Effects of Magnetic Field, Sheared Flow and Ablative Velocity on the Rayleigh - Taylor Instability

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Abstract. It is found that magnetic field has a stabilization effect whereas the sheared flow has a destabilization effect on the RT instability in the presence of sharp interface. RT instability only occurs in the long wave region and can be completely suppressed if the stabilizing effect of magnetic field dominates. The RT instability increases with wave number and flow shear, and acts much like a Kelvin-Helmholtz instability when destabilizing effect of sheared flow dominates. It is shown that both of ablation velocity and magnetic filed have stabilization effect on RT instability in the presence of continued interface. The stabilization effect of magnetic field takes place for whole waveband and becomes more significant for the short wavelength. The RT instability can be completely suppressed by the cooperated effect of magnetic field and ablation velocity so that the ICF target shell may be unnecessary to be accelerated to very high speed. The growth rate decreases as the density scale length increases. The stabilization effect of magnetic field is more significant for the short density scale length.

1. Introduction

The Rayleigh-Taylor (RT) instability occurs in inertial confinement fusion (ICF) and core-collapse supernova when a heavy fluid is accelerated by a light fluid [1-5]. This instability is annoying because it obstructs the realization of ICF. Hence, it is important to seek physical mechanisms that can suppress such instability. In almost all the treatment, there is no relative velocity between heavy fluid and the light fluid. This is a reasonable assumption but not fully justified. In realistic situation, compression will inevitably give rise to an inhomogeneity along the direction perpendicular to the interface between the heavy and light fluids. This inhomogeneity can induce an equilibrium flow parallel to the interface. Thus, it is interesting to study the effect of the equilibrium flow and its shear on the RT instability. On the other hand, the growth rate of the instability is commonly written as $\gamma = \alpha\sqrt{kg/(1+kL_0)} - \beta k V_a$, where k is the wave number, g is the acceleration, V_a is the ablation velocity, and L_0 is the density scale length at the ablation surface. Unfortunately, the coefficients α and β are not universal constants and fitting their magnitudes in different numerical simulations has shown different values. Therefore, α and β must be functions of some parameters such as equilibrium parameters and the perturbed wavelength.

Recently, the self-generated magnetic fields in laser-produced plasmas have attracted much theoretical and experimental attention for their roles in the design of ICF [6]. The self-generated magnetic fields can be produced by processes such as filamentation, resonance absorption, thermal, and Weibel instabilities [7]. Simulations on the interaction of an ultraintense laser pulse with an overdense plasma target have shown extremely high magnetic field strength up to 10^3 MG. These extremely strong magnetic fields may have interesting effects on the physics of ICF, including the RT instability.

In this paper, we consider the effects of magnetic filed, shear flow, and ablation velocity on the linear growth of RT instability. The starting point of our work is the ideal magnetohydrodynamic (MHD) equations in the SI system as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{J} \times \mathbf{B} - \rho \mathbf{g} \tag{2}$$

$$\mathbf{J} = \mu^{-1} \nabla \times \mathbf{B} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{4}$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0 \tag{5}$$

where ρ is the mass density, *d/dt* is the convective time derivative, *u* the fluid velocity, *p* the plasma pressure, **J** the current density, and **B** and **E** are the magnetic and electric fields, respectively.

We are going to investigate the perturbation of the equilibrium and assume all quantities are of the form $f = f_0 + f_1 \exp(iky - i\omega t)$ where the subscripts '0' and '1' denotes respectively the equilibrium and small perturbation.

2. The effects of magnetic filed and sheared flow on RT instability

It is assumed that the inertial state of fluid is described as $\mathbf{u}_0 = u_{0x}(z)\mathbf{e}_x + u_{0y}(z)\mathbf{e}_y$, $\rho_0 = \rho_0(z)$, and $B_0 = B_{0x}(z)\mathbf{e}_x + B_{0y}(z)\mathbf{e}_y$ so that $\nabla \cdot \mathbf{B}_0 = 0$, and $\nabla \cdot \mathbf{u}_0 = 0$ are automatically satisfied. Assume the perturbation of velocity and magnetic filed respectively as $\mathbf{u}_1 = \mathbf{e}_x u_{1x}(z) + \mathbf{e}_y u_{1y}(z) + \mathbf{e}_z u_{1z}(z)$, $\mathbf{B}_1 = \mathbf{e}_x B_{1x}(z) + \mathbf{e}_y B_{1y}(z) + \mathbf{e}_z B_{1z}(z)$.

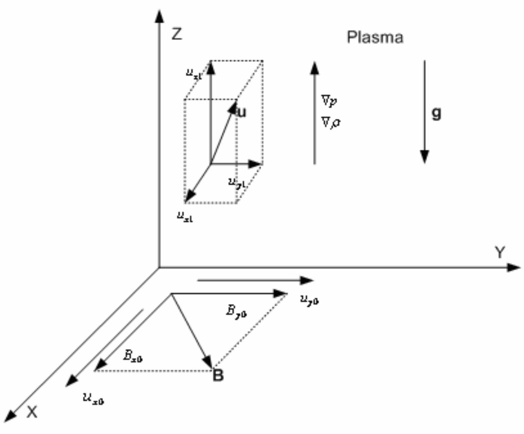

FIG. 1 The scheme of velocity, gravity, density , pressure, shear flow and magnetic field.

The linearized version of equations (1) - (5) for the perturbations can be combined as

$$\frac{d}{dz}\left\{\rho_0[(\omega - ku_{y0})^2 - k^2 u_A^2]\frac{d}{dz}\left(\frac{u_{1z}}{\omega - ku_{y0}}\right)\right\} = k^2\left\{\rho_0[(\omega - ku_{y0})^2 - k^2 u_A^2] + \frac{d\rho_0}{dz} g\right\}\left(\frac{u_{1z}}{\omega - ku_{y0}}\right)$$

(6)

where $u_A = B_{0y}/\sqrt{\mu\rho_0}$ is the Alfvén velocity. Let $\omega_* = \omega - ku_{y0}$, $u_z(z) = u_{1z}(z)\omega/\omega_*$ and $\rho_*\omega^2 = \rho_0(\omega_*^2 - k^2u_A^2)$, then the above equation becomes:

$$\frac{d}{dz}\left(\rho_*\frac{du_z}{dz}\right) = k^2\left(\rho_* + \frac{d\rho_0}{dz}\frac{g}{\omega^2}\right)u_z \tag{7}$$

If a sharp interface exists at $z = 0$, the equilibrium mass density, magnetic field and flow have a jump:

$$\rho_0(z) = \rho_- + (\rho_+ - \rho_-)h(z), \quad \rho_+ > \rho_-. \tag{8}$$

$$B_{y0}(z) = B_{y-} + (B_{y+} - B_{y-})h(z) \tag{9}$$

$$u_{y0}(z) = u_{y-} + (u_{y+} - u_{y-})h(z) \tag{10}$$

where $h(z)$ is the Heaviside function and defined as $h(z-\zeta) = \int_{-\infty}^{+\infty}\delta(x-\zeta)dx$. Hence, if all of ρ_-, ρ_+, u_{y-}, u_{y+}, B_{y-} and B_{y+} are constants, Eq. (7) can be simplified in either side of $z = 0$ as $d^2u_z/dz^2 = k^2u_z$. This equation has a neat solution:

$$u_z(z) = u_{z0}\exp(kz) + u_{z0}[\exp(-kz) - \exp(kz)]h(z) \tag{11}$$

By using Eqs. (8-11), integration of Eq. (7) over the sharp interface from 0_- to 0_+ gives

$$\rho_+(\omega - ku_{y+})^2 + \rho_-(\omega - ku_{y-})^2 = -kg(\rho_+ - \rho_-) + k^2(B_{y-}^2 + B_{y+}^2)/\mu \tag{12}$$

whose solution is complex and can be separated into two parts $\omega = \omega_r + i\gamma$, where $\omega_r = k(\rho_-u_{y-} + \rho_+u_{y+})/(\rho_- + \rho_+)$ stands for the pure oscillating frequency which results in a Doppler shift. Meanwhile, the imaginary part

$$\gamma = [gkA_T + k^2\delta_u^2(1 - A_T^2) - k^2u_{rA}^2]^{1/2} \tag{13}$$

is the growth rate of RT instability, where $A_T = (\rho_- - \rho_+)/(\rho_- + \rho_+)$ is the Atwood number, $\delta_u = |u_{y+} - u_{y-}|/2$ denotes the reduced difference of equilibrium flow across the sharp interface and $\mathrm{v}_{ra} = [(B_{y-}^2 + B_{y+}^2)/(\mu(\rho_- + \rho_+))]^{1/2}$ is the reduced Alfvén speed.

At first, we would like to discuss the effect of magnetic field on RT instability. It can be obtained from Eq. (13) that

$$\partial\gamma/\partial\mathrm{v}_{ra} = -k^2\mathrm{v}_{ra}/\gamma < 0 \tag{14}$$

The growth rate is a monotonic function that decreases as v_{ra} increases, which is confirmed by Fig. 2. In other words, RT instability can be suppressed by magnetic field, and even quenched when the magnetic field is strong enough that the v_{ra} is larger than the threshold

$v_T = [(1-A_T^2)\delta_u^2 + gA_T/k]^{1/2}$. For perturbations with large wave number or short wavelength, this threshold is relatively small and the RT instability can be quenched easily. This stabilizing effect is provided by the Lorentz force in the z-direction against the gravity, which is produced by the perturbed current J_{x1} and the equilibrium magnetic field B_{y0}.

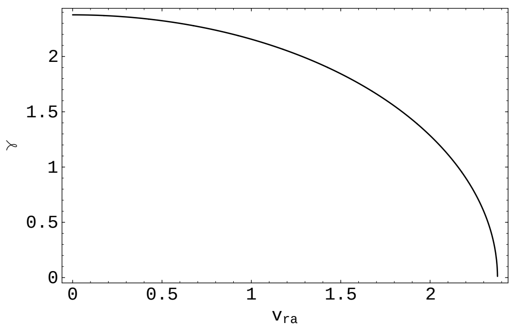

FIG. 2 Growth rate via reduced Alfven speed v_{ra} *for* $k = 1.0$, $A_T = 0.5$ *and* $\delta_u = 1.0$.

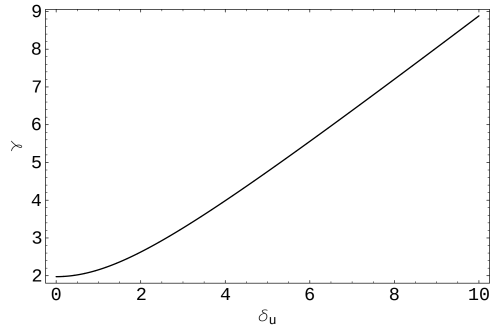

FIG. 3 Growth rate via reduced shear δ_u *for* $k = 1.0$, $A_T = 0.5$ *and* $v_{ra} = 1.0$

Secondly, we would like to discuss the effect of sheared flow on RT instability. Similarly, it can be obtained from Eq. (13) that

$$\partial\gamma/\partial\delta_u = k^2\delta_u(1-A_T^2)/\gamma > 0 \tag{15}$$

Hence, the growth rate increases monotonically with sheared flow. As shown in Fig. 3, the increment in growth rate goes into the linear stage for sufficiently large shear. Therefore, sheared flow reinforces the RT instability and is the governing drive when $k(1-A_T^2)\delta_u^2/A_T >> g$, which can be achieved when one of following conditions met. 1) Wave number is very large. 2) Flow shear is adequately strong. 3) Atwood number is small. When destabilizing effect of sheared flow dominates over that of gravity, the first term in the expression of growth rate γ can be ignored and the RT instability acts much like a Kelvin-Helmholtz instability.

Thirdly, we would like to discuss the effect of density gradient on RT instability. It is easy to find out from Eq. (13) that

$$\partial\gamma/\partial A_T = k(g - 2kA_T\delta_u^2)/(2\gamma) \tag{16}$$

which implies that the growth rate increases monotonically with A_T if $0 < A_T < g/(2k\delta_u^2)$. Therefore, growth rate γ reaches its maximum $\gamma_{m1} = (gk - k^2 v_{ra}^2)^{1/2}$ at $A_T = 1$ if $2kA_T\delta_u^2 < g$. Otherwise, it achieves a maximum $\gamma_{m2} = [g^2/(4\delta_u^2) + k^2(\delta_u^2 - v_{ra}^2)]^{1/2}$ at $A_T = g/(2k\delta_u^2)$ if $2k\delta_u^2 > g$. The dependence of growth rate γ on A_T is presented in Fig. 4 where v_{ra} is set fixed to 1.0. The solid line with $k = 1.0$ and $\delta_u = 1.0$ is the only curve increasing monotonically because the condition $g > 2k\delta_u^2$ is satisfied. The dash line with $k = 1.0$ and $\delta_u = 3.0$ and dot line with $k = 2.0$ and $\delta_u = 3.0$ are more unstable than the solid line due to the destabilizing effect of shear flow. Growth rate rises with A_T if

$2kA_T\delta_u^2 < g$. However, the growth rate is mainly driven by the shear flow if $2kA_T\delta_u^2 > g$, and will increase in the beginning and fall down as A_T increases.

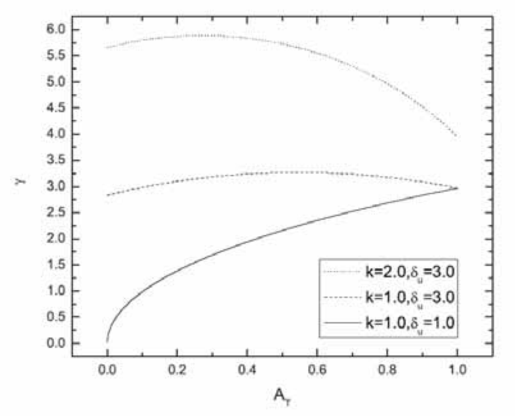

FIG. 4 Growth rate via the Atwood number A_T *for* $v_{ra} = 1.0$.

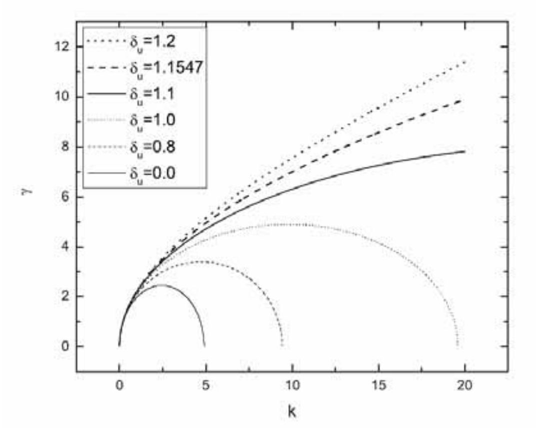

FIG. 5 Growth rate against wave number as $A_T = 0.5$ *and* $v_{ra} = 1.0$.

Finally, we would discuss the dependence of growth rate on wave number. From the partial derivative of the growth rate, it is easy to obtain

$$\partial\gamma / \partial k = [gA_T / 2 + k(\delta_u^2(1 - A_T^2) - v_{ra}^2)] / \gamma \tag{17}$$

We can find out that the parametric curve described by $\delta_u^2(1 - A_T^2) = v_{ra}^2$ divides the whole parametric space into two parts: one is the monotonically increasing part where $\Delta \equiv v_{ra}^2 - \delta_u^2(1 - A_T^2) < 0$, the destabilizing effect of sheared flow dominates, and growth rate increases with k and δ_u ; the other is the nonmonotonic part where $\Delta > 0$, the stabilizing effect of magnetic field dominates, and the growth rate approaches its maximum $\gamma_{\max} = gA_T / (2\Delta^{1/2})$ at $k_m \equiv gA_T / (2\Delta)$ and then falls down so that no instabilities occur in the short wave region when $k > gA_T / \Delta = 2k_m$. As an example, the relationship between γ and k is shown in Fig. 5 where $A_T = 0.5$ and $v_{ra} = 1.0$. Solid line ($\delta_u = 0.0$), dash line ($\delta_u = 0.8$), and dot line ($\delta_u = 1.0$) satisfy $\Delta > 0$ so that magnetic field dominates. Thick line ($\delta_u = 1.1$), thick dash line ($\delta_u = 1.1547$) and thick dot line ($\delta_u = 1.2$) satisfy $\Delta < 0$ so that flow shear dominates. The thick dash line with $\Delta = 0$ corresponds to the classical RT instability that ignores effects of magnetic and sheared flow, which is equivalent to the case where magnetic stabilization is balanced by shear flow.

3. The effects of magnetic filed and ablation velocity on RT instability

For simplicity, suppose that the plasma motion is incompressible and with a ablation velocity across the ablation front, namely $\nabla \cdot \mathbf{u} = 0$ and $\mathbf{u}_0 = u_{0z}(z)\mathbf{e}_z$. Consider that there is a magnetic field parallel to the ablation front, namely $\mathbf{B}_0 = B_{0y}\mathbf{e}_y$, where B_{0y} is a constant. Assume the perturbation of velocity and magnetic filed respectively as $\mathbf{u}_1 = u_{1z}\mathbf{e}_z$, and $\mathbf{B}_1 = B_{1z}\mathbf{e}_z$, where B_{1z} and u_{1z} are constants. The scheme of quantities is drawn in Fig. 6.

The linearized version of Equations (1) - (5) for the perturbations can be combined as

$$\begin{bmatrix} -i\omega + u_0' & \rho_0' \\ (u_0'u_0 + g)/\rho_0 & -i\omega + u_0' + ik^2u_A^2/\omega \end{bmatrix} \begin{bmatrix} \rho_1 \\ u_{1z} \end{bmatrix} = 0 \tag{18}$$

Let $\omega = i\gamma$, it is easy to obtain the dispersion relation as follows

$$\gamma(\gamma + u_0')^2 + k^2u_A^2(\gamma + u_0') - (u_0'u_0 + g)s^{-1}\gamma = 0 \tag{19}$$

where $s = (\rho_0'/\rho_0)^{-1}$ is the density scale length. If without magnetic filed, namely $u_A^2 = 0$, Eq. (19) is reduced to $\gamma_0 = \sqrt{(u_0'u_0 + g)/s} - u_0'$. Obviously, the growth rate decreases when the ablation velocity increases since we have $\partial\gamma_0/\partial u_0 = u_0'/2\sqrt{s(u_0'u_0 + g)} < 0$ for $u_0' < 0$.

It is straightforward to get a real solution from Eq. (19) as

$$\gamma = -\frac{2}{3}u_0' - \frac{2^{1/3}a}{3s(b + \sqrt{4a^3 + b^2})^{1/3}} + \frac{1}{3\times 2^{1/3}s}(b + \sqrt{4a^3 + b^2})^{1/3} \tag{20}$$

where $a = 3gs - u_0'^2 s^2 - 3u_0u_0's + 3k^2u_A^2s^2$, $b = -18u_0'gs^2 + 2u_0'^3 s^3 - 18u_0u_0'^2 s^2 - 9u_0'k^2u_A^2s^3$
In order to observe the dependence of growth rate on the ablation velocity and magnetic field, we take the partial derivative of growth rate with respect to u_0

$$\frac{\partial\gamma}{\partial u_0} = \frac{u_0'}{s[2\gamma^2(\gamma + u_0') - k^2u_A^2u_0']} < 0 \quad \text{if} \quad u_0' < 0 \quad \text{and} \quad \gamma > -u_0' \tag{21}$$

Hence, the growth rate decreases as the ablation velocity increases. It is shown in Fig. 7 that the RT instability can be completely suppressed by the cooperated effect of ablation velocity and magnetic field. Solid line is for $b = k^2V_A^2 = 0$, dash line for $b = k^2V_A^2 = 2$, dash-dotted line for $b = k^2V_A^2 = 6$ and dotted line for $b = k^2V_A^2 = 10$.

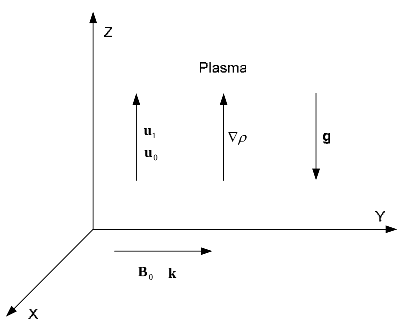

FIG. 6 The scheme of gravity, magnetic field, ablation velocity and density.

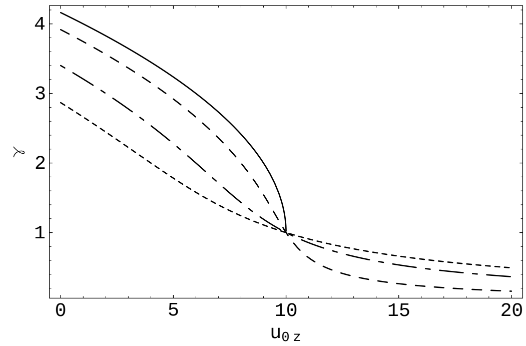

FIG 7 Growth rate against ablation speed for $u'_{0z} = -1$, $s = 1$ *and* $g = 10$*:*

The partial derivative of growth rate with respect to *k* is

$$\frac{\partial\gamma}{\partial k}=\frac{-2\gamma(\gamma+u_0')ku_A^2}{2\gamma^2(\gamma+u_0')-k^2u_A^2u_0'}<0 \quad \text{if} \quad u_0'<0 \quad \text{and} \quad \gamma>-u_0' \tag{22}$$

Therefore, the growth rate decreases as the wave number increases. In other words, the stabilization effect becomes more significant for the short wavelength. We can observe that the magnetic field has a stabilization effect on the RT instability for any k value in Fig. 8 where Solid line is for $V_{ra}=1$, dash line for $V_{ra}=2$, dash-dotted line for $V_{ra}=3$ and dotted line for $V_{ra}=4$.

The partial derivative of growth rate with respect to s is

$$\frac{\partial\gamma}{\partial k}=\frac{-(g+u_0'u_0)\gamma^2/s^2}{2\gamma^2(\gamma+u_0')-k^2u_A^2u_0'}<0 \quad \text{if} \quad g>-u_0'u_0 \quad \text{and} \quad \gamma>-u_0' \tag{23}$$

Thus, the growth rate decreases as the density scale length increases. We observe that the magnetic field has a stabilization effect on RT instability for any s value in Fig. 9 where Solid line is for $V_{ra}=1$, dash line for $V_{ra}=3$, dash-dotted line for $V_{ra}=5$ and dotted line for $V_{ra}=7$.

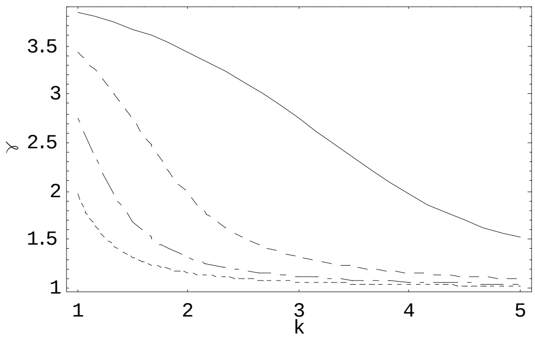

Fig 8 Growth rate against wave number for $u'_{0z}=-1$, $s=1$ & $g=10$.

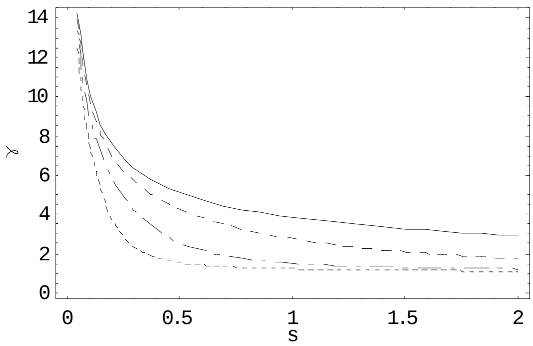

Fig 9 Growth rate against density scale length for $u_{0z}=1$, $u'_{0z}=-1$ *and* $g=9.8$.

4. Conclusion

4.1 The effect of magnetic field and sheared flow

An analytical growth rate is derived for the RT instability by taking into account the magnetic field and sheared flow. It is found that the magnetic field has a stabilization effect whereas the sheared flow has a destabilization effect on the RT instability in the presence of sharp interface.

1. RT instability only occurs in the long wave region and can be completely suppressed if the stabilizing effect of magnetic field dominates. .

2. The RT instability increases with wave number and flow shear, and acts much like a Kelvin-Helmholtz instability when destabilizing effect of sheared flow dominates.

3. If the wave number and the flow shear are relatively small, the growth rate of RT

instability rises monotonically with A_T. Otherwise, the growth rate firstly increases, achieves its maximum, and then falls down as density difference increases.

4.2 The effect of magnetic field and ablation velocity

By considering the effects of magnetic field and ablation velocity, an analytical growth rate is obtained for the Rayleigh-Taylor instability. It is shown that both of ablation velocity and magnetic filed have stabilization effect on RT instability in the presence of continued interface.

1. The stabilization effect of magnetic field takes place for whole waveband and becomes more significant for the short wavelength.

2. The RT instability can be completely suppressed by the cooperated effect of magnetic field and ablation velocity so that the ICF target shell may be unnecessary to be accelerated to very high speed.

3. The growth rate decreases as the density scale length increases. The stabilization effect of magnetic field is more significant for the short density scale length h.

Acknowledgments

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