Parametric Study of Equilibrium and Stability Analysis of HT-6M Tokamak in the Presence of Flow

Zahoor Ahmad 1), S.H.R. Rizvi 2)

1) National Tokamak Fusion Program, Pakistan Atomic energy commission, P.O. Box 3329, Islamabad, 44000, Pakistan
2) TPPD, PINSTECH, P.O. Nilore, Islamabad, 44000, Pakistan

Email: zahoor_a@yahoo.com

Abstract: It is experimentally observed that tokamak plasmas exhibits macroscopic flow in the toroidal and poloidal directions. Both the flow can considerably change the equilibrium parameters of tokamak. Equilibrium of HT-6M is revisited as a case study and its equilibrium parameters are simulated in the presence of toroidal and poloidal flows with the FLOW Code developed by L. Guazzotto at that University of Rochester (USA). FLOW code was originally designed for spherical tokamaks, which is modified to implement on the circular tokamak. Effect of toroidal and poloidal flow on poloidal flux, current density, particle density, temperature, beta profile, toroidal and poloidal magnetic fields are studied. A comparison is also made for no flow and with different flows. It is found that the plasma is squeezed against the outboard side of the tokamak producing an outward shift (Shafranov shift) because of centrifugal force. The presence of flow also pose problem with the stability of toroidal system. For our analysis of stability we studied effect of different toroidal flows on internal kink modes.

1 Introduction

In magnetic confinement research tokamak is considered as a potential candidate for future fusion reactor. It is experimentally recognized that axisymmetric devices exhibit the most attractive confinement properties. The equilibrium study is one of the fundamental problems of magnetically confined plasmas. Magnetohydrodynamic (MHD) equilibrium study of axisymmetric systems with flow have been carried out by many authors, including Chandrasekhar [1] and Woltjer [2] for the case of incompressible flow. Recently, Bogoyavlenskij [3] demonstrated the existence of exact axisymmetric magnetostatic equilibrium. These analyses are based on the Grad-Shafranov (GS) equation, derived originally to describe magnetostatic equilibria for nuclear fusion experiments. [4,5]. equilibria with flow, have been studied using MHD model [6,7,8], the two fluid model [9,10,11] and effect of pressure anisotropy [12,13]. In experiment flow is produced due to NBI (Neutral Beam Injection). MHD model gives the best description of equilibrium problem. Analytic equilibrium solutions for tokamak plasmas are difficult to find since the equations governing the equilibrium are highly nonlinear. Therefore numerical solutions are always useful. Moreover it has also been observed that strong toroidal rotation can suppress or mitigate many magnetohydrodynamic (MHD) instabilities and significantly improve plasma performance [14]. For example resistive wall modes can be stabilized by toroidal rotation [15]; ballooning modes are stabilized by sheared toroidal flows [16] where as the internal kink mode is also stabilized by rotation [17].The FLOW [18] and FLOS [19] codes are developed by Guazzotto *et al.* for the study of axisymmetric tokamak equilibrium in the presence of toroidal and poloidal flow for NSTX Tokamak. The present work is a step toward the understanding of tokamak equilibrium and stability in the presence of arbitrary flow, in the context of HT-6M tokamak. For this purpose we have modified the FLOW code for required circular cross-section HT-6M tokamak instead of spherical tokamak.

HT-6M was constructed and operated by Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, China. It is small air-core tokamak with following parameters: major radius $R = 65\ cm$, minor radius $a = 20\ cm$, magnetic field strength $B_T = 1.0\text{-}1.5T$, plasma current $I_p = 100\text{-}150\ kA$, discharge time$\tau_d = 50\text{-}100\ ms$, electron temperature $T_e = 600{\sim}800\ eV$, ion temperature $T_i = 200{\sim}400\ eV$, energy confinement time $\tau_E = 10\ ms$ and line averaged electron density $n_e = 0.5\text{-}4.0\times10^{19}\ m^{-3}$ [20,21].

In the present paper, the results of numerical study on HT-6M with purely toroidal flow and arbitrary flow, using code FLOW are described. The manuscript is organized as follows. The physical model and basic equations are presented in sec. 2, In sec. 3 and 4 numerical results are presented and in the last section 5.conclusions are presented.

2 Review of MHD Equilibrium Equations

The basic single fluid model for describing and determining the macroscopic equilibrium and stability properties of plasma is Ideal magnetohydrodynamics (MHD). The ideal MHD model is given by,

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}.\rho\vec{v}=0 \tag{2.1}$$

$$\rho\frac{d\vec{v}}{dt}=\vec{J}\times\vec{B}-\vec{\nabla}p \tag{2.2}$$

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right)=0 \tag{2.3}$$

$$\vec{E}+\vec{v}\times\vec{B}=0 \tag{2.4}$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \tag{2.5}$$

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J} \tag{2.6}$$

$$\vec{\nabla}.\vec{B}=0 \tag{2.7}$$

Taking the e_φ component of the momentum equation (2.2) and using the axsiymmetric properties of the equilibrium leads to the following expression [22] for the toroidal field

$$B_\Phi R=\frac{F(\psi)+\sqrt{\mu_o}R^2\Phi(\psi)\Omega(\psi)}{\left(1-\frac{\Phi^2(\psi)}{\rho}-\Delta\right)} \tag{2.8}$$

where $F(\Psi)$ is a free function of poloidal flux function Ψ. Above equation reduces to the standard form $B_\varphi R=F(\Psi)$ in the absence of poloidal flow ($\Phi(\Psi) = 0$) and anisotropy(**Δ** = 0). Here $\Phi(\Psi)$ and $\Omega(\Psi)$ are two free functions of Ψ describing the parallel and toroidal component of the velocity respectively. The poloidal component of the flow depends exclusively on $\Phi(\Psi)$ while the toroidal component is a function of both Φ and Ω

The next step is to take the $\vec{B}$ component of the momentum equation, which after a straight forward calculation yields the well known Bernoulli equation [22]**.**

The final step is to take the $\nabla\Psi$ components of the momentum equation (2.2) in order to derive the Grad-Shafranov equation for the poloidal magnetic flux.

We observe that the equilibrium model has been reduced to a system of three equations,

1. Toroidal Field Equation
2. Bernoulli equation.
3. A modified GS Equation

This system of equations can be solved numerically once the free functions of the system have been assigned.

3 Numerical Results and Discussion

The code FLOW solves above three equations simultaneously [18,22]. We have modeled equilibrium of HT-6M tokamak for cases: (i) pure toroidal flow and (ii) poloidal & toroidal flows.

3.1 Equilibria with Purely Toroidal Flow

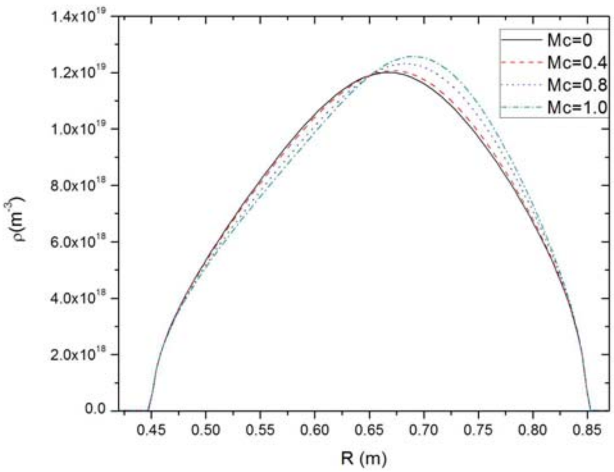

Figure 1: Density variation with varying toroidal flow

For HT-6M tokamak calculation for density variation is shown in Figure 1 which shows that density profile shifted outward with the increase of toroidal flow. Flow is varied by changing toroidal Mach number. Where M_c= 0 corresponds to no flow and M_c=1 corresponds to sonic flow. Line plot shows that there is small variation corresponding to subsonic flow and large variation near sonic flow. It may be noted initial density profile from ref 20 is used.

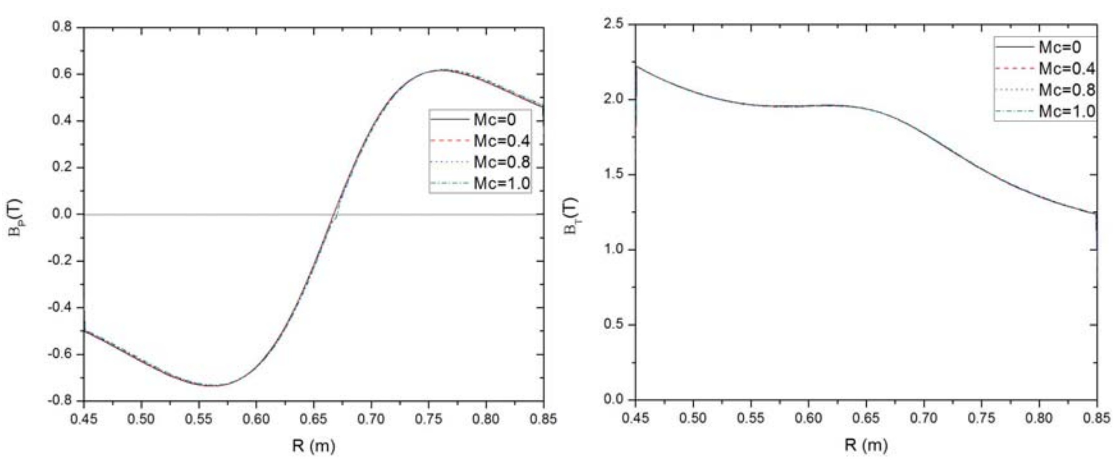

Figure 2: Poloidal B and Toroidal B with with varying toroidal Mach speed

Toroidal and polidal fields are plotted in Figure 2 for different toroidal Mach numbers. There is no significant effect on toroidal magnetic field, while poloidal magnetic field decreases at inboard side and increases at outboard side of tokamak.

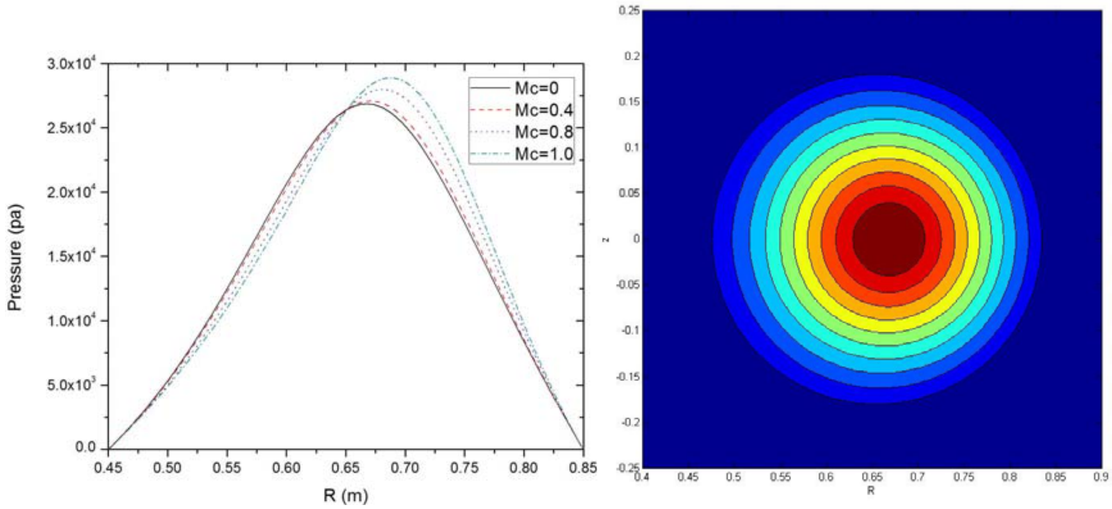

Figure 3: Variation of Plasma pressure (left) and (right) shows the shifted contours of poloidal flux function corresponding to sonic toroidal flow (M_c=1)

Increase in plasma pressure is observed due to toroidal flow and is plotted in Figure 3 (right), however toroidal flow also produces unwanted large outward shift. Figure 3 (right) represents the shifted contours of poloidal flux function at M_c=1. Toroidal flow has important effect on beta ratio as shown in Figure 4, this shows that beta value is increased from 13.8% to 16.4% by the corresponding change of toridal flow from M_c=0 to M_c=1.

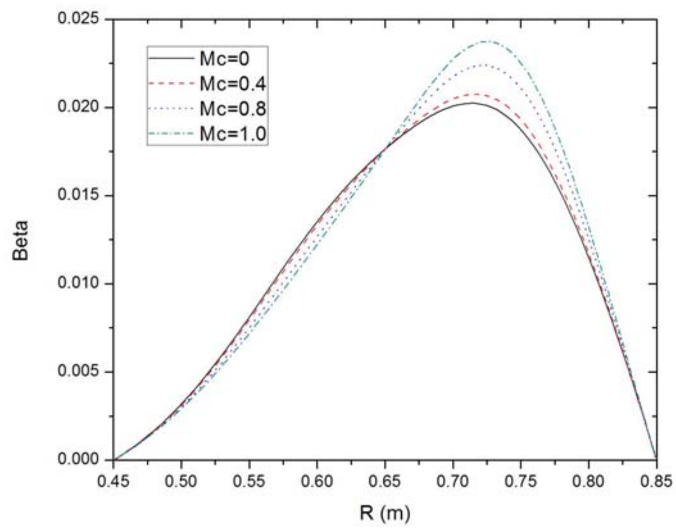

Figure 4: Beta variation with purely toroidal flow.

3.2 Equilibria in the presence of toroidal and poloidal flow

Motivation for the Poloidal flow is that, in tokamak poloidal flow is induced by Reynolds Stress [23], turbulence [24], Mirnov oscillations [25], radial electric field and also due to poloidal magnetic field [26]. Poloidal flow in HT-6M is in moderate range, so it's good enough to discuss here only poloidal flow of the order of subsonic range.

It is interesting to observe that a toroidal plasma cannot sustain a purely poloidal flow [27]. This is because in toroidal plasma there is net momentum transfer between the plasma and magnetic field, which leads to finite toroidal velocity as well.

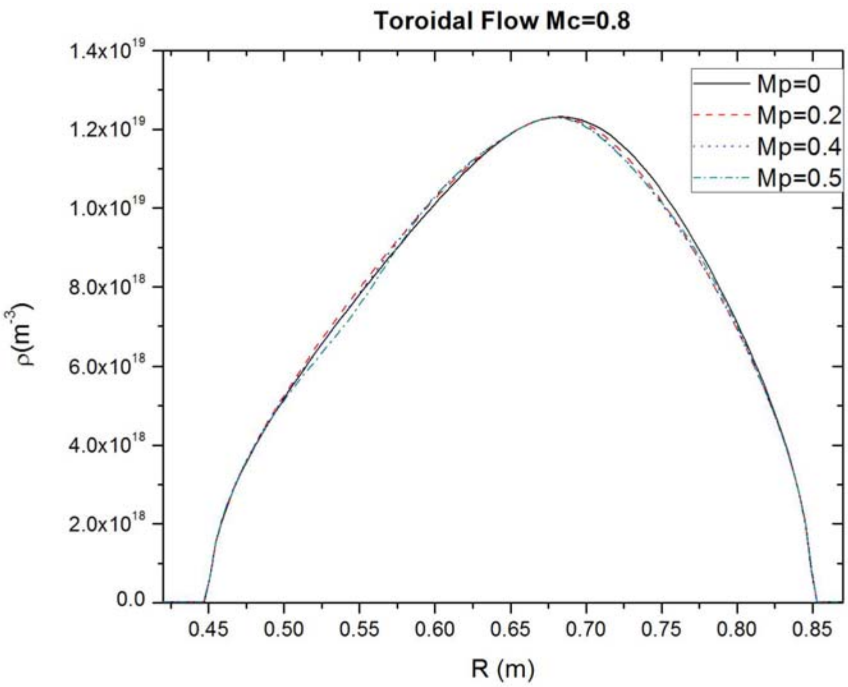

Figure 5: Density variation with varying poloidal flow at constant toroidal flow (M_c=0.8)

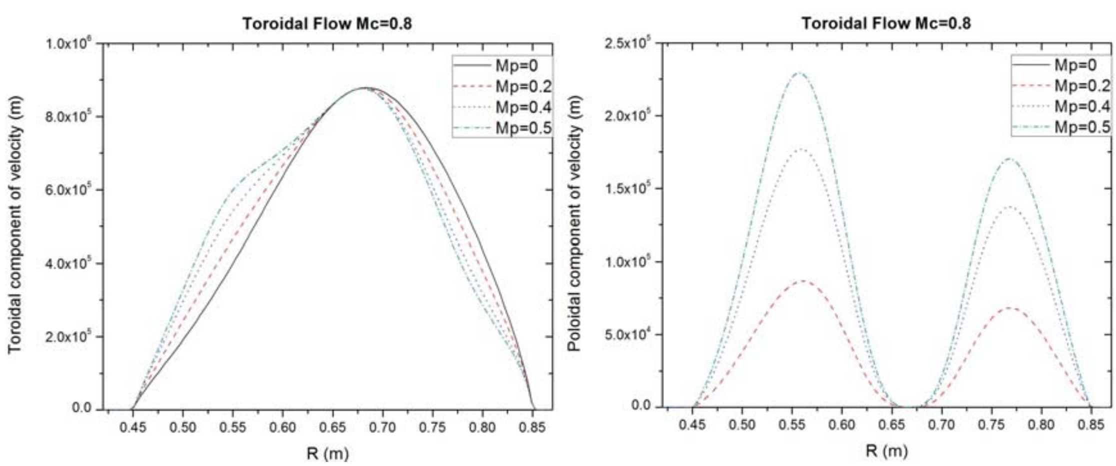

Figure 6: Toroidal velocity (left) and poloidal velocity (right) variation with poloidal flow at constant toroidal flow (M_c=0.8).

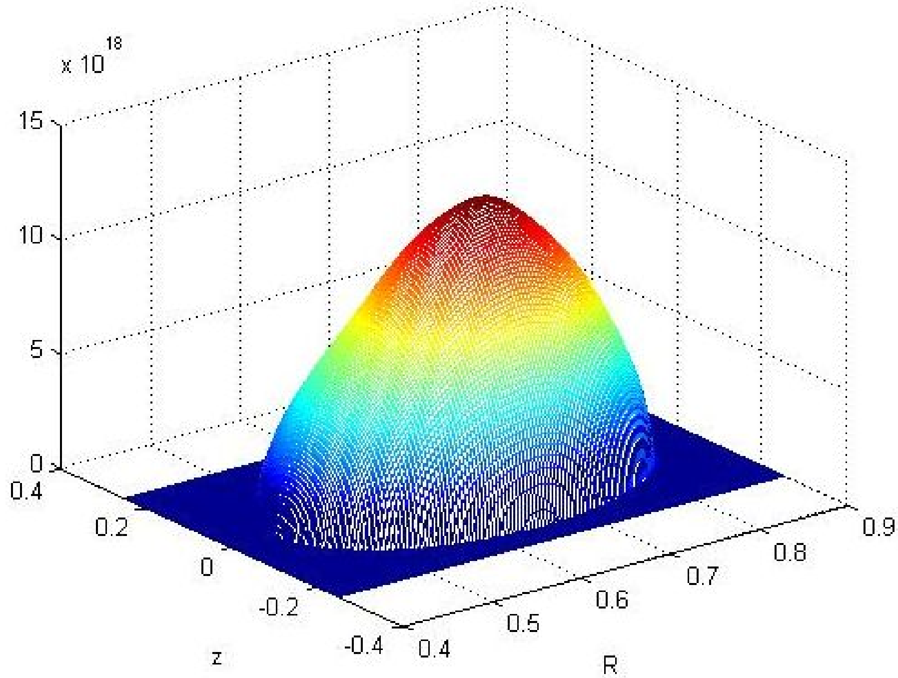

Figure 7: 3D density plot for equilibrium arbitrary flow (where density is in #/m^3)

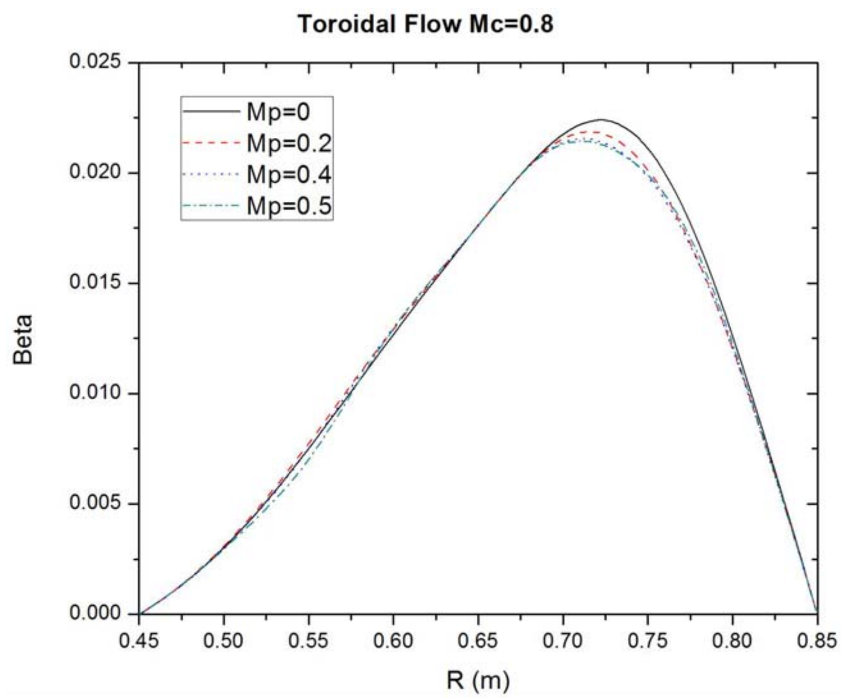

Figure 8: Effect of poloidal flow on beta profile, at sonic speed

Taking toroidal component in the range of subsonic speed i.e. M_c=0.8 and varying M_p from 0.0 to 0.5, all the equilibrium variables are calculated and corresponding profiles are shown in Figure 5 to Figure 8, Figure 6 shows plots of toroidal and polidal velocities in the presentce of arbitrary flow. One can clearly notice that with the inclusion of polidal flow toroidal velocity is shifted from outboard side to inboard side. Figure 7 shows the 3D plot of density representing the 2D variation of the plasma density. Beta profile shows (Figure 8) that by introducing strong troroidal flow along with subsonic poloidal flow, beta is decreased and have less outward shift hence have effects on the confinement.

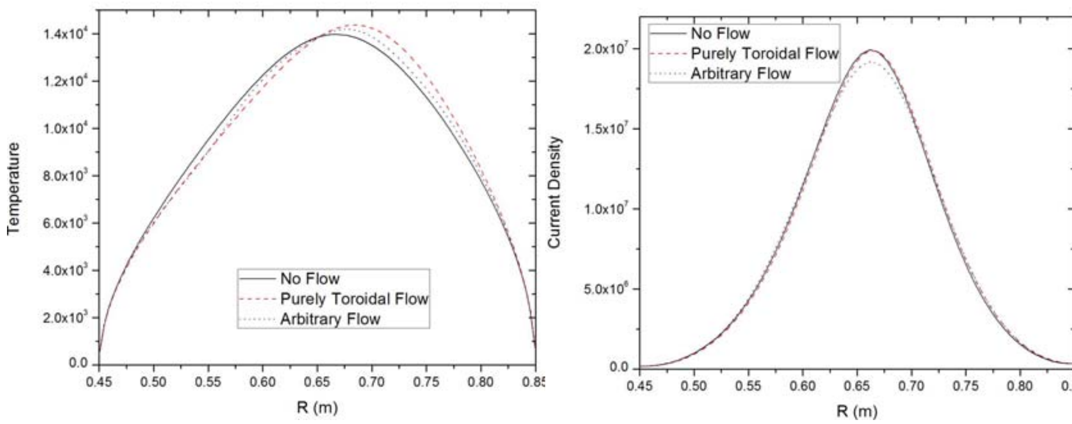

Figure 9: Variation of Density (Left) and current density (Right) with purely toroidal flow and with arbitrary flow

Figure 9 is plotted for the temperature and Current density without flow, with toroidal flow and arbitrary flow. It can be clearly seen here that toroidal flow enhance the parameters and increase Grad-Shafranov shift where as arbitrary flow increase the parameter but less as compared with purely toroidal flow. However it also reduces Sharanov shift.

4. Sability

We are not presenting detailed calculations of stability in this version how ever in the literature it found that internal kink is stabilized if the following criteria is satisfied;

$$\Omega M \approx \gamma_0$$

Where Ω is plasma rotation, γ_0 growth rate of kink instability in the absence of any flow. In the figure 10 we have plotted normalized growth rate of internal kink instability for different toroidal flows. It can be observe that growth is reduced with the inclusion of flow.

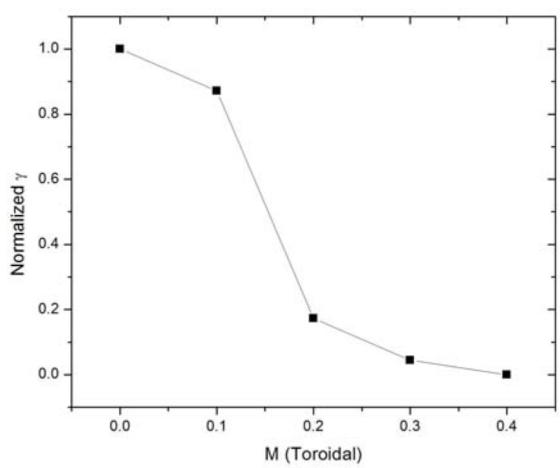

Figure 10: Instability growth rate as a function Toroidal flow

5. Conclusion

The findings of our study are the effect of toroidal and poloidal flow on Shafranov shift and on equilibrium profiles. Toroidal and poloidal magnetic field and toroidal currents are less sensitive to the toroidal flow effects where as density and beta were found affected by increase of both toroidal and poloidal flow. Temperature is also increased with the flow. The important of them is the inward shift of all profiles corresponding to sonic toroidal flow and subsonic poloidal flow component. This inward shift reduces the outward Shafranov shift produced by pure Toroidal flow. Toroidal flow also reduces growth rate of internal kink instability.

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