Modeling of High Density and Strong Magnetic Field Generation by Plasma Jet Compression

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Abstract. In magnetized target fusion (MTF), an imploding, conducting liner compresses a magnetized plasmoid, such as a spheromak or field-reversed configuration. The increasing magnetic field of the target reduces thermal conduction, and the liner's inertia provides transient plasma stability and confinement. Almost all of the early research focused on solid or liquid liners. The present research explores the kinetics of alpha particles and gradient drift instabilities in non-uniform magnetic field of the recently invented concept of using plasma jets to form the liner. This study points to solution of problem connected with theoretical investigation of methods supplying with magneto inertial fusion (MIF). Evolution of traditional inertial and magnetic confinement fusion is the concept with high density plasma ($n > 10^{27}$ m^{-3}) in strong magnetic field ($B > 500$ T). MIF with plasma liner uses multiple high kinetic energy plasma jets carrying both the main D-T fuel followed by the high density liner. Results of numerical simulation are presented.

1. Introduction

Magneto-inertial fusion (MIF) is one of alternative confinement concepts that has a long history dating back to early years of pulsed-power research in USA and USSR [1-3]. Several experimental and simulation groups worked and continue to work now in this area [4-6]. Interest in research on plasma liner driven magneto-inertial fusion has recently been stimulated by (a) plasma jet driven magnetic flux compression experiments, (b) the progress of high β (beta is the ratio of plasma to external magnetic pressure) magnetic systems, and (c) advantages in plasma guns and liners.

Compression of target in MIF includes four stages: i) The system uses a solenoid operated with a pulsed voltage power supply to generate initial (seeded) magnetic field in the imploding target plasma of MIF; ii) Magnetic field embedded in the fuel thermally insulates it from the plasma liner; iii) Thus, magnetic field compression leads to increasing of the plasma pressure (dynamic high-pressure on a target), heating target plasma to ultrahigh temperature; and iiii) Fuel temperature and pressure both increase to extremely high values with increasing of compression ratio, producing ultrahigh magnetic fields. Direct compression of the magnetized plasma (target) by plasma liner (plasma jets) schematically shown in Fig. 1.

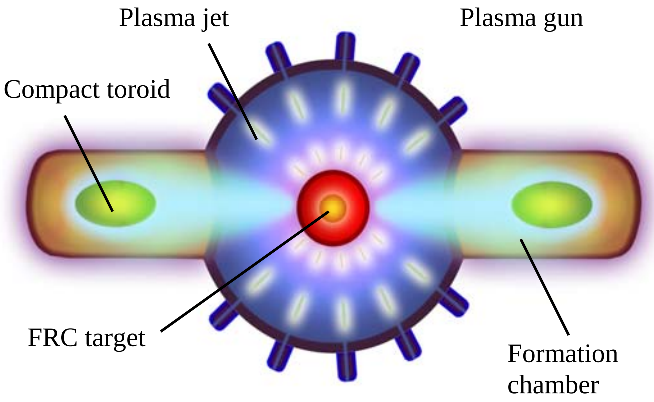

FIG. 1. Magnetic flux compression by plasma jet driven magneto-inertial fusion.

Plasma confinement with $\beta \approx 1$ is an important advantage (relative to the classical tokamak) from the viewpoint of MIF approach. The class of such systems includes open traps and cusps, field reversed configuration (FRC) combining the field of open and closed magnetic field lines, mirror based system. In this work we have investigated the electromagnetic drift instability on the basis of the model [7, 8] recently developed specifically for such systems. Studies have shown that the parameters of the drift instabilities depend strongly on the nature of the magnetic field non-uniformity. The non-uniformity of the magnetic field is associated with a transverse gradient of magnetic induction and the curvature of the magnetic field lines.

In the magneto-inertial fusion (MIF) devices, plasma dynamics is mainly determined by collisions, which is associated with high plasma density in the typical MIF regimes. Typical density value in MIF systems is many orders greater than in magnetic confinement devices, but it essentially smaller than in inertial fusion. At the final stage of the target implosion magnetic field achieves extremely high value $B \sim 100–1000$ T. Nevertheless, some important processes are collisionless in nature and require a kinetic consideration. Among such processes is slowdown of fast (or high-energy) alpha particles produced in the D–T-reaction. In this case, the collisionless regime is realized due to high energies of these particles.

Alpha particle energy is the source of self-heating of fusion plasma. It is therefore desirable to choose the parameters of the magnetized target so as to ensure effective retention of fast alpha particles. On the other hand, accumulation of thermalized alpha particles in the plasma leads to additional radiation losses and increases the energy expended. Therefore, analysis of the kinetics of alpha particles is important from the viewpoint of optimal regimes.

Another process, whose influence on the energy balance of the plasma deserves attention is the anomalous transport due to collisionless drift instabilities. Instabilities of this type define the transport properties of plasma in magnetic confinement systems. In high-β plasma with inhomogeneous magnetic field drift instability growth rates may be comparable with the inverse time of the target compression. Then the transport losses due to drift turbulence can play a decisive role. Otherwise, these losses are insignificant. To determine the regimes corresponding to the second case, it is useful to estimate the growth rates of drift instabilities in the collisionless limit.

During plasma compression by high-density, high Mach-number plasma jets for MIF (a time scale of micro-seconds) the accumulation of α-particles may occur. The slowdown time is small, but magnetic field is very high, up to 300-1000 T or even higher. The drift of a particle lead to the transport, which may pose a risk to these systems. Qualitative and quantitative assessment is required, taking into account that they (particles) are kept well. And therefore our main task is to determine the maximum role of α-particles in MIF systems.

2. The kinetics of alpha particles

Let's estimate the level of magnetic field at which alpha particles with energy $W_0 \approx 3.5$ MeV are magnetized in the target with radius $a \approx 1$ cm. It means that the Larmor radius of the alpha particle ρ_α is less than the radius of the target:

$$\rho_\alpha \approx \sqrt{m_\alpha W_0}\,/(2eB) < a\,, \tag{1}$$

where m_α is the alpha particle mass, e is the electron charge, B is the magnetic field induction.

This condition [Eq. (1)] is satisfied if $B > \sqrt{m_\alpha W_0}/(2ea)$ ≈ 20 T. Thus, at the final sage of the compression ($a \approx 1$ cm, $B \approx 200$ T) alpha particles can be considered as magnetized. Similar calculations for the initial stage ($a \approx 1$ cm, $B \approx 200$ T) give the following result: $\rho_\alpha < a$, if $B > 2$ T. Under such conditions, the orbits of major portion of alpha particles are out of the target.

In magnetic configurations with loss regions in velocity space, such as, for example, mirror traps and the cusps, a part of the alpha particles are lost immediately after the birth (the so-called first orbit losses). Angular scattering into the loss region during the slowdown is relatively slow process at high energies. In the idealized conditions under which the losses in the process of slowing down are not available, the function of the velocity distribution of fast alpha-particles is [9]

$$f_0(v) = \frac{q\tau_s}{4\pi(v^3 + v_c^3)}\,. \tag{2}$$

Here

$$\tau_s = 6\pi\sqrt{2\pi}\,\frac{m_\alpha}{\sqrt{m_e}}\,\frac{\varepsilon_0^2\left(k_B T_e\right)^{3/2}}{\Lambda_{\alpha/e} Z_\alpha^2 e^4 n_e} \tag{3}$$

is the slowdown time due to the Coulomb collisions; q is the number of particles appearing in the unity volume during the unity time (particle source) excluding first orbit losses; $v_c = \left(\frac{3\sqrt{\pi}}{4}\,\frac{\Lambda_{\alpha/i}}{\Lambda_{\alpha/e}}\,\frac{m_e}{n_e}\sum_i \frac{Z_i^2 n_i}{m_i}\right)^{\frac{1}{3}}\left(\frac{2k_B T_e}{m_e}\right)^{\frac{1}{2}}$ is so-called critical velocity which characterize the equality of slowdown rates for the electron and ion collisions, the summation is performed on all kinds of ions; Z_i and m_i present the charge and the mass of corresponding ions; ε_0 is the magnetic permittivity of the vacuum; k_B is the Boltzmann constant; T_e in the electron temperature; m_e is the mass of the electron; $\Lambda_{\alpha/i}$ and $\Lambda_{\alpha/e}$ is the Coulomb logarithm for collisions of the alphas with ions and electrons; $Z_\alpha = 2$; n_e is the electron density.

In Ref. 10, the analysis of alpha particles in MIF systems was carried out neglecting losses due to the angular scattering with corresponding velocity distribution function [Eq. (2)]. Numerical study of fast particle kinetics [11–13] based on Fokker–Planck equation [14, 15] showed the possibility of using high-energy approximation with angle-averaged loss time. This approximation is available in the following velocity range: $v_{Ti} < v < v_{Te}$, where v_{Ti} and v_{Te} are thermal velocities of Maxwellian ions and electrons. The angle-averaged Fokker–Planck equation has the following form in this range [9, 16]:

$$\frac{1}{\tau_s}\frac{1}{v^2}\frac{\partial}{\partial v}\left[\left(v^3 + v_c^3\right)f\right] + \frac{q}{4\pi v_0^2}\,\delta(v - v_0) - \frac{f}{\tau(v)} = 0\,. \tag{4}$$

To estimate the losses associated with scattering of fast alpha particles into the loss region, one can use the angle-average loss time [13]

$$\tau_L(v) = C\tau_s[z]\left(\frac{v}{v_c}\right)^3 \tag{5}$$

where $[z] = \sum_i \frac{Z_i^2 n_i m}{n_e m_i}$; $C \approx 0.9$ for the mirror ratio range 3..10.

The solution of Eq. (4) is the following

$$f(v)=\frac{q\tau_s}{4\pi(v^3+v_c^3)}\exp\left[-\int_v^{v_0}\frac{\tau_s}{\tau(v)}\frac{v^2}{v^3+v_c^3}dv\right]=\frac{q\tau_s}{4\pi(v^3+v_c^3)}\left(\frac{v^3}{v_0^3}\frac{v_0^3+v_c^3}{v^3+v_c^3}\right)^{\frac{C}{3[z]}}. \quad (6)$$

We use this solution to calculate parameters of fast alpha particles for typical MIF regimes. The temperatures of Maxwellian components were assumed to be equal: $T_i = T_e = T$. Note that energy balance estimations show the possible difference between ion and electron temperatures as low as 1–3 keV at $T_i \approx T_e$ = 10–20 keV.

In Figs. 2 and 3, results of the calculations of the following parameters are presented: alpha particle density n_α, pressure, p_α, integral loss time τ_α, and energy loss per particle W_L due to the angular scattering. As scales of mentioned parameters we use $n_0 = n_\mathrm{D} + n_\mathrm{T}$ ($n_\mathrm{D} = n_\mathrm{T}$, n_D and n_T are deuterium and tritium density), $p_0 = (n_\mathrm{D} + n_\mathrm{T} + n_e)k_BT$, slow-down time τ_s, and alpha particle burn energy W_0.

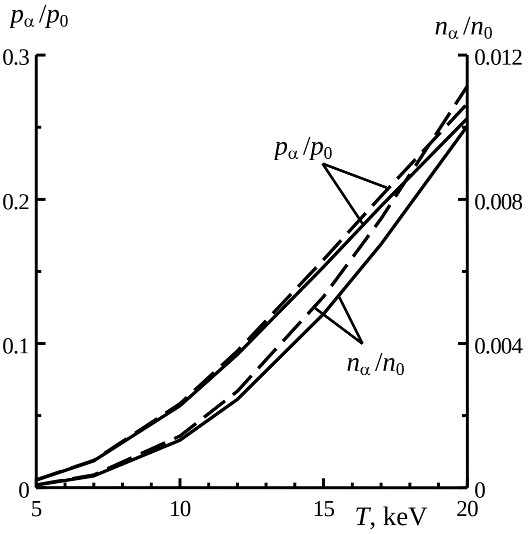

FIG. 2. Pressure and density of fast alpha particles in D–T mirror plasma vs temperature taking into account losses due to angular scattering (solid) and neglecting angular scattering (dashed).

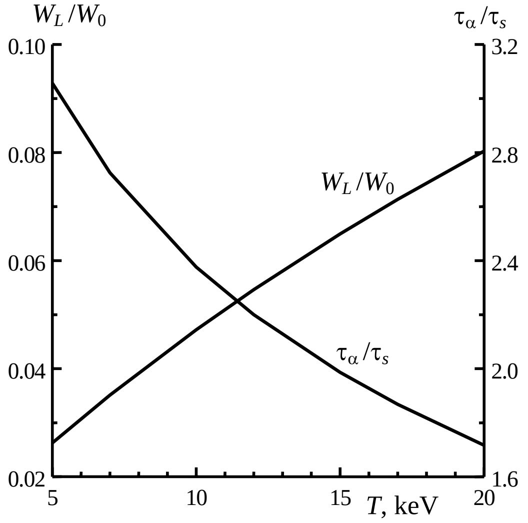

FIG. 3. Energy losses due to angular scattering and corresponding energy confinement time.

As one can see, the losses of fast alpha particles during the slowing down slightly reduce their density and pressure in comparison with the case of ideal confinement. Anomalous mechanisms of particle transport across the magnetic field can significantly change this situation, if the corresponding loss time $\tau_\perp$ for fast particles is small compared with the slowdown time τ_s.

3. Gradient drift instabilities in non-uniform magnetic field

Here we consider collisionless drift instabilities which are related to the problem of turbulent anomalous transport across magnetic field under conditions of high-β magnetized plasma. “High-β” means that the local parameter β (β = plasma pressure / magnetic pressure) inside the plasma can be larger than unity.

The usual reasons of the plasma transport across magnetic field are the gradient drift instabilities associated with ion temperature gradient (ITG) and electron temperature gradient (ETG) modes. For a strongly collisional regime the problem of drift instability in MIF plasma was considered in Ref. 17 as a modification of collisionless case of "infinite" β wall-confinement regime [18]. Here we study collisionless ITG/ETG induced turbulence and transport in small-size (L ~ 1 cm) plasma with magnetic field $B \sim 100$ T and $\beta \sim 1$. Because of high density the confinement time required for high efficiency is relatively short and cross-field transport plays not very important role in confinement. But the ITG/ETG induced penetration of strong magnetic field into the plasma during the implosion can be significant.

In traps with strong plasma diamagnetism due to high β (mirrors, FRCs) magnetic field increases from the central regions to the periphery of the plasma in accordance with the relationship $B \approx B_e \sqrt{1-\beta_0}$, where B_e is the external magnetic field, $\beta_0 = 2\mu_0 p / B_e^2$ is the ratio of plasma pressure p to the external magnetic field pressure. The local beta parameter $\beta = 2\mu_0 p / B^2$ is calculated using the field inside plasma B. Two mentioned beta parameters are connected as follows: $\beta = \beta_0 /(1-\beta_0)$.

In the cusp magnetic field lines are concave inward. There are regions of decreasing field in systems with internal conductors. The results of calculations of the drift instabilities can be used as the basis for model of plasma transport. Two typical ranges of the transversal wave number $k_\perp$ are considered in mentioned model: i) $k_\perp \sim 1/\rho_{Ti}$, and ii) $k_\perp \sim 1/\rho_{Te}$, where ρ_{Ti} and ρ_{Te} are the ion and electron thermal gyro-radiuses. The first range corresponds to the ion temperature gradient (ITG) driven instability and the instability due to the plasma density gradient. Such relatively large scale convective motion in composition with sheared plasma flows can form non-linear low-frequency turbulent structures. The second range corresponds to electron temperature gradient (ETG) instability. Oscillations with $k_\perp >> 1/\rho_{Te}$ usually are not interesting from the viewpoint of plasma confinement and anomalous turbulent transport in magnetic fusion devices.

The model [7, 8] includes ITG/ETG instabilities taking into account non-adiabatic responses of both ions and electrons in whole range of $k_\perp$ under the consideration. The analysis is carried out in the framework of the local electromagnetic kinetic approach. Low frequency ($\omega << \omega_{ci}$, ω_{ci} is the ion cyclotron frequency) drift instabilities are studied on the basis of the linearized Vlasov–Maxwell equations. This system includes gyro-kinetic solution of the Vlasov equation, quasi-neutrality condition, and Ampere's law for parallel and perpendicular perturbations of the magnetic field [19–21]. Each solution of dispersion equation $\omega = \omega_R + i\gamma$ (ω_R is the real frequency, γ is the growth rate) depends on the following parameters: parallel wave number $k_\|$, transversal wave number $k_\perp$, $\eta_i = L_n/L_{Ti}$, $\eta_e = L_n/L_{Te}$, T_e/T_i, parameter of the transversal non-uniformity of the magnetic field $\alpha_B = L_n/L_B$, relative curvature parameter $\alpha_R = L_n/R$; here $L_n = -\, n/\nabla_\perp n$, $L_{Ti} = -\, T_i/\nabla_\perp T_i$, $L_{Te} = -\, T_e/\nabla_\perp T_e$, $L_B = B/\nabla_\perp B$, n is the plasma density, T_i is the ion temperature, T_e is the electron temperature, B is the static magnetic field inside the plasma, R is the magnetic field line curvature radius.

Narrow class of modes of practical importance enclosing perturbations with $k_\| = 0$. In this case dispersion equation describes two kinds of perturbations: extraordinary and ordinary waves. Extraordinary wave can be interpreted as temperature gradient drift mode coupled with compressional perturbations [22]. The perturbed electric field of this mode $\mathbf{E}_1$ is perpendicular to the unperturbed magnetic field **B**. The structure of the ordinary-wave electromagnetic perturbations is the following: $\mathbf{E}_1$ is parallel to **B**.

As the scale of the growth rate γ and real frequency ω_R we use $\omega_0 = k_B T_i /(eBL_n \rho_{Ti})$; dimensionless wave numbers are $k_{||}L_n$, and $k_\perp \rho_{Ti}$.

In general case $\alpha_B = L_n/L_B$ is independent parameter but for the traps such as mirrors and FRCs it is connected with other parameters as $\frac{L_n}{L_B} = \sum_j \frac{(1+\eta_j)\beta_j}{2}$, where $\beta_j = 2\mu_0 n_j k_B T_j / B^2$ is the local beta parameter for component *j*.

In Fig. 4, results of calculations are presented for the case when α_B and β are independent parameters. These solutions are obtained using full electromagnetic dispersion relationship [7, 8]. For high-β regimes practically interesting range of transversal non-uniformity of the magnetic field is $L_n/L_B > 1$. If L_n/L_B increases, pure β effect is practically negligible *(see FIG.4,a)*, and electrostatic dispersion equation can be used for the analysis. Growth rates are maximal at $L_n/L_B \approx 1.5$ that corresponds to $\beta \approx 1$ ($\beta_0 \approx 0.5$).

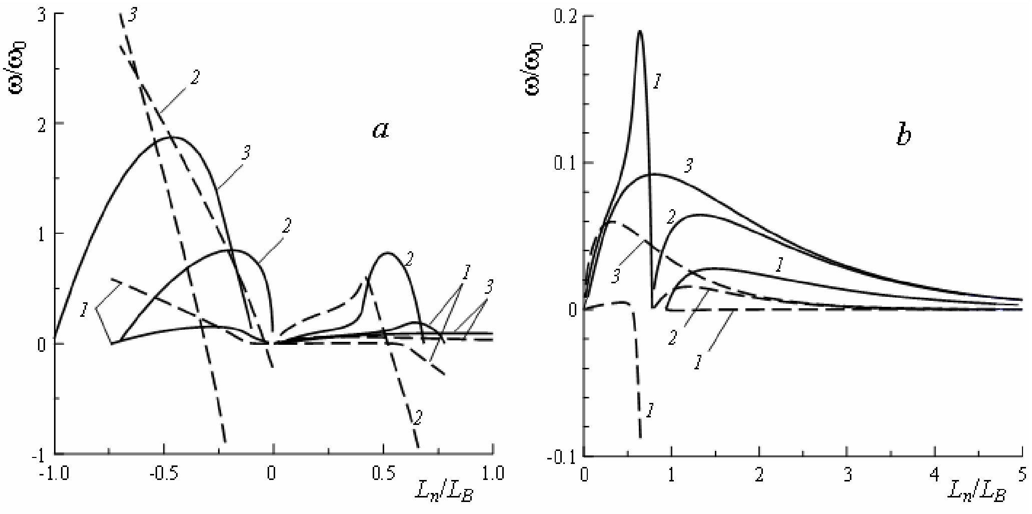

FIG. 4. Growth rates (solid) and real frequencies (dashed) vs L_n/L_B at $\eta_i = \eta_e = 2$, $\tau = 1$, $L_n/R = 0$, $k_{||}L_n = 0$: 1 – $k_\perp\rho_{Ti} = 1$, $\beta = 1$; 2 – $k_\perp\rho_{Ti} = 5$, $\beta = 1$; 3 – $k_\perp\rho_{Ti} = 5$, $\beta = 0$.

For $\beta \approx 1$ we estimate that collisionless approximation for drift instabilities is available for not very high compression of the magnetic field: $B < T^{3/2}/\sqrt{L}$, where units of *T, B*, and *L* are keV, Tesla, and meter, respectively. If this condition is satisfied the instability growth rate exceeds the rate of electron-electron collisions. Collisionless approximation gives an upper bound for the growth rate. For example, at $L = 0.001$ m and $T = 20$ keV upper boundary of collisionless drift wave transport regime is $B \approx 3000$ T. In the present work we consider the conditions close to the cusp-based magneto-inertial fusion system proposed in Ref. 23. It was estimated that for prospective regimes of this system $B \approx 200$–400 T [24]. Presented results correspond to the case of moderate density of surface plasma of the target. They complete the picture of drift instabilities considered in Refs. 17 and 18.

Conclusion

This research is aimed to the evaluate attractiveness of plasma jet driven magneto-inertial fusion. Magneto-inertial approach to a fusion combines the advantages of magnetic and inertial confinement and provides low-cost simple fusion schemes. Plasma jets can merge to form a plasma liner and compress targets to fusion temperatures. Early analysis shows that the liner needs to have a density exceeding 10^{29} per m^3, that is, close to or exceeds solid density. Features of such MIF system depend primarily on the final compression and can be checked by an exact and suitable modeling. A new quasi-monotone numerical method and model for both laser-driven magneto-inertial fusion and plasma liner magneto-inertial fusion are proposed in Ref. 25. We are developing a simple quasi-steady state model with the estimation of plasma transport. The principal theoretical result is a demonstration of the feasibility of plasma jet driven magneto-inertial fusion. We shall explore further the implosion and energy gain physics of multi-layered (inner “afterburner” and outer layer) plasma liners, using a combination of analytical models and numerical simulations.

Acknowledgements

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