Collisionless Evolution of Isotropic Alpha-Particle Distribution in a Tokamak

E.A. Sorokina 1,2), V.I. Ilgisonis 1,2)

1) Russian Research Center "Kurchatov Institute", 1 Kurchatov Sq., Moscow 123182, Russia
2) Peoples' Friendship University of Russia, 3 Ordzhonikidze Str., Moscow 117198, Russia

E-mail contact of main author: sorokina.ekaterina@gmail.com

Abstract. Evolution of initially isotropic α-particle velocity and space distribution resulting from collisionless motion of particles in tokamak magnetic field is considered. The analysis is based on fully three-dimensional calculations of charged particle trajectories without simplifying assumptions typical for drift and neoclassical approaches. Spontaneous anisotropization of the distribution function, which results, in particular, in non-inductive current generation, is demonstrated. The current density is calculated over the entire cross section of the plasma column, including the magnetic axis. It is shown that the current density is not a function of a magnetic surface and is strongly polarized over the poloidal angle. On the magnetic axis the current density is nonzero due to asymmetry of the phase-space boundary between trapped and passing particles and its scaling is sensitive to the spatial distribution of α-particles. The role of non-uniform safety factor (including inverse shear configurations) and radial electric field in process of current generation is also investigated.

1. Introduction

Generation of the bootstrap current as a result of spontaneous anisotropization of the charged particles distribution function is a peculiar phenomenon having its origin in the specific configuration of the tokamak's non-uniform magnetic field possessing toroidal–helical structure. The current generated by α-particles, which born in thermonuclear reactions having the same energy and being isotropically distributed in the velocity space, could play the role of a "seed" current [1] being alternative to the existing expensive and energy-consuming methods of noninductive current generation [2].

Starting from classical studies (see, e.g., [3–4]), most part of both analytical and numerical methods of bootstrap current calculation in a varying degree employ the assumptions of standard neoclassical theory (SNT). Taking into account the collisional processes makes the verification of the obtained data rather complicated: simplest collisional operators don't conserve the total momentum and energy of the system, artificial operators amplification in purpose of invariants conservation makes the calculation very complicated. As a result, the solution of steady-state kinetic problem not only sensitive but often is determined by the choice of the collisional operator [5]. The variety of factors affecting the bootstrap current, such as the difference in values and profiles of the electron and ion temperatures, viscosity, and ion inertia, hinder obtaining reliable results and their subsequent verification. Moreover, these factors are usually taken into account not quite self-consistently and require various additional simplifying assumptions. In this case, the nature of the bootstrap effect, which is attributed to the specific character of the toroidal–helical structure of the tokamak magnetic field, is obscured.

It should be emphasized, however, that the effect of the α-particle distribution function anisotropization could be calculated quite correctly without any simplifying assumptions inherent in the SNT and free from the choice of collisional operator (time scales for high-energy α-particles are much shorter than the characteristic collision time). The present study is just aimed at such calculations. The problem is formulated in such a way that collisionless generation of the current could be calculated in the entire volume of the tokamak, including the axial region of the plasma column, which is the most important from the standpoint of "seed" current generation and, simultaneously, the most difficult as concerns particles drift motion. In our analysis, the magnetic surfaces will be assumed to be circular and concentric and the safety factor q is non-uniform ($B_r = z/(q(\rho)r)$, $B_\varphi = R/r$, $B_z = -(r-R)/(q(\rho)r)$, here and after $\{r,\ \varphi,\ z\}$ – cylindrical coordinate system connected with geometrical tokamak center, $\{\rho,\ \theta,\ \varphi\}$ – polar coordinate system connected with magnetic axis, R – the major radius of tokamak). The proposed procedure of calculating the bootstrap current can also be applied to any other axisymmetric magnetic field configuration and to configuration with electric field.

In what follows, the density of the toroidal current is defined as a moment of the distribution function, which is a solution to the Vlasov collisionless equation with an α-particle source on the right-hand side. For the known initial positions of particles, the distribution function is found by reverse integration over real three-dimensional particle trajectories, which, in turn, are determined by numerically solving exact equations of motion. This procedure allows one to correctly (with allowance for the entire variety of particle trajectories and without any additional assumptions on the smallness of their deviations from magnetic surfaces) calculate the current on the axis and also to verify the calculation procedure by controlling the accuracy of exact invariants conservation. In our case (calculation of trajectories by fourth-order Runge-Kutta method) the accuracy of the energy and the toroidal component of canonical momentum conservation was better then 10^{-4}%. It should also be noted that numerical calculations of the distribution function allow us to abandon averaging over magnetic surfaces, which is obligatory in analytical calculations and is performed in most studies on bootstrap current generation.

2. Method for calculating the current density

Distribution function of α-particles $f(\mathbf{r}, \mathbf{v}, t)$, depends on the coordinate, velocity, and time, is a solution of collisionless kinetic equation with a given source S on the right-hand side,

$$S = \delta(t) n_0(\mathbf{r}) \phi(v^2). \tag{1}$$

Here, $\delta(t)$ is the delta-function of time, $n_0(\mathbf{r})$ is the initial density of particles, and $\phi(v^2) = \delta(v^2 - v_0^2)/(2\pi v_0)$ is the monoenergetic distribution function, which depends only on the absolute value of the starting particle velocity v_0 and in the absence of electric field does not change in time. Such source corresponds to the situation when α-particles are born at the instant $t = 0$ being isotropically distributed in velocity space and with spatial profile determined by the parameters of the background plasma. The distribution function thus defined is fundamentally non-steady-state and is specified by the initial conditions (source). The solution of kinetic equation with source S is $f = \Theta(t) n_0(\mathbf{r}_0) \phi(v_0^2)$, where $\Theta(t)$ is the Heaviside function, $\mathbf{r}_0 = \mathbf{r}(t = 0)$, $\mathbf{v}_0 = \mathbf{v}(t = 0)$. In the absence of collisions, the particle distribution function is constant along a particle trajectory. If the particle trajectories $\mathbf{r} = \mathbf{r}(\mathbf{r}_0, \mathbf{v}_0, t)$, $\mathbf{v} = \mathbf{v}(\mathbf{r}_0, \mathbf{v}_0, t)$ are known (i.e., if we managed to integrate the equations of motion), then, we can reconstruct the dependencies $\mathbf{r}_0 = \mathbf{r}_0(\mathbf{r}, \mathbf{v}, t)$ and

$\mathbf{v}_0 = \mathbf{v}_0(\mathbf{r}, \mathbf{v}, t)$ and represent the distribution function as

$$f(\mathbf{r}, \mathbf{v}, t) = n_0(\mathbf{r}_0(\mathbf{r}, \mathbf{v}, t))\phi(v^2). \qquad (2)$$

Integrating (2) over $\mathbf{v}$-space with a weight of toroidal velocity v_φ, we can calculate the density $j_\varphi(\mathbf{r}, t)$ of the toroidal α-particles current (bootstrap current) at a point $\mathbf{r}$. Since particles move along conditionally periodic trajectories, the time dependence of j_φ should be an oscillating function with a period determined by the largest characteristic time of the system. Therefore, there is a reason to average $j_\varphi(\mathbf{r}, t)$ over the time. For a sufficiently large averaging interval T, the oscillations will be smoothed, and the averaged current density is given by

$$J_\varphi(\mathbf{r}, T) = \frac{1}{T}\int_0^T j_\varphi(\mathbf{r}, t)dt = Z_\alpha e \int v_\varphi \langle n_0 \rangle \varphi(v^2) d^3v, \qquad (3)$$

$$\langle n_0 \rangle(\mathbf{r}, \mathbf{v}, T) = \frac{1}{T}\int_0^T n_0(r_0(\mathbf{r}, \mathbf{v}, t))dt.$$

Z_α – charged number of α-particle, e – elementary charge. In the limit $T \to \infty$, the quantity $\langle n_0 \rangle$ is time-independent and represents an analog of a steady-state α-particles distribution in phase space. This distribution depends only on the invariants of motion. At a given spatial point, the density $< n_0 > (T)$ approaches its steady-state value corresponding to the above asymptotically steady-state distribution over several bounce periods, i.e., over a time much shorter than the collision time ($T \approx (2–3)\tau_b \ll \tau_{st}$).

To get $J_\varphi(\mathbf{r}, T)$, we proceed the integration over velocity space calculating the average concentration of the particles along the corresponding trajectories. The value of current density calculating by this way is the maximum possible current density generated due to spontaneous anisotropy of the ensemble of α-particles as a result of their interaction with the tokamak magnetic field. Deceleration of α-particles by the bulk plasma particles should only slightly decrease the value of v in Eq. (3), due to which it becomes smaller than v_0. As a result, the absolute value of the generated current somewhat decreases, its spatial distribution being preserved. Scattering of α-particles by particles of the bulk plasma, which, at $E \gg T_{i,e}$ (E – α-particles energy, $T_{i,e}$ – ions and electrons temperature), is considered isotropic with good accuracy, could only decrease the degree of anisotropy of the distribution function in velocity space. To conclude, we also note that, along with the current caused by drift motion of particles, our calculations automatically (owing to calculations of actual trajectories) take into account the diamagnetic current arising due to particle rotation along Larmor orbits, as well as the current generated due to toroidal precession of banana orbits.

3. Distribution function on the magnetic axis

The time-averaged distribution function of α-particles on the magnetic axis as a function of the cosine of the initial pitch angle is shown in Fig. 1. One can see, that as the result of particles movement in the magnetic field of tokamak, initially isotropic (but non-uniform) source of α-particles becomes essentially anisotropic. The level of this anisotropy (and as a consequence the value of generated current) strongly depends on the initial inhomogeneity of the source (compare curves for $n_0 = n_2 = n_a(1 - x^2)^2$ and $n_0 = n_4 = n_a(1 - x^4)^2$, $x = \rho/a$, a – the minor radius of tokamak). Note that obtained time-averaged distribution

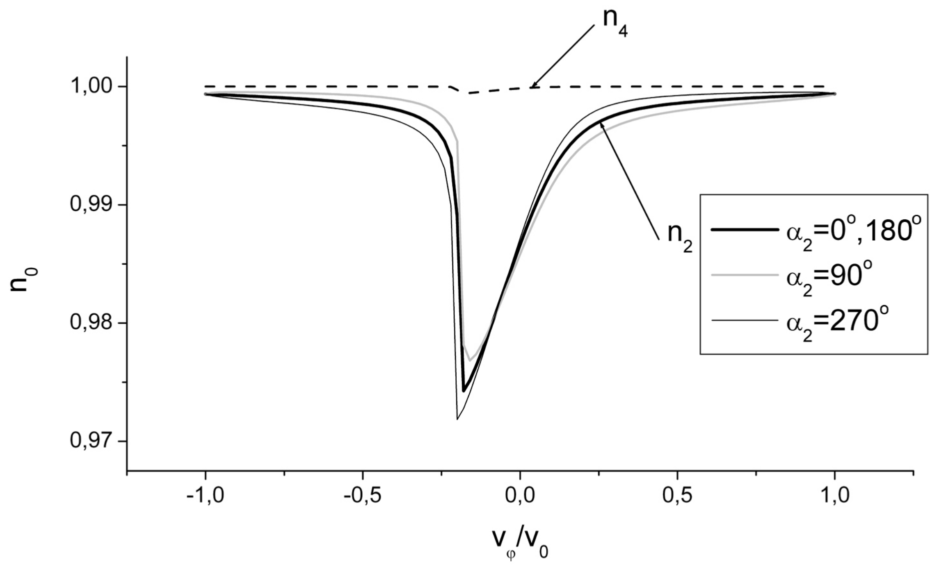

FIG. 1. Time-averaged distribution function of α-particles on the magnetic axis as a function of the initial pitch angle for the density profiles n_2 (solid curves) and n_4 (dashed curve). Different solid curves correspond to different starting phases of Larmor rotation α_2.

function looks very similar to steady-state distribution function for collisional regime [6].

From the anisotropy of distribution function one can see that, although the gradient of initial concentration on the magnetic axis equals zero, the current density exists here. The reason is that particles feel the difference in concentration along the whole trajectory and under the condition of finite banana-width (in contrast to SNT assumptions) the current density cannot be unambiguously characterized by the initial density gradient on the starting magnetic surface, but depends non-locally on the source inhomogeneity. Nevertheless, for plain enough profiles of initial concentration one can suggest model scalings for current density on the magnetic axis taking into account higher derivatives of concentration. Because the trajectories in the central region deviate substantially from magnetic surfaces these scaling are not universal and depend on the concentration profile. Instead of typical for SNT proportionality to ∇n, current density near the magnetic axis is mainly determined by the first non-zero derivative of n:

$$J_{\varphi_{n_2|x=0}} = 0.6ev_0\left\{\frac{\zeta^2 A^2}{4}\frac{d^2 n_2}{dx^2}_{|x=0}\right\}, \qquad J_{\varphi_{n_4|x=0}} = 0.6ev_0\left\{\frac{\zeta^{7/2} A^{7/2}}{8}\frac{d^4 n_4}{dx^4}_{|x=0}\right\}. \quad (4)$$

Here, $\zeta = 2q\rho_L/R$, ρ_L is the Larmor radius calculated for the field on the magnetic axis B_0 and v_0, $A = R/a$ is the tokamak aspect ratio.

Analysis of particle trajectories in the central part of tokamak shows that in this area (the area with small gradient of initial concentration) the current is mainly generated due to asymmetry of the phase-space boundary between trapped and passing particles starting with opposite velocities from a spatially non-uniform source. It may be said that, in the present paper, the asymmetry current predicted in [7] is calculated correctly.

4. Poloidal distribution of the current density

Our calculations show, that the density of the toroidal current generated due to spontaneous anisotropization of α-particles distribution function is crucially not a function of a magnetic surface and is strongly polarized in the poloidal cross section – see Fig. 2. The

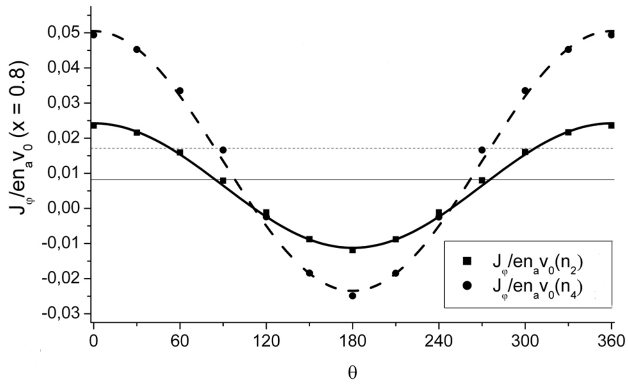

FIG. 2. Current density as a function of the poloidal angle at distances $x = 0.8$ from the axis. The symbols (squares – $n_0 = n_2$, circles – $n_0 = n_4$) and curves show the calculated data and analytical fits, respectively.

calculated points nearly coincide with the curve $\cos\theta$ shifted upward with respect to zero (it is this shift that yields a nonzero current density when averaging over the magnetic surface). More precise calculations indicate that in the poloidal dependence of current density the second harmonic of cosine is also present. Taking into account the principal difference between central region of tokamak and its periphery one can write scaling for bootstrap current working in the whole plasma column:

$$J_\varphi = ev_0 \Big\{ J_{|x=0} \exp\left(\frac{-x}{\zeta A}\right) - \left(1 - \exp\left(\frac{-x}{\zeta A}\right)\right) \zeta A \frac{dn}{dx} \{A(x) + B(x)\cos\theta + C(x)\cos 2\theta\} \Big\}, \quad (5)$$

where coefficients $A(x),\ B(x),\ C(x)$ are

$$A(x) = \sqrt{\frac{x}{A}}(0,18(\frac{1}{q}+2) - 0,24\frac{x}{A}), \quad (6)$$

$$B(x) = 0,68 - 0,2\frac{x}{A}, \quad (7)$$

$$C(x) = -0,14(\frac{x}{A} + 0,02). \quad (8)$$

Here we used the exponentially decreasing function with the half-width proportional to the size of banana orbit to separate scalings for center and periphery of a tokamak. The dependencies $A(x)$ and $B(x)$ are presented in Fig. 3. Far away from the magnetic axis the current density averaged over the magnetic surface $A(x)$ corresponds well with usual neoclassical formulas [8]. At that the amplitude of current density oscillations is substantially larger then mean part of the current density on the magnetic surface. This leads, in particular, to the generation of a negative current on the inner side of the plasma column. The negative current is generated by passing particles which born on the inner side of

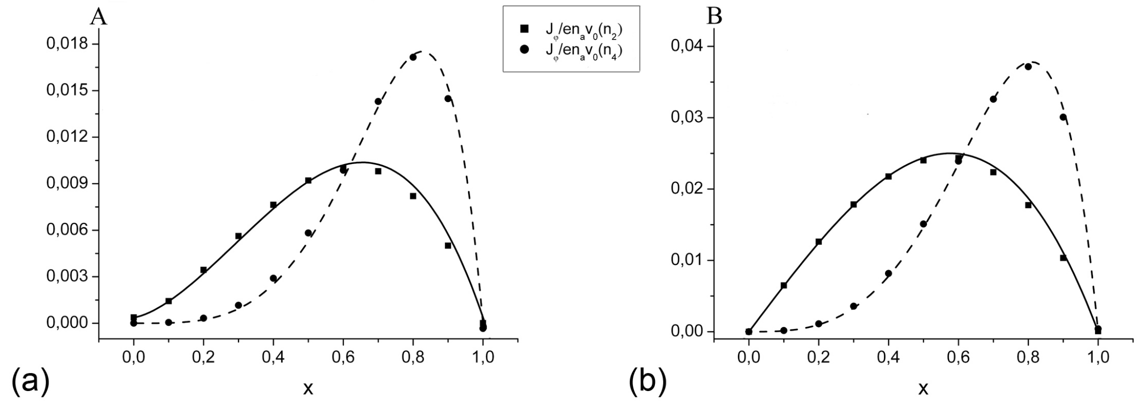

FIG. 3. (a) Normalized current density A(x) averaged over the magnetic surface and (b) amplitude B(x) of oscillations of the normalized current density for the density profiles $n_0 = n_2$ (squares) and $n_0 = n_4$ (circles). The symbols and curves show the calculated data and analytical fits, respectively.

torus and substantially deviate from magnetic surfaces on its outer side. Earlier, such current density polarization was found in α-particles bootstrap-current numerical calculations by Monte-Carlo method [9], but in [9] following the neoclassical theory the authors analyzed only the current density averaged over magnetic surface. However, the above mentioned strong current polarization requires the bootstrap-current calculations to be performed together with the plasma equilibrium. The formal averaging of the generated current over a magnetic surface seems to be insufficient.

4. Current density in configurations with reverse shear

All above mentioned results are valid for $q = \text{const}$. It is clear that due to deviations of the particle orbits from magnetic surfaces the current density is sensitive to the integral variation of the safety factors value over their typical trajectories (if q is non-uniform). It appears, that the influence of q-factor inhomogeneity on current generation is big enough to change the scaling of bootstrap current only in case of non-uniform gradient of concentration, i.e., when the profile of initial concentration has the second derivative, or in case of non-uniform gradient of q, but the influence of the second factor is much smaller then the influence of the first of them. Otherwise the inhomogeneity of q-factor could be accounted by substitution of instantaneous q value in ordinary scaling for bootstrap current. The sign of addition in current density scaling due to q inhomogeneity is determined by the sign of $d^2n/dx^2dq/dx$. If $d^2n/dx^2dq/dx > 0$ the effect is negative, i.e., when the gradient of q is nonzero, the current density is lower than that calculated in the assumption that $q = \text{const}$. For $d^2n/dx^2dq/dx < 0$, the effect is positive. For conventional profiles of n and q sensible deviation of current density from ordinary scalings is observed on the periphery of plasma column (because of the growth of q- and dn/dx-gradients with x) and it is negative. If the gradients are constant the effect is maximal on the magnetic axis (because of the large deviations of the particle orbits from magnetic surfaces).

Study of q-profile with a minimum in plasma column (reverse shear configuration) shows that the current density also obeys the scalings found for monotonic profiles of q with substitution $q(x)$ instead of $q = \text{const}$ in (5). Nevertheless, the spatial distribution of the current density differs from the case of monotonic q – see Fig. 4. In the point of minimum q the amplitude of current oscillations $B(x)$ is minimal.

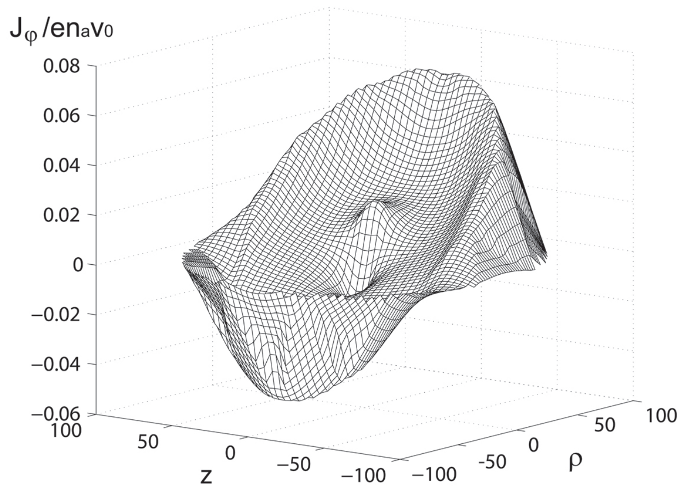

FIG. 4. Current density distribution in the poloidal plane $\varphi = const$ *for* $n_0 = n_1 = n_a(1-x)$, $q = 1 + 4(x-0.3)^2/(1-0.3)^2$.

5. The role of the radial electric field

The most remarkable consequence of the radial electric field is the change in the character of the particle toroidal motion. We compare differences of trajectories of the particles starting in the same magnetic configuration from the same point with the same velocities with and with no electric field. With no electric field, trapped particle shifts in the toroidal direction extremely weak just according to banana precession. In radial electric field the particle possesses different velocity on the opposite sides of the banana-orbit, so its toroidal displacement during one bounce-period is not compensated. The value of toroidal velocity acquired by the trapped particle in the electric field with a good accuracy is equal to the toroidal component of the electric drift velocity in the poloidal magnetic field: $\triangle v_{tr} =< v_\varphi >_{|E\neq 0} - < v_\varphi >_{|E=0} = c < |[\mathbf{E}\times\mathbf{B}_\theta]_\varphi|/B_\theta^2 >$ and has the same sign. Here $<>$ denotes averaging over the trajectory; $\triangle v_{tr} < 0$ for $\mathbf{E}$ directed to the magnetic axis (negatively charged plasma). As an ordinary electric drift in straight fields, the value of this velocity depends neither the mass nor the charge of the particle. Passing particles also receive additional toroidal velocity in the electric field but $\triangle v_{pas}$ for these particles doesn't follow the formula of electric drift in the poloidal magnetic field. For passing particles the electric drift in the toroidal magnetic field becomes important: it changes the poloidal cross-section of the particle trajectory and, as a result, the energy of the toroidal rotation. Thus, particles starting on the outer side of torus obtain $\triangle v_{pas}$ in the direction opposite to the $[\mathbf{E}\times\mathbf{B}_\theta]$-drift. On the inner side of torus additional velocity of passing particles follows again the $[\mathbf{E}\times\mathbf{B}_\theta]$-direction, but doesn't correspond it in absolute value. The absolute value of $\triangle v_{pas}$ also depends neither on mass nor on charge that indicates its origin from the electric drift.

In Fig. 5 the dependencies of $\triangle v$ on starting v_φ/v_0 for two spatial points on the axis: $x = 0.2$ and $x = 0.9$ are presented. Here one can see the tendency described above. Two spikes on curves correspond to transition of trapped particle in fraction of passing particles under the influence of the electric field directed to the center. Although the addition to the toroidal velocity depends on mass neither for trapped nor for passing, the value of $\triangle v$, appeared as a result of transition between this two types of orbits, changes with mass in the central region of plasma.

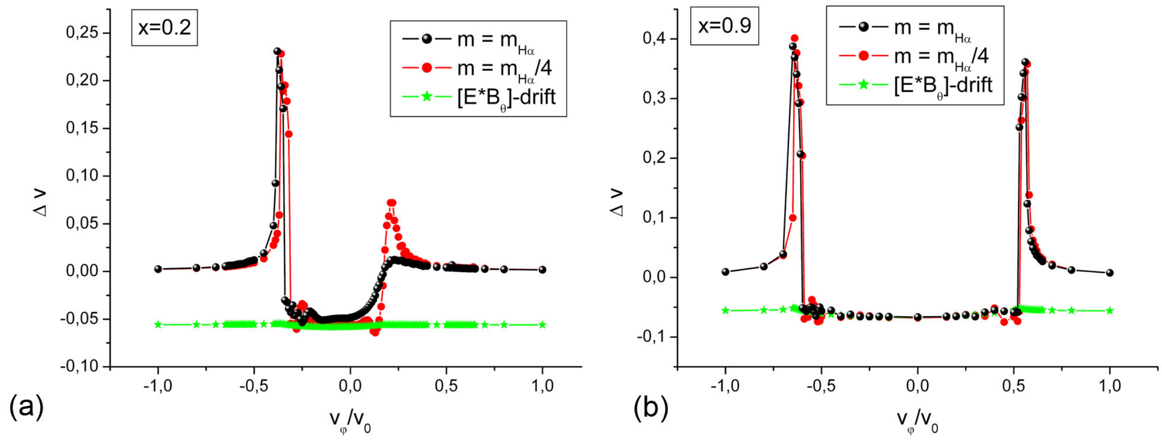

FIG. 5. Dependence of $\triangle v =< v_\varphi >_{|E\neq 0} - < v_\varphi >_{|E=0}$ *on* v_φ/v_0 *in two spatial points: (a) –* $x = 0.2,\ \theta = 0$*, (b) –* $x = 0.9,\ \theta = 0$.

6. Conclusions

Collisionless evolution of α-particles distribution function in a tokamak magnetic configuration is calculated for different profile of initial particles concentration. The time averaged distribution function is shown to reveal a form in the velocity space typical for neoclassical theory. No restrictions of SNT-applicability were involved to calculate the scalings for bootstrap current, which cover plasma center as well. The scalings are suitable both for positive and negative shear configurations. The peculiarities of the electric field influence on the particle trajectories in a tokamak are demonstrated.

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