening of both pressure and toroidal flow have been observed in experiments in the presence of a large magnetic island.[10] At the complete reconnection, both pressure and toroidal flow patterns are flattened. Thus m=1 reconnection normally goes through to completion in the MHD model by flattening the toroidal flow profile with it.

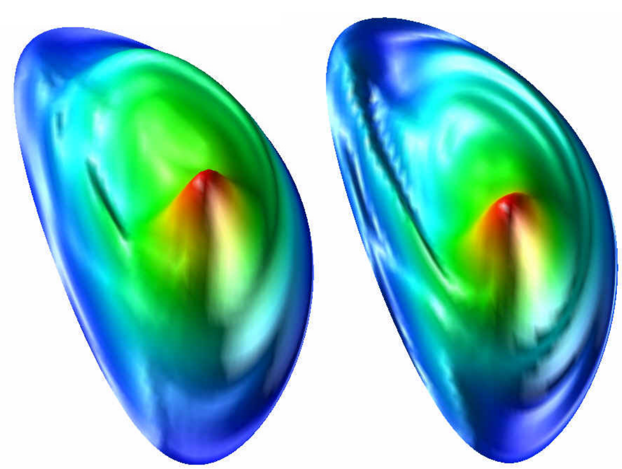

Figure 8. Pressure profile and toroidal velocity profile show close similarities. A large m=1 island is located at the upper region.

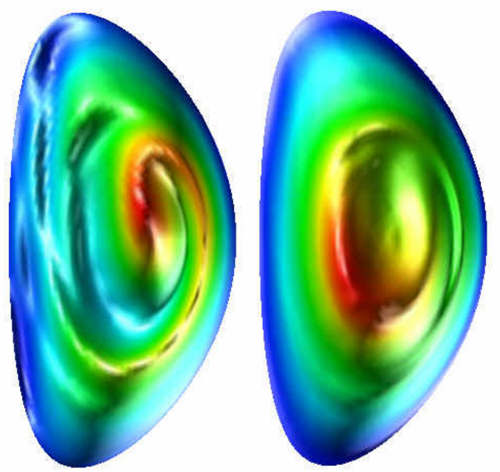

Figure 9. Density and temperature profiles, with density (and pressure) peak inside the island and temperature peak inside the original hot spot.

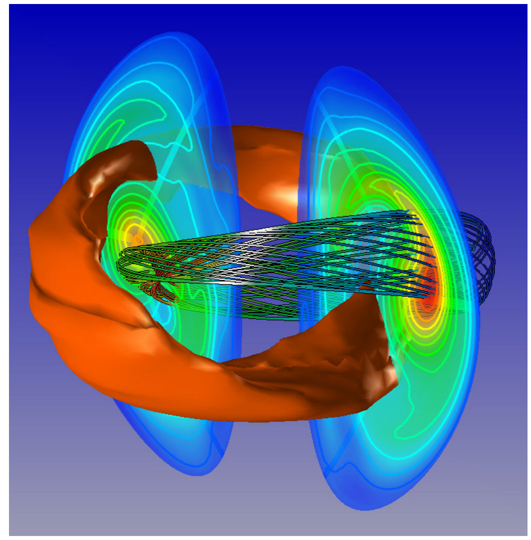

Figure 10. A saturated state. A temperature iso-surface, density contours, and a field line are shown.

If the plasma survives a crash, sometimes a subsequent reconnection process can saturate by spontaneously generating a curious profile shown in Fig. 9 and Fig. 10. The temperature is highest in the original 'hot' spot, but the density and pressure peak inside the island, probably due to the centrifugal force effect on density. Such a profile has been known to be stabilizing to a m=1 reconnection process, and leads to a saturated state.[11] The driving free energy of the usual m=1 reconnection comes partly from the outward shift of the central high pressure region inside the q=1 surface, while in the present case such a shift would not be energetically favorable because the pressure in the island which would be moving inward is the highest.

Thus MHD physics with shear flow could explain some of m=1 saturation cases seen in NSTX. At the least, such effects would contribute with additional ω^* and energetic particle stabilization effects to eventually cause the saturation of the m=1 mode. Such studies are currently being carried out using the two-fluid and hybrid particle/MHD levels of the code.

4. Hot particle/MHD hybrid studies of energetic ion driven modes

Energetic-particle driven MHD modes are studied for ST plasmas using the hybrid model in M3D.[4] In the hybrid model, the plasma is divided into the bulk plasma and an energetic particle component. The bulk plasma, which includes thermal electrons and ions, is treated as a single fluid and the energetic particles are described by gyrokinetic particles. The effects of hot particles enter through the momentum equation via the hot particle stress tensor (pressure coupling). The δf method is used in calculating the stress tensor from the hot particles which are advanced in a self-consistent electromagnetic field.

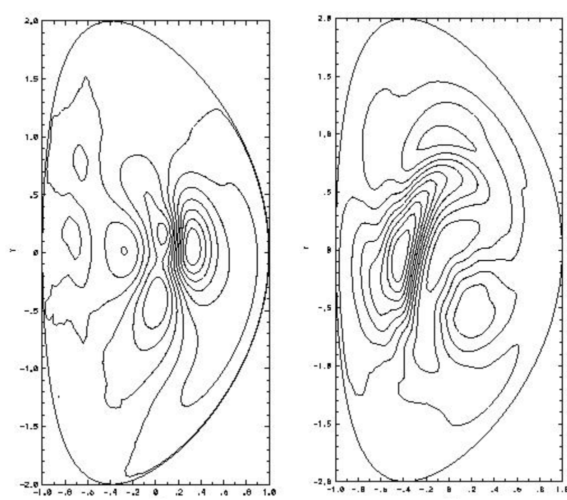

Figure 11. TAE mode on the left. BAE mode on the right is destabilized at higher β.

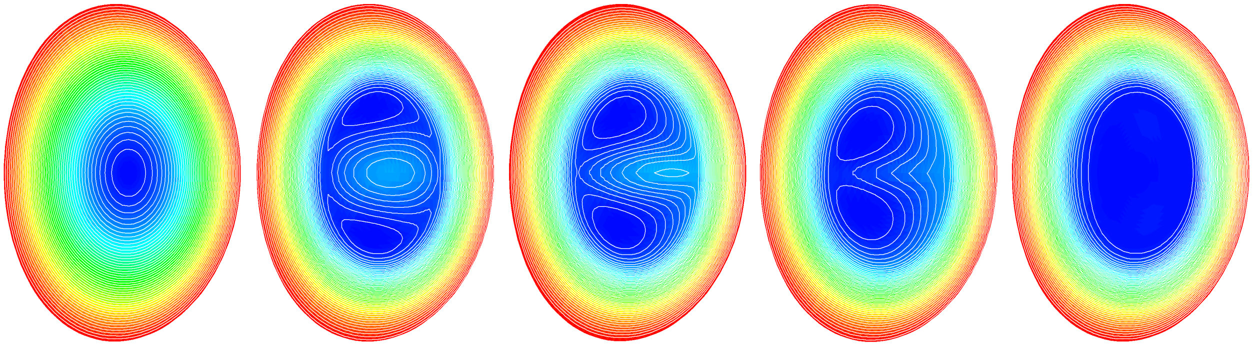

Figure 12. Time sequence of poloidal flux contours during a n=0 reconnection process which clamps core current near zero.

We consider parameters and profiles similar to the NBI-heated NSTX plasmas: aspect ratio A=1.3, elongation κ=2.0, triangularity δ=0.3, central and edge safety factor q(0)=1.4 and q(a)=7.3, ratio of hot ion speed and Alfven speed is 3.9, and the normalized hot ion gyroradius, ρ_h/a =0.09. Figure 11 shows a linearly unstable TAE-like eigenmode with its mode frequency comparable to the expected TAE frequency at the q=1.5 surface. This unstable TAE is driven by hot ions at hot ion beta value of β_h=3.8% and zero bulk plasma beta. However, when the bulk plasma beta is increased to β_{bulk}=24.5%, the mode frequency is reduced by a factor of 5 to ω= 0.07 $v_A(0)/R$. The corresponding eigenmode structure is shown in Fig. 11 on the right. The frequency of this new mode is located below the beta-induced continuum gap near the center of the plasma and thus it may be called Beta-induced Alfven eigenmode (BAE). To test whether this mode is indeed beta driven, the ratio of the specific heat, Γ, is changed to zero (i.e., zero sound speed) from Γ=5/3, and the mode has reverted to a TAE mode. The mode frequency is higher than ω* of bulk ions. The stabilizing effect of bulk ion Landau damping on this BAE mode is currently being studied using the ion-gyrokinetic-particles/electron-fluid hybrid level of M3D code.

5. Interpretation of tokamak experiments with a current hole

Recent experiments in JET and JT-60 with a fast current ramp-up and external current drive exhibit a central region with virtually zero current density.[12] Simulation of these current hole discharges indicate that the current clamping is due to sawtooth like crashes, but with toroidal mode number n=0, rather than n=1. Fig. 12 shows the poloidal flux contours during one such crash, using Lundquist number S=10^4. (Rationale for using smaller S is given in Section 3.) It shows a complete reconnection with predominant m=1 mixed with m=2. The crash free energy comes from two sources; 1) the 2D state has no regular equilibrium and gives rise to a singular current sheet with finite jump of magnetic field strength (related to the Shafranov shift), and

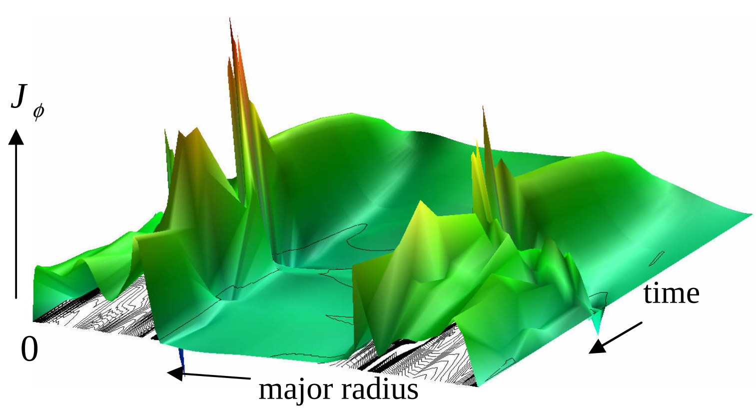

Figure 13. Time history of toroidal current density, showing multiple n=0 reconnections, and some 3D instability.

2) a free energy analogous to the usual n=1 internal kink. Fig. 13 shows the time history of the toroidal current density across the horizontal midplane, clearly revealing the clamping of the core current density to near zero. This simulation includes a general 3D perturbation, and shows that some 3D modes are also unstable. The implications of such modes are currently being studied.

6. Conclusions

Various physics levels of the M3D code are used to study ST and tokamak phenomena. The magnitude of the outboard shift of density profile relative to electron temperature profile seen in NSTX under strong toroidal flow is explained. IRE's in ST discharges can be classified depending on the crash mechanism, just as in tokamak discharges - sawtooth crash, disruption due to stochasticity, or high-β disruption. Toroidal shear flow can reduce the linear growth of the internal kink. It has strong stabilizing effects nonlinearly and can cause mode saturation if its profile is maintained, e.g., through a fast momentum source. Normally however, the flow profile itself flattens during the reconnection process, allowing a complete reconnection. In some instances, the maximum density and pressure spontaneously occur inside the island and cause a mode saturation. Hot particle/MHD hybrid simulation of NSTX has produced a BAE mode. Finally, the current clamping in tokamak experiment with a current hole is explained as due to an n=0 sawteeth reconnection process. Even though much less stiff cases than the actual experimental parameters are used in the studies presented here, many useful insights are gained. Future simulation studies will have increasingly realistic parameters with appropriate physical and numerical resolutions for the parameters used.

Acknowledgements
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