Paleoclassical Electron Heat Transport

J.D. Callen, University of Wisconsin, Madison, WI 53706-1609 USA
callen@engr.wisc.edu, http://homepages.cae.wisc.edu/∼callen

Abstract. Radial electron heat transport in low collisionality, magnetically-confined toroidal plasmas is shown to result from paleoclassical Coulomb collision processes (parallel electron heat conduction and magnetic field diffusion). In such plasmas the electron temperature equilibrates along magnetic field lines a long length L, which is the minimum of the electron collision length and a maximum effective half length of helical field lines. Thus, the diffusing field lines induce a radial electron heat diffusivity $M \simeq L/(\pi R_0 q) \sim 10 >> 1$ times the magnetic field diffusivity $\eta/\mu_0 \simeq \nu_e (c/\omega_p)^2$. The paleoclassical electron heat flux model provides interpretations for many features of "anomalous" electron heat transport: magnitude and radial profile of electron heat diffusivity (in tokamaks, STs, and RFPs), Alcator scaling in high density plasmas, transport barriers around low order rational surfaces and near a separatrix, and a natural heat pinch (or minimum temperature gradient) heat flux form.

1. Introduction: Paleoclassical Physical Mechanism

The fastest and dominant Coulomb-collision-induced transport processes in magnetically-confined plasmas occur on the electron collision time scale $1/\nu_e$ and will be called paleoclassical [1,2]: parallel electron heat conduction and magnetic field diffusion. On this time scale, the electron distribution is Maxwellianized and electron heat conduction equilibrates the electron temperature T_e over long distances parallel to the magnetic field $\mathbf{B}$ — up to the electron collision length $\lambda_e \equiv v_{Te}/\nu_e$ in which $v_{Te} \equiv (2T_e/m_e)^{1/2}$. Magnetic field diffusion is induced by the plasma electrical resistivity η. It causes magnetic flux (bundles of field lines) to diffuse perpendicular to $\mathbf{B}$ with a diffusion coefficient $D_\eta \simeq \eta_0/\mu_0 \equiv \nu_e (c/\omega_p)^2 \sim (\Delta x)^2/\Delta t$, which implies a diffusive radial step $\Delta x \simeq \delta_e \equiv c/\omega_p$ [the electromagnetic (em) skin depth] in a collision time $\Delta t \simeq 1/\nu_e$.

Electron gyromotion about magnetic field lines causes the electron guiding center to be identified with the small amount of magnetic flux associated with field lines penetrating the gyroorbit. However, since those field lines diffuse radially due to D_η, the guiding center position becomes a radially diffusing "stochastic variable." To account for this effect a spatial Fokker-Planck operator [3] is added to the usual drift-kinetic equation — see (17), (18). If λ_e is longer than the length of a helical field line on a $q_* \equiv m/n$ rational surface or the effective parallel length of diffusing field lines [for $n \leq n_{\max} \sim 10$ — see (23)], the parallel equilibration length L is reduced to these lengths — see (26). The effect of the T_e equilibration over a length L along radially diffusing helical rational field lines that are longer than the poloidal periodicity half length ($\sim \pi R_0 q$) is that the effective electron heat diffusivity is a multiple $M \sim L/(\pi R_0 q) \sim 10$ of the magnetic field diffusivity D_η — see (22), (25), (27), and (29).

2. Magnetic Field Geometry

The paleoclassical model is developed using a full axisymmetric magnetic field model for arbitrary aspect ratio ($A \equiv R_0/r \equiv 1/\epsilon$ where R_0, r are the major, minor radii of the torus) to facilitate application of the theory to most types of axisymmetric toroidal plasmas — large aspect ratio tokamaks ($A >> 1$) and regions of spherical tokamaks (STs, $A \gtrsim 1$), spheromaks, and reversed field pinches (RFPs) where $\epsilon^2, B_p^2/B_t^2 << 1$. Approximate results for large aspect ratio tokamaks are indicated at the end of many equations after an approximate equality ($\simeq$).

Paleoclassical transport is concerned with diffusion of magnetic flux (bundles of magnetic field lines). Since for axisymmetric toroidal plasmas with $\epsilon^2, B_p^2/B_t^2 << 1$ the toroidal magnetic flux ψ_t is less mobile than the poloidal magnetic flux ψ [4-6], diffusion of the poloidal flux surfaces (and field lines) will be determined relative to ψ_t and hence a dimensionless, cylindrical-type radial variable ρ: $\rho \equiv [\psi_t/\psi_t(a)]^{1/2} \simeq r/a$, $\psi_t(\rho, t) \equiv (1/2\pi) \iint d\mathbf{S}(\zeta) \cdot \mathbf{B}_t \simeq r^2 B_0/2$. The appropriate magnetic field model [4-6] has toroidal (t) and poloidal (p) components: $\mathbf{B} = \mathbf{B}_t + \mathbf{B}_p = I\, \boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\psi = \boldsymbol{\nabla}\psi \times \boldsymbol{\nabla}(q\,\theta - \zeta)$. As usual, $I = I(\rho, t) \equiv R B_t \simeq B_0 R_0$. Also, ζ is the toroidal angle and $\psi(\rho, t) \equiv (1/2\pi) \iint d\mathbf{S}(\theta) \cdot \mathbf{B}_p$, $\partial\psi/\partial\rho \simeq a R_0 B_p$. Further, θ is the straight-field-line (in the ψ = constant plane) poloidal angle and q is the winding number or pitch ("safety factor" for kink stability) of magnetic field lines on a flux surface: $q(\rho, t) \equiv (\partial\psi_t/\partial\rho)/(\partial\psi/\partial\rho) = \#$ toroidal transits/# poloidal transits $\simeq r\, B_t/R_0 B_p$. For an axisymmetric magnetic field $q(\rho, t) = q(\psi, t)$ and $\mathbf{B} \cdot \boldsymbol{\nabla}\theta = I/qR^2 \simeq B_t/R_0 q = B_p/r$.

The Jacobian for transforming from the original Eulerian coordinates to the curvilinear set $u^i \equiv (\rho, \theta, \zeta)$ is $\sqrt{g} \equiv 1/\boldsymbol{\nabla}\rho \cdot \boldsymbol{\nabla}\theta \times \boldsymbol{\nabla}\zeta = (\partial\psi/\partial\rho)/\mathbf{B} \cdot \boldsymbol{\nabla}\theta \simeq r a R_0$. The radial differential of the volume is

$V' \equiv \partial V(\rho,t)/\partial\rho = 2\pi\int_{-\pi}^{\pi}\sqrt{g}\,d\theta \simeq a(2\pi r)(2\pi R_0)$. The average of an axisymmetric ($\partial f/\partial\zeta = 0$) scalar function $f(\mathbf{x},t)$ over a flux surface is $\langle f(\mathbf{x},t)\rangle = (2\pi/V')\int_{-\pi}^{\pi}\sqrt{g}\,d\theta\; f(\mathbf{x},t)$. The flux-surface-average is an annihilator for the parallel gradient operator: $\langle\mathbf{B}\cdot\boldsymbol{\nabla} f\rangle = 0$, for any function $f(\mathbf{x},t)$ that is periodic in both θ and ζ. For a similarly periodic vector field $\mathbf{A}(\mathbf{x},t)$, the flux-surface-average of its divergence, defined by $\boldsymbol{\nabla}\cdot\mathbf{A} \equiv \sum_i(1/\sqrt{g})(\partial/\partial u^i)(\sqrt{g}\mathbf{A}\cdot\boldsymbol{\nabla} u^i)$, becomes $\langle\boldsymbol{\nabla}\cdot\mathbf{A}\rangle = \partial\langle\mathbf{A}\cdot\boldsymbol{\nabla} V\rangle/\partial V$.

Flux surfaces are rational or irrational depending on whether q is the ratio of integers (m,n):

$$q(\rho,t)\;\begin{cases} = m/n, & \text{rational surface,}\\ \neq m/n, & \text{irrational surface.}\end{cases} \tag{1}$$

The irrational surfaces form a dense set while the rational surfaces are a set of measure zero and radially isolated from each other. Rational surfaces are of interest here because their (helical) magnetic field lines close on themselves after m toroidal (or n poloidal) transits.

The differential length $d\ell$ along magnetic field lines obtained from the poloidal ($\boldsymbol{\nabla}\theta$) projection of the field line equation $d\mathbf{x}/d\ell = \mathbf{B}/B$ is $d\ell = (B/\mathbf{B}\cdot\boldsymbol{\nabla}\theta)\,d\theta \simeq R_0 q\,d\theta$. The half length ℓ_* of a closed helical field line on a $q_* \equiv q(\rho_*) \equiv m/n$ rational surface is [2]:

$$\ell_* \equiv \frac{1}{2}\int_{-n\pi}^{n\pi}\frac{B\,d\theta}{\mathbf{B}\cdot\boldsymbol{\nabla}\theta} = \pi\bar{R}q_* n, \quad \text{rational field line length;} \quad \bar{R} \equiv \frac{\langle B\rangle\, V'}{4\pi^2 q_*\partial\psi/\partial\rho} \simeq R_0. \tag{2}$$

While helical field lines on medium order rational surfaces with $n \sim 10 >> 1$ are long ($>> \pi\bar{R}q_*$), those with low n ($\equiv n^\circ = 1,2$) are short ($\sim \pi\bar{R}q_*$).

Radial distances between medium order rational surfaces can be estimated using a Taylor series expansion of $q(\rho,t)$ about a rational surface at $\rho = \rho_*$: $q(\rho,t) \simeq q_* + x\,q' + \mathcal{O}(x^2)$, in which $q_* \equiv q(\rho_*,t) = m/n$, $x \equiv \rho - \rho_*$ (dimensionless radial distance from rational surface), and $q' \equiv |\partial q/\partial\rho|_{\rho_*}$. The distance between rational surfaces with $m\pm 1$ but the same n is obtained from $1/n = q - q_* \simeq xq'$:

$$\Delta \simeq 1/nq', \quad \text{distance between same } n \text{ rational surfaces.} \tag{3}$$

Defining $q(\rho_{\rm max}) = m_{\rm max}/n_{\rm max}$ and expanding $q(\rho) = (m_{\rm max}n + 1)/n_{\rm max}n$ about $\rho = \rho_{\rm max}$ yields the distance between a $q_* \equiv m/n$ rational surface and the nearest $n \leq n_{\rm max}$ rational surface:

$$\delta x(n) \equiv \rho_* - \rho_{\rm max} \simeq \frac{1}{n_{\rm max} n\, q'}, \quad \text{minimum spacing for } q' \neq 0,\ n \leq n_{\rm max}; \quad \text{or,} \tag{4}$$

$$\delta x_{\rm min}(n) \equiv \rho_* - \rho_{\rm max} \simeq \left(\frac{2}{n_{\rm max} n\, q''}\right)^{1/2}, \quad q'' \equiv \left.\frac{\partial^2 q}{\partial\rho^2}\right|_{\rho_*}, \quad \text{spacing near minimum in } q. \tag{5}$$

For $n_{\rm max} \gtrsim 10$, $q' \sim 1$, and $q'' \sim 1$, all of these distances are small fractions of the minor radius: $\Delta \sim 1/n_{\rm max} << 1$ for $n \sim n_{\rm max}$, and $\delta x(n) \lesssim 1/n_{\rm max} << 1$, $\delta x_{\rm min}(n) \lesssim 1/\sqrt{n_{\rm max}} < 1$.

3. Magnetic Flux, Field Line Diffusion

Evolution equations for ψ_t and ψ obtained from Faraday's law ($\partial\mathbf{B}/\partial t = -\boldsymbol{\nabla}\times\mathbf{E}$) are [2,5,6]:

$$\frac{d\psi_t}{dt} \equiv \left.\frac{\partial\psi_t}{\partial t}\right|_{\mathbf{x}} + \langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi_t\rangle = 0, \quad \text{toroidal flux evolution,} \quad \langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi_t\rangle \equiv q\,\frac{\langle\mathbf{E}\cdot\mathbf{B}_p\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle}; \tag{6}$$

$$\frac{d\psi}{dt} \equiv \left.\frac{\partial\psi}{\partial t}\right|_{\mathbf{x}} + \langle\mathbf{u}_g\cdot\boldsymbol{\nabla}\psi\rangle = \frac{\langle\mathbf{E}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} - \frac{\partial\Psi}{\partial t}, \quad \text{poloidal flux evolution.} \tag{7}$$

Toroidal flux ψ_t is advected radially by the grid velocity $\mathbf{u}_g$ induced by the poloidal electric field, but conserved in a Lagrangian frame. In (7) $\partial\Psi/\partial t \equiv V_{\rm loop}^{\zeta}(t)/2\pi$ is the (positive) constant of a spatial integration. It represents the toroidal loop voltage induced by the rate of change of the magnetic flux in the central solenoid of a tokamak. The poloidal flux ψ and hence poloidal magnetic field lines move relative to ψ_t [compare (6) and (7)] because of departures from ideal MHD (i.e., a nonzero parallel electric field $\langle\mathbf{E}\cdot\mathbf{B}\rangle$) or a temporally changing magnetic flux in the central solenoid (i.e., $\partial\Psi/\partial t \neq 0$).

A parallel Ohm's law for $\langle\mathbf{E}\cdot\mathbf{B}\rangle$ is obtained [2] from the flux-surface-average of the parallel ($\mathbf{B}\,\cdot$) component of the electron momentum equation including inertial and viscosity effects:

$$\frac{\langle\mathbf{E}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} = \left(\frac{\eta_\parallel^{\rm nc}}{\mu_0} + \delta_e^2\frac{d}{dt}\right)\frac{\langle\mu_0\mathbf{J}\cdot\mathbf{B}\rangle}{\langle\mathbf{B}\cdot\boldsymbol{\nabla}\zeta\rangle} + \frac{\mu_e}{\nu_e}\eta_0\frac{1}{\langle R^{-2}\rangle}\frac{dP}{d\psi}, \quad \text{parallel Ohm's law.} \tag{8}$$

Here, the terms on the right indicate: magnetic flux diffusion [see (12) below] induced by the neoclassical parallel resistivity, electron inertia, and the neoclassical bootstrap current. The (neoclassical) parallel electrical resistivity (neglecting poloidal electron heat flow effects) is

$$\boxed{\frac{\eta_\parallel^{\rm nc}}{\eta_0} \simeq \frac{\eta_\parallel^{\rm Sp}}{\eta_0} + \frac{\mu_e}{\nu_e}, \quad \text{neoclassical resistivity;}} \quad \frac{\eta_\parallel^{\rm Sp}}{\eta_0} \simeq \frac{\sqrt{2}+Z}{\sqrt{2}+13Z/4}, \quad \text{Spitzer resistivity.} \tag{9}$$

The reference ($\perp$) resistivity η_0 and electron viscous drag frequency μ_e adapted from [5,6] are

$$\frac{\eta_0}{\mu_0} \equiv \frac{m_e \nu_e}{n_e e^2 \mu_0} \simeq \frac{1.4\times 10^3\, Z}{[T_e(\text{eV})]^{3/2}} \left(\frac{\ln\Lambda}{17}\right) \frac{\text{m}^2}{\text{s}}, \quad \frac{\mu_e}{\nu_e} \simeq \frac{Z+\sqrt{2}-\ln(1+\sqrt{2})}{Z\,(1+\nu_{*e}^{1/2}+\nu_{*e})} \frac{f_t}{f_c} \overset{\nu_{*e}=0,\ Z=1}{\Longrightarrow} 1.5\,\frac{f_t}{f_c}. \tag{10}$$

Here, Z ($\to Z_{\rm eff} \equiv \sum_i n_i Z_i^2/n_e$ for multiple ion species) is the (effective) ion charge, $f_c \simeq 1-1.46\epsilon^{1/2}+\mathcal{O}(\epsilon^{3/2})$ is the circulating particle fraction [6], $f_t \equiv 1-f_c$, and $\nu_{*e} \equiv \nu_e/[\epsilon^{3/2}(v_{Te}/R_0 q)] = R_0 q/\epsilon^{3/2}\lambda_e$ is the electron collisionality parameter. The $\eta_\parallel^{\rm nc}$ in (9), (10) ranges from being equal to (for $\mu_e/\nu_e << 1$) to twice (for $\mu_e/\nu_e >> 1$) the most precise neoclassical resistivity results [5,6].

From Ampere's law $\mu_0 \mathbf{J} \equiv \boldsymbol{\nabla}\times\mathbf{B} = (\partial I/\partial\psi)\boldsymbol{\nabla}\psi\times\boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta\,\Delta^*\psi$, in which the usual magnetic differential operator is $\Delta^*\psi \equiv (1/|\boldsymbol{\nabla}\zeta|^2)\boldsymbol{\nabla\cdot}|\boldsymbol{\nabla}\zeta|^2\boldsymbol{\nabla}\psi$. Dotting this $\mu_0\mathbf{J}$ with $\mathbf{B}$, flux surface averaging, and using $|\boldsymbol{\nabla}\zeta|^2 = R^{-2}$ yields [2,6]

$$\Delta^+\psi \equiv \frac{\langle \mu_0 \mathbf{J\cdot B}\rangle}{\langle \mathbf{B\cdot}\boldsymbol{\nabla}\zeta\rangle} = \frac{I}{\langle R^{-2}\rangle V'}\frac{\partial}{\partial\rho}\left[\left\langle \frac{|\boldsymbol{\nabla}\rho|^2}{R^2}\right\rangle \frac{V'}{I}\frac{\partial\psi}{\partial\rho}\right] \simeq \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\psi}{\partial r}. \tag{11}$$

Substituting the Ohm's-law-determined $\langle \mathbf{E\cdot B}\rangle$ in (8) using $\langle \mathbf{J\cdot B}\rangle$ from (11) into the poloidal flux evolution equation (7), one obtains a diffusion-type (at least for $\delta_e^2\Delta^+ << 1$) equation for ψ:

$$\frac{d}{dt}\left(1-\delta_e^2\Delta^+\right)\psi = D_\eta\,\Delta^+\psi - S_\psi; \quad \boxed{D_\eta \equiv \frac{\eta_\parallel^{\rm nc}}{\mu_0}, \quad \text{magnetic field diffusivity.}} \tag{12}$$

Sources of poloidal flux in $S_\psi \equiv \partial\Psi/\partial t - (\mu_e/\nu_e)\eta_0(1/\langle R^{-2}\rangle)dP/d\psi$ arise from the "current-drive" effects due to a changing flux in the central solenoid ($\partial\Psi/\partial t \equiv V_{\rm loop}^\zeta/2\pi$) and the bootstrap current.

In equilibrium in the Lagrangian frame, $d/dt \to 0$ and the equation for the stationary poloidal flux ψ_0 becomes $0 = D_\eta\,\Delta^+\psi_0 - S_\psi$. Thus, in equilibrium the diffusion of ψ (poloidal field lines) is balanced by the source S_ψ of poloidal magnetic flux, field lines; the Poynting flux represented by $\partial\Psi/\partial t$ brings poloidal field lines into the plasma and the magnetic field diffusivity D_η diffuses them out of the plasma — even for a stationary poloidal magnetic field $\mathbf{B}_p$, which will be henceforth assumed.

To determine the properties of the small bundle of magnetic flux $\delta\psi(x,t)$ penetrating an electron gyroorbit, one substitutes an Ansatz of $\psi \to \psi_0 + \delta\psi$ into (12) to obtain (for $x^2 << 1$) $(\partial/\partial t + \bar{u}_g\,\partial/\partial x)(1-\bar{\delta}_e^2\,\partial^2/\partial x^2)\,\delta\psi \simeq \bar{\nu}_e\bar{\delta}_e^2\,\partial^2\delta\psi/\partial x^2$, in which the following normalized variables have been defined: $\bar{u}_g \equiv \langle \mathbf{u}_g\boldsymbol{\cdot\nabla}\rho\rangle$, $\bar{\delta}_e \equiv \delta_e/\bar{a}$, $1/\bar{a}^2 \equiv (1/\langle R^{-2}\rangle)\langle|\boldsymbol{\nabla}\rho|^2/R^2\rangle \simeq 1/a^2$, $\bar{D}_\eta \equiv D_\eta/\bar{a}^2$, $\bar{\nu}_e \equiv \nu_e(\eta_\parallel^{\rm nc}/\eta_0)$. For $|x| < \delta_e$ (or $k_x^2\delta_e^2 > 1$), the $\delta\psi$ solution of this equation is spatially constant [2]; hence, it produces no field lines or diffusion of them in this region (i.e., $\delta\mathbf{B}_p \equiv \boldsymbol{\nabla}\zeta\times\boldsymbol{\nabla}\delta\psi = \mathbf{0}$ there).

For radial scale lengths longer than the em skin depth δ_e (for which $k_x^2\bar{\delta}_e^2 << 1$ so that $\delta_e^2\Delta^+ << 1$ can be neglected), the evolution equation for $\delta\psi$ becomes a simple diffusion equation:

$$\frac{d\,\delta\psi}{dt} \equiv \left(\frac{\partial}{\partial t} + \bar{u}_g\frac{\partial}{\partial x}\right)\delta\psi = \bar{D}_\eta\,\frac{\partial^2\delta\psi}{\partial x^2}. \tag{13}$$

Its (Green function) solution for a delta function of flux initially located at $x=0$ is

$$\delta\psi(x,t) = e^{-(x-\bar{u}_g t)^2/4\bar{D}_\eta t}/(4\pi\bar{D}_\eta t)^{1/2}, \quad \int_{-\infty}^{\infty}\!dx\,x\,\delta\psi = \bar{u}_g\,t, \quad \int_{-\infty}^{\infty}\!dx\,x^2\,\delta\psi = 2\bar{D}_\eta t = 2\,\bar{\nu}_e t\,\bar{\delta}_e^2. \tag{14}$$

Note that $\delta\psi$ indicates a temporally evolving probability distribution for the radial location of a unit quanta of poloidal flux (field lines) that was initially at $x=0$. As indicated, the average radial displacement and spread (variance) of the flux (field lines) grow linearly with time. This advection and diffusion process occurs even when the magnetic field $\mathbf{B}$ is in stationary equilibrium (i.e., $d\psi/dt = 0$).

In the next section a Fokker-Planck model will be used to include effects of radial advection and diffusion of field lines in a kinetic analysis. Relevant Fokker-Planck coefficients deduced from (14) are

$$\frac{\langle \Delta x \rangle}{\Delta t} = \bar{u}_g, \quad \frac{\langle (\Delta x)^2 \rangle}{2\,\Delta t} = \bar{D}_\eta \quad \Longrightarrow \quad \frac{\langle \Delta \mathbf{x} \rangle}{\Delta t} \equiv \frac{\langle \Delta x \rangle}{\Delta t}\mathbf{e}_\rho, \quad \frac{\langle \Delta \mathbf{x} \Delta \mathbf{x} \rangle}{\Delta t} \equiv \frac{\langle (\Delta x)^2 \rangle}{\Delta t}\mathbf{e}_\rho \mathbf{e}_\rho. \tag{15}$$

In the last form the Fokker-Planck coefficients have been written in a general vectorial form in terms of the covariant base vector in the "radial" direction $\mathbf{e}_\rho \equiv \partial \mathbf{x}/\partial \rho = \sqrt{g}\,\boldsymbol{\nabla}\theta \times \boldsymbol{\nabla}\zeta$, for which $\mathbf{e}_\rho \cdot \boldsymbol{\nabla}\rho = 1$.

To consider diffusion of helical flux (field lines) in the vicinity of a rational surface at $\rho = \rho_*$ where $q_* \equiv q(\rho_*) = m/n$, one uses a local helical coordinate system with helical angle [7] $\alpha \equiv \zeta - q_*\theta = \zeta - (m/n)\,\theta$. Since $\boldsymbol{\nabla}\theta \times q_* \boldsymbol{\nabla}\theta = \mathbf{0}$, the Jacobian $\sqrt{g} \equiv (\boldsymbol{\nabla}\rho \cdot \boldsymbol{\nabla}\theta \times \boldsymbol{\nabla}\alpha)^{-1}$ is the same as before. Thus, one writes $\mathbf{B}$ in the local helical form $\mathbf{B} = \boldsymbol{\nabla}\alpha \times \boldsymbol{\nabla}\psi + \boldsymbol{\nabla}\psi_* \times \boldsymbol{\nabla}\theta \equiv \mathbf{B}_h + \mathbf{B}_*$, in which

$$\partial \psi_*/\partial \rho = (q - q_*)\,\partial \psi/\partial \rho, \quad \text{helical flux definition} \quad \Longrightarrow \quad \psi_{*0} \simeq (x^2/2)\,q'\,\psi'. \tag{16}$$

Here, the last form has been obtaied using $q(\rho) \simeq q_* + xq'$. Integrating the general form in (16) over ρ near $\rho = \rho_*$, taking its total time derivative, and using $d\psi_t/dt = 0$ from (6), one obtains [2] (again neglecting em skin depth effects for $x^2 > \bar{\delta}_e^2$ and $t > 1/\bar{\nu}_e$) $d\psi_*/dt = -\,q_*\,d\psi/dt$. Thus, the helical flux ψ_* diffuses like the poloidal flux ψ does. Also, writing $\psi_* \to \psi_{*0} + \delta\psi_*$, one finds [2] that $\delta\psi_*$ obeys the same diffusion-type equation as $\delta\psi$ does, i.e., (13). Hence helical flux (field lines) in the vicinity of rational surfaces also advect and diffuse with the Fokker-Planck coefficients given in (15).

4. Paleoclassical Kinetics, Analysis

In plasma kinetic theory magnetic flux surfaces and field lines are usually assumed to be fixed in space; however, as discussed in the preceding section, when $\eta\,\langle \mathbf{J} \cdot \mathbf{B} \rangle \neq 0$ they diffuse radially. Thus, the bundle of magnetic flux (field lines) penetrating the electron gyroorbit becomes a stochastic (diffusing) variable. The field line diffusion is, like most stochastic processes [3], governed (at least for $x^2 > \bar{\delta}_e^2$ and $t > 1/\bar{\nu}_e$) by a diffusion equation and representable by Fokker-Planck coefficients — as given in (15). The relevant electron kinetic equation is the gyro-averaged one, which is called the drift-kinetic equation [5]. Adding the Fokker-Planck-type effects [3] of magnetic flux (field line) diffusion of the electron guiding centers, the magnetic-field-diffusion-Modified Drift-Kinetic Equation (MDKE) is

$$\boxed{\frac{\partial f}{\partial t} + \frac{v_\parallel}{B}\mathbf{B} \cdot \boldsymbol{\nabla} f + \mathbf{v}_D \cdot \boldsymbol{\nabla} f + \dot{\varepsilon}\,\frac{\partial f}{\partial \varepsilon} = \mathcal{C}\{f\} + \mathcal{D}\{f\}.} \tag{17}$$

Here, $f = f(\mathbf{x}_g, \varepsilon, \mu, t)$ is the guiding center distribution function, $\mathcal{C}\{f\}$ is the Coulomb collision operator, and the other notation is standard. Effects due to magnetic field line diffusion are indicated by the Fokker-Planck spatial diffusion operator ($\mathcal{D}$), which in general is [3] $\mathcal{D}\{f\} \equiv -\boldsymbol{\nabla} \cdot [(\langle \Delta \mathbf{x} \rangle/\Delta t) f] + \boldsymbol{\nabla} \cdot [\boldsymbol{\nabla} \cdot (\langle \Delta \mathbf{x} \Delta \mathbf{x} \rangle / 2\,\Delta t) f]$. Using $\boldsymbol{\nabla} \cdot \mathbf{A} \equiv \sum_i (1/\sqrt{g})(\partial/\partial u^i)(\sqrt{g}\mathbf{A} \cdot \boldsymbol{\nabla} u^i)$ and the Fokker-Planck coefficients in (15), when f is solely a function of a magnetic flux coordinate (i.e., ρ, ψ_* or x), the flux-suface-average of this operator becomes [neglecting $\langle \boldsymbol{\nabla}\rho \cdot \partial \mathbf{e}_\rho/\partial \rho \rangle = \langle \boldsymbol{\nabla}\rho \cdot \partial^2 \mathbf{x}/\partial \rho^2 \rangle \sim \mathcal{O}(\epsilon^2)$]

$$\boxed{\langle \mathcal{D}\{f(\rho)\} \rangle \simeq \frac{1}{V'}\frac{\partial}{\partial \rho}\left(-\,V'\frac{\langle \Delta x \rangle}{\Delta t} f + \frac{\partial}{\partial \rho} V' \frac{\langle (\Delta x)^2 \rangle}{2\,\Delta t} f\right) = \frac{1}{V'}\frac{\partial}{\partial \rho}\left(-\,V'\bar{u}_g f + \frac{\partial}{\partial \rho} V' \bar{D}_\eta f\right).} \tag{18}$$

Next, consider Fourier expansion of the distribution function in poloidal (θ) and toroidal (ζ) angles:

$$f(\psi, \theta, \zeta) = \sum_{m,n} f_{mn}(\psi)\,e^{im\theta - in\zeta} = \sum_m f_{m0}(\psi)\,e^{im\theta} + \sum_{m,n\neq 0} f_{mn}(\psi) e^{im\theta - in\zeta} \equiv f_{\rm a} + f_{\rm na}. \tag{19}$$

The $n = 0$ contributions represent the axisymmetric distribution function f_a that yields the usual neoclassical transport [5,6]. The electron energy transport equation including both neoclassical and axisymmetric paleoclassical effects is obtained from the flux-surface-average of the kinetic energy moment of the axisymmetric part of (17), approximating f in $\mathcal{D}\{f\}$ by a Maxwellian $f_M(\psi)$:

$$\frac{3}{2}\frac{\partial}{\partial t}(n_e T_e) + \frac{\partial}{\partial V}\langle (\mathbf{q}_e^{\rm nc} + \frac{5}{2} T_e \boldsymbol{\Gamma}_e^{\rm nc}) \cdot \boldsymbol{\nabla} V \rangle + \frac{\partial}{\partial V}\langle \mathbf{Q}_e^{\rm pc} \cdot \boldsymbol{\nabla} V \rangle = Q_e, \tag{20}$$

Here, the electron entropy-producing processes are: the neoclassical conductive ($\mathbf{q}_e^{\rm nc}$) and convective $[(5/2)T_e\mathbf{\Gamma}_e^{\rm nc}]$ heat fluxes, the paleoclassical heat flux ($\mathbf{Q}_e^{\rm pc}$) which is induced by $\mathcal{D}\{f_M\}$, and the heating (Q_e) due to collisional effects (joule heating, electron viscosity, and collisions with ions).

Near a $q_* = m/n$ rational surface $f_{\rm na}$ can be put into a form that isolates its poloidal (θ) and helical $[\alpha \equiv \zeta - q_*\theta = \zeta - (m/n)\,\theta]$ angle dependences: $f_{\rm na}(\psi,\theta,\alpha) = \sum_{n\neq 0} e^{-in\alpha} \sum_{\tilde m} f_{m+\tilde m,n}(\psi)\, e^{i\tilde m\theta}$. Further, since near a rational surface the magnetic field can be represented by its helical and magnetic shear components as $\mathbf{B} = \mathbf{B}_h + \mathbf{B}_*$, the parallel-streaming differential operator in (17) becomes [7]: $\mathbf{B}\cdot\boldsymbol{\nabla} f = (\mathbf{B}\cdot\boldsymbol{\nabla}\theta)[\partial f/\partial\theta|_{\psi_*,\alpha} + \ (q-q_*)\,\partial f/\partial\alpha|_{\psi_*,\theta}]$. Thus, near the $q_* \equiv m/n$ rational surface $f \to f(\psi_*,\theta,\alpha,\varepsilon,\mu)$ and applying this parallel-streaming operator to $f_{\rm na}$ yields

$$\mathbf{B}\cdot\boldsymbol{\nabla} f_{\rm na} = (\mathbf{B}\cdot\boldsymbol{\nabla}\theta) \sum_{n\neq 0} e^{-in\alpha} \sum_{\tilde m} e^{i\tilde m\theta} \times i\,[\tilde m - n(q-q_*)]\, f_{m+\tilde m,n}(\psi). \qquad (21)$$

Since the parallel-streaming term $(v_\parallel/B)\,\mathbf{B}\cdot\boldsymbol{\nabla} f_{\rm na}$ is dominant in (17), it causes the Fourier coefficients $f_{m+\tilde m,n}$ to be small unless $\tilde m - n(q-q_*)$ is small. Near the $q_* \equiv m/n$ rational surface $q \simeq q_* + xq'$ and this coefficient becomes $\tilde m - n(q-q_*) \simeq \tilde m - nxq'$. It will be small and lead to the largest $f_{m+\tilde m,n}$ for $\tilde m = 0$ and $|nxq'| << 1$. The resulting "helically resonant" Fourier coefficient (near $q = q_*$) will be $f_*(x) \equiv f_{m,n}(\psi_*)$. Here, the argument has been changed from the poloidal (ψ) to the helical (ψ_*) flux, which is the appropriate flux (radial) label near the given rational surface. Using (3), the criterion $|nxq'| << 1$ is $|x| << \Delta$. Hence, $f_*(x)$ solutions will be highly peaked near the q_* rational surface.

Developing a useful (i.e., one-dimensional) representation for $f_{\rm na}$ near a $q_* = m/n$ rational surface for $n >> 1$ is analogous to the development of ballooning mode theory [8]. The basic issue is: how does one maintain periodicity of the solutions in the poloidal (θ) and helical (α) angles as one moves radially away (i.e., to $x \neq 0$ in a sheared magnetic field structure) from a rational surface composed of helically symmetric field lines. For "flute-like" behavior extending long distances ($|\ell| >> \pi\bar R q$) along large n helical field lines, one assumes q is locally a linear function of x (i.e., $q \simeq q_* + xq'$) and employs the procedure Lee and Van Dam [8b] used to develop a ballooning representation, to obtain [2]

$$\boxed{f_{\rm na} \simeq \sum_{n\neq 0} e^{-in\alpha} \sum_{p=-\infty}^{\infty} \hat f_*(\theta+\lambda+2\pi p)\, e^{inxq'(\theta+\lambda+2\pi p)} \simeq \sum_{n\neq 0} e^{-in\alpha} \int_{-\ell_*}^{\ell_*} \frac{d\ell}{2\pi\bar R q_*} \hat f_*(\ell)\, e^{i\,k_\parallel(x)\,\ell}.} \qquad (22)$$

Here, $\hat f_*(\ell)$ is the Fourier transform [8b] of $f_*(x)$. Note that this $f_{\rm na}$ is a periodic function of both the poloidal (θ) and helical (α) angles. In the last form the discrete sum over p has been converted into a continuous integral in which $\ell \simeq (\theta+\lambda+2\pi p)\bar R q_*$ represents extension of the poloidal angle θ into a field line variable along $\mathbf{B}$. For the sheared ($q' \neq 0$) magnetic field $k_\parallel(x) \equiv nxq'/\bar R q_*$.

The ℓ integration limits in (22) are the half length of a helical field line: $\ell_* = \pi\bar R q_* n$ from (2). These limits also imply the sum over p in (22) only extends from $-n/2$ to $n/2$ — to represent summing over n poloidal transits of the field line. Since $\hat f_*(\ell)$ is usually nearly constant for $|\ell| \leq \ell*$ [2], (22) yields a factor $\sim \ell_*/\pi\bar R q_* = n >> 1$, which produces the multiplier M [see (27), (29)] in the paleoclassical electron heat diffusivity — physically because contributions of n poloidal passes of the rational helical field line are summed to obtain the net response for one poloidal period of the plasma. In the "ballooning representation" the parallel distance ℓ is proportional to the Fourier transform variable $k_x(\ell)$ for the x (radial) variation of $f_*(x)$. Also, note that $k_\parallel(x)\,\ell = k_x(\ell)\,x$, where $k_x(\ell) \equiv nq'(\ell/\bar R q_*) = nq'(\theta+\lambda+2\pi p)$, which is the usual [8] $k_x = k_\theta s\,\theta$ with $k_\theta \equiv nq/\rho a$ and $s \equiv \rho\, q'/q$.

Satisfying the criterion $k_x^2(\ell)\bar\delta_e^2 < 1$ (or $|x|^2 > \bar\delta_e^2$) for diffusing field lines requires $|\ell| < \ell_\delta \equiv \bar R q_*/(n\bar\delta_e q')$. Requiring ℓ_δ to be longer than the helical field line length $\ell_* \equiv \pi\bar R q_* n$ in (2) yields a maximum n and length of field lines that are diffusing over their entire length:

$$\boxed{n_{\rm max} \equiv 1/(\pi\bar\delta_e q')^{1/2}, \quad \text{maximum } n; \quad \ell_{\rm max} \equiv \pi\bar R q_* n_{\rm max}, \quad \text{maximum diffusing length.}} \qquad (23)$$

Solutions of the nonaxisymmetric MDKE in (17) are sought [2] using an ordering scheme in which the transit frequency $\omega_t \sim v_\parallel(\mathbf{B}\cdot\boldsymbol{\nabla}\theta)/B \sim v_{Te}/R_0 q$ is larger than all other frequencies. To lowest order $\partial f_{*0}/\partial\theta|_{\psi_*,\alpha} = 0$; hence f_{*0} must be independent of the poloidal angle θ. The next order kinetic equation includes parallel streaming along ψ_* surfaces and collisions. Bounce-averaging it annihilates a $\partial f_{*1}/\partial\theta$ term to yield $\omega_t\,(q-q_*)\,\partial f_{*0}/\partial\alpha|_{\psi_*} = \langle\mathcal{C}\{f_{*0}\}\rangle_\theta$, whose solution [2] (for $q-q_*\neq 0$, $\lambda_e > \ell_*$) is a Maxwellian constant along ψ_* surfaces (i.e., closed field lines with the pitch of rational field lines):

$$f_{*0} = f_M(\psi_*,\varepsilon,t) = n_e(\psi_*,t)\left(\frac{m_e}{2\pi T_e(\psi_*,t)}\right)^{3/2} e^{-\mathcal{E}/T_e(\psi_*,t)}. \tag{24}$$

When $\lambda_e < \ell_*$, finite parallel electron heat conduction limits the electron temperature equilibration to the region $|\ell| \lesssim \lambda_e$. Thus, the f_{*0} in (24) is applicable for $|\ell| \leq \ell_{fM*} \equiv \min\{\ell_*,\,\lambda_e\}$.

The paleoclassical radial electron heat transport induced by the diffusion of the nonaxisymmetric, helical magnetic flux (field lines) near $q_* = m/n$ is obtained by taking the helical average of the energy moment of the $\mathcal{D}\{f_{na}\}$ term in (17) and using the $f_{\rm na}$ representation in (22) with f_{*0} from (24):

$$-\langle\boldsymbol{\nabla\cdot}\mathbf{Q}^{\rm pc}_{e*}\rangle \equiv \int d^3v\,\frac{mv^2}{2}\int_{-\pi}^{\pi}\frac{d\alpha}{2\pi}\,e^{in\alpha}\,\langle\mathcal{D}\{f_{\rm na}\}\rangle \simeq \langle\mathcal{D}\{\int_{-L}^{L}\frac{d\ell}{2\pi\bar{R}q_*}\,e^{ik_\parallel(x)\ell}\,\frac{3}{2}n_eT_e\}\rangle. \tag{25}$$

The Fokker-Planck coefficients in (15) are only applicable for $x^2 > \bar{\delta}_e^2$. Thus, the maximum half length of helical field lines is the $\ell_{\rm max}$ in (23). Hence, the limits of the ℓ integration in (25) are given by $\pm L$, in which L is the minimum of the lengths over which f_{0*} is Maxwellian (ℓ_{fM*}) and field lines are diffusing ($\ell_{\rm max}$):

$$\boxed{L \equiv \min\{\,\ell_{\rm max},\,\lambda_e,\,\ell_{n^\circ}\},\quad \text{equilibration length.}} \tag{26}$$

Since very near a rational surface $|k_\parallel(x)L| << 1$, one can set $e^{ik_\parallel\ell}\simeq 1$ [for $|x| << \bar{R}q_*/nq'L \leq 1/(\pi n^2q') = \Delta/\pi n$], perform the ℓ integration in (25), and obtain for the total (advective plus diffusive) paleoclassical electron heat flux near the q_* flux surface

$$\langle\mathbf{Q}^{\rm pc}_{e*}\boldsymbol{\cdot\nabla}V\rangle = V'M\,\bar{u}_g\,\frac{3}{2}n_eT_e - \frac{\partial}{\partial\rho}\left(V'M\,\bar{D}_\eta\,\frac{3}{2}n_eT_e\right),\quad \boxed{M\equiv\frac{L}{\pi\bar{R}\,q_*},\quad\text{helical multiplier.}} \tag{27}$$

Considering radial profile effects [2], the net helical paleoclassical electron heat flux (after summing over all possible rational surfaces) varies little with radius. Thus, q_* can be replaced by $q(\rho)$ in (27). However, in the vicinity of low order rational surfaces where $q(\rho_*^\circ)\equiv m^\circ/n^\circ$ with $n^\circ = 1,\,2$ it also varies little with radius, but is smaller because L and hence M are smaller there [up to a distance from ρ_*° of order $\delta x^\circ \equiv \delta x(n^\circ)$ or, around a minimum in q about q°, $\delta x^\circ_{\rm min}\equiv\delta x^\circ_{\rm min}(n^\circ)$].

5. Paleoclassical Radial Electron Thermal Transport

The total paleoclassical electron heat flux ($\mathbf{Q}^{\rm pc}_e$) in the rest frame of the toroidal flux surfaces (i.e., removing u_g contributions) for a stationary poloidal magnetic field $\mathbf{B}_p$ is the sum of axisymmetric ($M\to 1$ in (27) [2]) and the nonaxisymmetric (quasi-helical symmetric) transport flux in (27):

$$\boxed{\langle\mathbf{Q}^{\rm pc}_{e}\boldsymbol{\cdot\nabla}V\rangle = -\frac{\partial}{\partial\rho}\left(V'(M+1)\,\bar{D}_\eta\,\frac{3}{2}n_eT_e\right),\quad\text{total paleoclassical electron heat flux.}} \tag{28}$$

The diffusive part of this heat flux indicates a paleoclassical electron heat diffusivity of

$$\boxed{\chi_e^{\rm pc}\equiv\frac{3}{2}(M+1)D_\eta\simeq\frac{3}{2}M\frac{\eta_\parallel^{\rm nc}}{\mu_0} = \frac{3}{2}M\bar{\nu}_e\delta_e^2,\quad\text{paleoclassical electron heat diffusvity }\chi_e.} \tag{29}$$

Comparing this $\chi_e^{\rm pc}$ with the magnetic flux diffusivity D_η in (12), one sees that T_e diffuses a factor of order M faster than ψ does — because T_e is equilibrated over the long length L of helical field lines, compared to the poloidal periodicity length $\pi\bar{R}q$. Thus, the paleoclassical model may be able to explain the experimentally observed T_e "profile resiliency" [9], which was originally called "profile consistency" [10] and has often been linked to the q profile.

There are two collisionality regimes of paleoclassical electron heat diffusion. For most toroidal plasmas the collision length λ_e is longer than $\ell_{\rm max}$; then, $M = n_{\rm max} >> 1$ yields

$$\chi_e^{\rm pc}\simeq\frac{3}{2}\left(\frac{1}{\pi\bar{\delta}_e|q'|}\right)^{1/2}\frac{\eta_\parallel^{\rm nc}}{\mu_0},\quad \lambda_e > \ell_{\rm max}\equiv\pi\bar{R}q\,n_{\rm max},\quad\text{collisionless paleoclassical regime.} \tag{30}$$

As an example of the magnitude of this "collisionless" electron heat diffusivity, for a typical ohmically-heated TFTR plasma [10b] with $T_e \simeq 1.2$ keV, $n_e \simeq 3 \times 10^{19}$ m^{-3}, $Z_{\rm eff} \simeq 2$, $R_0 \simeq 2.55$ m, $q \simeq 1.6$, and $a/q' \simeq 0.4$ m at $r/a \simeq 0.4/0.8 = 0.5$, one obtains $\eta_0/\mu_0 \simeq 0.067$ m^2/s, $\eta_\parallel^{\rm nc}/\eta_0 \simeq 2.2$ (neglecting ν_{*e} effects, which would make the results a factor of 0.6 smaller), $\delta_e \simeq 10^{-3}$ m, $n_{\rm max} \simeq 11$, and $\lambda_e \simeq 300$ m $> \pi R_0 q\, n_{\rm max} \simeq 140$ m, so that $L \simeq \pi R_0 q\, n_{\rm max}$, $M = n_{\rm max} \simeq 11$, and the estimated $\chi_e^{\rm pc}$ is 2.5 m^2/s $\sim \chi_e^{\rm exp}$. Since this $\chi_e^{\rm pc} \propto n_e^{1/4}/(q'T_e^3)^{1/2} \propto T_e(r)^{-3/2}$, the radial dependence of $\chi_e^{\rm pc}$ increases strongly as T_e decreases with increasing r, in qualitative agreement with inferences from experiments.

In high density, more collisional plasmas where $L = \lambda_e$, $M = \lambda_e/(\pi\bar{R}q) >> 1$ yields

$$\chi_e^{\rm pc} \simeq \frac{3}{2}\frac{\eta_\parallel^{\rm nc}}{\eta_0}\frac{v_{Te}}{\pi\bar{R}q}\frac{c^2}{\omega_p^2}, \quad \pi\bar{R}q < \lambda_e < \pi\bar{R}q\, n_{\rm max}, \quad \text{collisional paleoclassical regime.} \qquad (31)$$

In typical high density toroidal plasmas $Z_{\rm eff} \simeq 1$ and $\nu_{*e} >> 1$; for such plasmas $(3/2)(\eta_\parallel^{\rm nc}/\eta_0) \simeq (3/2)(0.51)$. Thus, the collisional $\chi_e^{\rm pc}$ implies an overall electron energy confinement time $\tau_{Ee} \sim a^2/4\chi_e^{\rm pc} \simeq 0.27\,(n_e/10^{20}{\rm m}^{-3})\,a^2 R_0 q\,(T_e/500{\rm eV})^{1/2}$ s, which approximately reproduces (in both magnitude and scaling for the highest performance pellet-fueled Alactor C plasmas [11] that had $a = 0.165$ m and $R_0 = 0.64$ m) the "neo-Alcator scaling" deduced empirically primarily from ohmically-heated tokamak plasma data in the 1970s and early 1980s [12]: $\tau_E^A \sim 0.07\, n_e a R_0^2 q_a$.

In the closed field line region near the magnetic separatrix region where q and q' become very large, one can have $\lambda_e \lesssim \pi\bar{R}q$. In this region, the paleoclassical electron heat diffusivity is [2]

$$\chi_{\rm es}^{\rm pc} \simeq \frac{3}{2}\frac{\eta_\parallel^{\rm Sp}}{\mu_0}\left(1 + \frac{\eta_\parallel^{\rm nc}}{\eta_\parallel^{\rm Sp}}\frac{\lambda_e}{\pi\bar{R}q}\right), \quad \pi R < \lambda_e < \pi\bar{R}q\ \max\{1, n_{\rm max}\}, \quad \text{near-separatrix region.} \qquad (32)$$

For $\lambda_e/\pi\bar{R}q > (\eta_\parallel^{\rm Sp}/\eta_\parallel^{\rm nc}) \sim 1$, this yields the collisional $\chi_e^{\rm pc}$ in (31). In the opposite limit one obtains a smaller $\chi_{\rm es}^{\rm pc} \simeq (3/2)(\eta_\parallel^{\rm Sp}/\mu_0) \simeq [100/T_e({\rm eV})]^{3/2}$ m^2/s for $Z_{\rm eff} \simeq 1$. There are some experimental indications in DIII-D [13] that within about 2 cm of the separatrix $\boldsymbol{\nabla} T_e$ is significantly larger, which implies $\chi_e^{\rm exp}$ is reduced there; the maximum $\boldsymbol{\nabla} T_e$ apparently occurs at about the $\rho^s \sim 0.95$–0.97 predicted by the paleoclassical model: $q(\rho^s) \sim (\lambda_e/\pi\bar{R})\,(\eta_\parallel^{\rm nc}/\eta_\parallel^{\rm Sp}) \sim 5$–$10$.

The paleoclassical model applies to all types of axisymmetric toroidal plasmas in regions where $\epsilon^2, B_p^2/B_t^2 << 1$. For $R_0 \simeq 1$ m STs with $T_e \sim 1$ keV and $n_e \sim 3 \times 10^{19}$ m^{-3}, the prediction at $r/a \sim 0.5$ is $\chi_e^{\rm pc} \sim 5$–10 m^2/s, which, in reasonable agreement with experimental results [14], is large because for STs $q' << 1$ is small and $\eta_\parallel^{\rm nc}/\eta_0 \gtrsim 3$ is large in the plasma confinement region ($r/a \sim 0.5$). For quiescent RFP plasmas in the Madison Symmetric Torus (MST) Pulsed Poloidal Current Drive (PPCD) experiments [15], at $r/a \sim 0.3$–0.5 one obtains $\chi_e^{\rm pc} \sim 5$–10 m^2/s (large because $q < 0.2$ and $|q'| \lesssim 0.2$ are small), which is close to the effective χ_e's inferred from global ($\bar{\chi}_e^{\rm exp} \equiv a^2/4\tau_E \sim 7.5$ m^2/s [14a]) and local ($\chi_e^{\rm exp} \sim 10$–30 m^2/s [14b]) measurements. In quasi-symmetric stellarator plasmas there would be no paleoclassical transport if there is no flux-surface-average parallel current $\langle \mathbf{J} \cdot \mathbf{B} \rangle$; however, net flux-surface-average parallel currents in a stellarator would induce a $\chi_e^{\rm pc}$.

As indicated by (26), (27), and (29), the predicted $\chi_e^{\rm pc}$ is much smaller for the "short" helical field lines [see (2) and discussion thereafter] in the vicinity of low order rational surfaces with $q^\circ = m^\circ/n^\circ$: $\chi_e^{\rm pc} \sim (3/2)(n^\circ + 1)\,\eta_\parallel^{\rm nc}/\mu_0$. The estimated width of the low $\chi_e^{\rm pc}$ "transport barriers" is $2\,\delta x^\circ \equiv 2\,\delta x(n^\circ)$ for $q' \neq 0$, or, if q is near a minimum at the rational surface, $2\,\delta x_{\rm min}^\circ \equiv 2\,\delta x_{\rm min}(n^\circ)$. These barrier widths can be compared to some key tokamak results. First, as experiments in RTP [16] slowly moved highly localized electron cyclotron heating (ECH) radially outward, a "stair-step" reduction in the central T_e was observed as the ECH passed low order rational surfaces. It was thus inferred [16] that transport barriers existed with about a factor of 10 reduction in χ_e over relative (to a) barrier widths of order 0.04 (0.1 for $q = 1/1$). For RTP parameters $2\,\delta x^\circ \sim 0.06$–$0.12$ (0.17 for $q = 1/1$). Next, jumps in T_e (over radial widths ~ 0.2) have been observed in evolving DIII-D L-mode plasmas [17] as an off-axis minimum in $q(\rho, t)$ passes through low order rational surfaces. For the DIII-D parameters, $2\,\delta x_{\rm min}^\circ$ gives a similar estimate (~ 0.3) for the transport barrier width. Finally, a strong $\boldsymbol{\nabla} T_e$ and internal transport barrier were created in the pioneering JT-60U experiments [18]; the experimentally inferred barrier width was ~ 0.11, close to the paleoclassical prediction of 0.14 near an assumed $q_{\rm min} = 3$.

Note that the paleoclassical electron heat flux in (28) is not in a normal (diffusive) Fourier heat flux law form (i.e., $\mathbf{q}_e = -\kappa_e \boldsymbol{\nabla} T_e \equiv -n_e \chi_e \boldsymbol{\nabla} T_e$). Rather, it can be written in general as:

$$\langle \mathbf{Q}_e^{\rm pc} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle = -\, V' n_e \chi_e^{\rm pc} \frac{\partial T_e}{\partial \rho} - \langle \mathbf{q}_e^{\rm pi} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle; \quad \langle \mathbf{q}_e^{\rm pi} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle \equiv T_e \frac{\partial}{\partial \rho} \left(V' n_e \chi_e^{\rm pc} \right), \quad \text{heat pinch.} \qquad (33)$$

The electron heat pinch heat flux $\langle \mathbf{q}_e^{\rm pi} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle$ is usually positive (inward) and increases with ρ, in qualitative agreement with JET experimental results [9]. Also, a heat pinch effect implies [9] a "power balance" χ_e ($\chi_e^{\rm pb}$), which is defined as the net electron heat flux divided by $-n_e \boldsymbol{\nabla} T_e$, less than $\chi_e^{\rm pc}$. Alternatively, in qualitative agreement with some tokamak experimental data [19], $\langle \mathbf{Q}_e^{\rm pc} \boldsymbol{\cdot} \boldsymbol{\nabla} V \rangle$ can be written in the form of a heat flux proportional to the degree to which $-\boldsymbol{\nabla} \ln T_e \equiv 1/L_{Te}$ exceeds a critical magnitude $\simeq \partial \ln(V' n_e \chi_e^{\rm pc})/\partial r$. If this paleoclassical critical gradient is approximately constant over the confinement region, it would agree with the experimental observations that $\boldsymbol{\nabla} \ln T_e$ in the "confinement region" ($0.3 \lesssim \rho \lesssim 0.8$) is nearly constant [20] and usually close to its critical value.

6. Summary and Discussion

Equations (28)–(33) are the main paleoclassical results. As indicated in the preceding section, they provide interpretations for many features of "anomalous" electron heat transport. Because the results were obtained by a large n asymptotic analysis and the characteristic lengths in L have been only approximately determined, M (and hence all M-dependent results) should be interpreted as scaling results. More detailed studies could introduce numerical coefficients of order unity in L, M, and $\chi_e^{\rm pc}$.

Paleoclassical electron heat transport is based on the primitive Coulomb collision processes of parallel electron heat conduction and plasma resistivity leading to magnetic field diffusion. Thus, it is an "irreducible, ubiquitous" transport process, just as classical and neoclassical transport [5,6] are.

Plasma turbulence induced by microinstabilities could induce additional transport. Since $D_\eta \propto \eta \propto 1/T_e^{3/2}$, the "collisionless" paleoclassical electron heat diffusion coefficient decreases with increasing electron temperature; for $n_{\rm max} \lesssim 10$ and $\eta_\parallel^{\rm nc}/\eta_0 \lesssim 2$, $\chi_e^{\rm pc}$ becomes less than 1 m^2/s for $T_e \gtrsim 2$ keV. Thus, if microturbulence-induced transport due to trapped-electron or electron-temperature-gradient instabilities induce $\chi_e^{\mu\rm turb} \gtrsim 1$ m^2/s, they could become dominant transport mechanisms for electron temperatures above a few keV. Such microturbulence would apparently not directly affect paleoclassical processes since it usually does not affect the parallel Ohm's law much [21] and the parallel correlation length for magnetic microturbulence usually exceeds the relevant paleoclassical length L.

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