Interaction of turbulence and magnetic islands

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Abstract. We have investigated the role of turbulence for two problems, the effect of the polarization current on the NTM and the effect of diamagnetic drifts on mode penetration. For the NTM problem, we find that the fluctuations exert a drag force on the island that results in a stabilizing slowdown in the propagation velocity. For the mode penetration problem, we have investigated the influence of polarization and diamagnetic rotation in the error field penetration threshold by comparing the results of electromagnetic simulations using a two-fluid model with theory. For rotation velocities between the ion and electron diamagnetic velocity, drift-waves are excited and the solution becomes delocalized. We find that the existing linear theories that neglect the drift wave radiation effect overestimate the penetration threshold.

1. Introduction

Magnetic islands may arise as a consequence of the Neoclassical Tearing Mode (NTM) instability in tokamaks [1, 2], where they can limit the duration of high beta discharges. Alternatively, they may appear as a result of currents flowing in external conductors, either applied intentionally as in the ergodic magnetic divertor or as result of errors in the alignment of the coils [3]. In either case, they significantly affect the plasma by modifying its local features, as for example, its pressure gradient and velocity profile in the neighborhood of the island. Island formation and evolution in tokamaks occur in a turbulent environment, since the steep pressure gradients present in the plasma naturally excite short-wavelength instabilities such as drift-waves. Previous efforts at predicting the onset and growth of magnetic islands have modelled the effects of turbulence through the use of anomalous transport coefficients. In most cases of interest, however, the widths of the magnetic islands and their phase velocities are comparable to the corresponding scales for plasma turbulence, so that the mutual interaction between islands and turbulence cannot properly be described through the use of macroscopic transport coefficients. In the last few years, a number of numerical and theoretical works [4, 5, 7, 6, 8] have studied different aspects of the interaction between magnetic islands and turbulence. In this paper we investigate the role of turbulence for two problems, the effect of the polarization current on the NTM and the effect of diamagnetic drifts on mode penetration.

The polarization current can be responsible for the observed threshold in the growth of neoclassical tearing modes. In order to be stabilizing, however, the relative velocity between the island and the surrounding plasma must lie between the ion and electron diamagnetic drift velocities, with the strength of the stabilization decreasing as either drift velocity is approached [9]. We have examined the effects of resistive drift-wave turbulence on an island by using a fluid model that extends the Hasegawa-Wakatani model by including the effects of magnetic shear (and thus of the magnetic island) on the turbulence [8]. The work reported here forgoes the use of an electromagnetic model such as that used by Ishizawa [7] in favor of an electrostatic model. In an electrostatic model the island can be thought of as a perturbation of the magnetic field of fixed shape and amplitude. The island's rotation velocity u, by contrast, can be allowed to evolve dynamically. The value of u is determined by the vorticity equation and by the boundary conditions. The advantage of the electrostatic approximation is that it allows the island width to be set to any desired value without external forcing (which would affect the rotation frequency as well as the width of the island) and without the long simulation times needed for Ohmic relaxation. The electrostatic approximation facilitates the study of the dependence on the island width of various quantities and phenomena of interest such as the mode frequency, polarization current, background density-flattening, and turbulence.

In the second part of the paper, we describe how the penetration in the plasma of resonant magnetic perturbations (i.e. error fields) is modified by diamagnetic effects. Driven magnetic islands, resulting from such penetration, exert a braking force on the plasma and thus play an important role in momentum transport, causing qualitative changes to the profile of plasma velocity. In the standard model [3], as the error field amplitude increases, a mode penetration threshold is encountered above which the plasma rotation is arrested and the island grows to a size comparable to or greater than its size in vacuum. It has been shown with quasi-linear MHD models, however, that plasma rotation can effectively suppress the error field penetration. We have investigated the influence of polarization and diamagnetic rotation in the error field penetration threshold by comparing the results of two-fluid electromagnetic simulations with previously-developed analytic theories. For moderate rotation velocities drift-waves are excited and the solution becomes delocalized. We find that the existing linear theories that neglect the drift-wave radiation effect overestimate the penetration threshold.

2. Model Equations

Our investigation is carried out in a 2D slab configuration, for low-β plasmas with a strong magnetic field B_z in the ignorable direction, $\mathbf{e_z}$. We assume constant electron temperature and cold ions. The magnetic field and the plasma $\mathbf{E} \times \mathbf{B}$ velocity can be written as: $\mathbf{B} = B_z\mathbf{e_z} + \mathbf{e_z} \times \nabla\psi$ and $\mathbf{v} = \mathbf{e_z} \times \nabla\varphi$, where ψ is the normalized magnetic flux and φ is the normalized electric potential. The fluid model that we analyze is a reduced version of that obtained by Hazeltine *et al.* in Ref.[10]. The normalized equations are:

$$\frac{\partial U}{\partial t} + [\varphi, U] = [J, \psi] + \mu\nabla^2 U, \tag{1}$$

$$\frac{\partial \psi}{\partial t} + [\varphi, \psi] = [n, \psi] - \eta(J - J_{eq}), \tag{2}$$

$$\frac{\partial n}{\partial t} + [\varphi, n] = \rho^2[J, \psi] + D\nabla^2 n. \tag{3}$$

Equation 1 is the curl of the plasma (ion) momentum balance projected along the confining magnetic field direction and evolves the normalized plasma vorticity, $U = \nabla^2\varphi$. Equation 2 is Ohm's law, obtained taking the projection of the electron momentum balance along the magnetic field (and neglecting electron inertia). Lastly, Eq.3 is the conservation equation for the normalized plasma density, n (without loss of generality, we assume that n is the plasma density minus its value at the reconnecting surface). The system is closed by Ampère's law, $J = -\nabla^2\psi$ where J is the normalized current. In our convention, all the transverse lengths scale (along x) with L, a typical equilibrium length scale, and all the velocities with $v_A = (B_z\epsilon)/\sqrt{4\pi m_i n_c}$, the transverse Alfvén velocity. Here n_c is a typical density, m_i is the ion mass and ϵ is the inverse aspect ratio, The "radial" and "poloidal" directions are labelled x and y respectively. Consequently, the transverse Alfvén time is: $\tau_A = L/v_A$. The operator $[A, B] = \partial_x A\partial_y B - \partial_x B\partial_y A$ is the Poisson bracket of two generic scalar fields A and B. Note that $[\varphi, \cdots] = \mathbf{v}\cdot\nabla\cdots$ represents the $\mathbf{E}\times\mathbf{B}$ advection and $[\cdots, \psi] = \mathbf{B}/B_z \cdot \nabla \cdots$ is the parallel gradient, $\nabla_\parallel$. In Eq.(3) $\rho = \rho_s/L$, $\rho_s = c_s/\Omega$ with $c_s = \sqrt{T/m_i}$ the ion sound speed calculated with the electron temperature, T, and $\Omega = eB_z/m_i c$ measures the ion gyrofrequency (e is the electric charge and c is the speed of light). The model includes several (normalized) dissipative coefficients such as the electrical resistivity, η, the particle diffusivity, D, the perpendicular ion viscosity, μ, and the parallel viscosity. At this point, it is useful to shortly describe the forces that equilibrate the plasma in the poloidal direction. These forces determine the dynamics (in particular the rotation frequency) of the island in the y direction and play an important role in both the NTM and error field penetration problem. In a frame of reference moving with the island, we write the poloidal plasma momentum balance equation averaged over y (to eliminate the pressure term) and integrated over x around the reconnecting surface:

$$\int dS\,(\partial_t + \mathbf{v}\cdot\nabla)\,v_y = -\widetilde{\psi}k\int dS J\sin(ky) + \mu\int dS\partial_x^2 v_y + \int dS\widehat{F}_y, \tag{4}$$

where $\int dS = \int_{x_-}^{x^+} dx \oint \frac{d(ky)}{2\pi}$. The previous equation balances all the forces acting on a stripe of plasma included between $0 \le ky \le 2\pi$ (the full "poloidal" direction) and $x^- \le x \le x^+$. The term on the left-hand side of Eq.4 is the plasma inertia (v_y is the "poloidal" component of the plasma velocity), the second term on the right-hand side is the viscous force and the last term is a constant homogeneous force acting on the plasma (i.e. unbalanced NBI). The first term on the right-hand side represents the electromagnetic $\mathbf{J}\times\mathbf{B}$ force, and is obtained by using a constant-ψ approximation to describe the magnetic island: $\psi = x^2/2 + \widetilde{\psi}\cos(ky)$ (this implies that the island width is $w = 4\widetilde{\psi}^{1/2}$).

3. Mutual interaction between NTMs and turbulence

To investigate the mutual interaction between magnetic islands and turbulence it is convenient to introduce an electromagnetic approximation, which can be justified as follows. For the short wavelength components (the turbulence), the magnetic fields generated by the turbulent polarization currents must be negligibly small. This is the case whenever $\widehat{\beta} = \beta L_s^2/L_n^2 \ll 1$ (β is the ratio between the thermal and the magnetic pressure, L_s is the magnetic shear length, and L_n the density scale length). For the long wavelength components (such as the magnetic island), by contrast, the electrostatic assumption requires that the reconnection rate is slow [11], that is: $C(\Delta' + \Delta_{\rm pol})\rho_s/\widehat{\beta} \ll 1$, where $C = \eta\omega_*/\rho^4$ is a measure of the plasma collisionality, Δ' is the standard linear stability parameter for the tearing mode [12], and $\Delta_{\rm pol}$ is the contribution of the internal polarization currents to the island evolution. Here $\omega_* = v_* k$ is the electron diamagnetic frequency and $v_* = cT/(eB_z L_n)$. Within these limit we ensure a separation of scale between the rate of change of the island width and the drift frequency characterizing the turbulent modes. This implies that we can set $\partial/\partial t \cong 0$ in Eq.2, thus describing the turbulent fluctuations

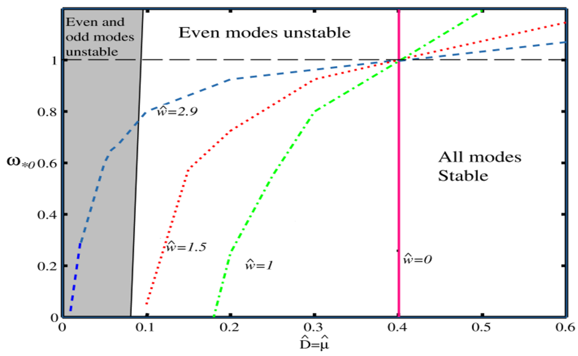

Figure 1: Stability thresholds for the even and odd modes. The shaded region represents the domain of instability of the odd modes and the thin, nearly vertical line bordering this shaded region represents $\widehat{D}_{\rm odd}(\omega_{*0}, w)$. The remaining lines represent $\widehat{D}_{\rm evn}(\omega_{*0}, w)$ for $\widehat{w} = 0$ (vertical solid line), $\widehat{w} = 1$ (dash-dotted line), $\widehat{w} = 1.5$ (dotted line) and $\widehat{w} = 2.9$ (dashed line).

with an Hasegawa-Wakatani model while the magnetic flux is given by the constant-ψ approximation.

We first investigate how the turbulence is affected by the presence of a magnetic island. A key question is how the presence of an island modifies the linear stability properties of resistive drift waves (and therefore regulates the turbulence). A well-known effect of magnetic islands is the tendency for the profiles of density and pressure to flatten inside the separatrix [13]. Since the radial gradient of the background density is the source of free-energy for the drift-wave instability, one expects the local flattening caused by the presence of an island to have a stabilizing effect on the turbulence. In order to quantify the consequences of this stabilizing effect without carrying out a 2D stability analysis, we have numerically investigated equilibria with a 1D radial density profile, $n_{eq}(x)$, that is partially flattened in a region of width w around the neutral line. We have used the model profile $n_{eq}(x) = -x + (1 - \omega_{*0})xe^{-2(x/w)^2}$ and $\varphi_{eq}(x) = -ux$, where ω_{*0} represents the local diamagnetic frequency at the O-point. This profile can be thought as an average over y of the actual 2D flattened profile. In self-consistent nonlinear simulations, we expect that the degree of density flattening ω_{*0} will depend on the island width as well as the other parameters. Since this dependence is unknown a priori, however, we will treat ω_{*0} as an independent parameter. Figure 1 shows the dependence on D (for simplicity, always assumed equal to μ), for selected values of w, of the critical inner density gradients $\omega_{*0}^{\rm odd}$ and $\omega_{*0}^{\rm evn}$ for marginal stability against modes that are odd and even with respect to the reconnecting surface. This figure shows that the stabilization threshold for the even modes, $\omega_{*0}^{\rm evn}(D; w)$, depends sensitively on the dissipation D, the sensitivity becoming greater as the island width increases. The stability boundary for the odd modes is best characterized in terms of the critical diffusion $D_{\rm odd}$, which is almost independent of ω_{*0} because these unstable modes and the flattened component of the equilibrium field share the same parity. Therefore, we find that for $w \neq 0$ the stability of the drift-waves is determined by the island width w (the width of the flattened region) and the locally reduced diamagnetic drift frequency ω_{*0} (measuring the amount of density flattening) in addition to the dissipation $D = \mu$.

In order to gain some understanding of the nonlinear effects of turbulence on the is-

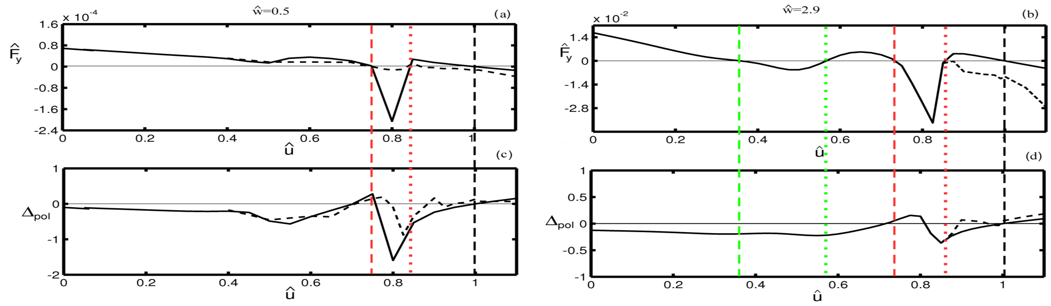

Figure 2: (a)-(b)Perpendicular force $\widehat{F}_y$ acting on the island and (c)-(d) Δ_{pol} versus imposed velocity $\widehat{u}$ for $\widehat{w} = 0.5$ and $\widehat{w} = 2.9$, respectively. The solid lines represent quiescent states (parity enforced numerically) and the dashed lines are the time-averaged values in the turbulent states. The vertical dashed (dotted) lines indicate rotationally stable (unstable) unforced states for the quiescent island

land, we begin by investigating the response of the system to an island moving with an imposed constant velocity, u. In a convenient frame of reference moving with the magnetic island, this is equivalent to impose a constant plasma velocity, $v_y = -u$, far from the resonant surface. With appropriate boundary conditions (see Ref.[8]) the LHS and the viscous term at the RHS of Eq.4 disappear and the electromagnetic force induced by the imposed island rotation is balanced by the volumetric external forces acting on the plasma. As a consequence, when $\widehat{F}_y = 0$, the u corresponds to the "natural" island velocity. The effect of the turbulence on the island stability is mediated by the turbulent fluctuations of the polarization current and is measured by the parameter $\Delta_{\rm pol}(u,w) \equiv -(\pi\widetilde{\psi})^{-1}\int_{-l_x}^{l_x} dx \int_{-\pi}^{\pi} dy\,(J - \langle J\rangle_\psi)\cos y$, where $\langle\cdot\rangle_\psi$ represents the average across an infinitesimal tube of flux ψ, and the radial integration is extended all over the numerical box (of width $2l_x$). In our study we use an initial-value, finite-difference code which employs a fully implicit multi-grid algorithm (4^{th} order in space and 2^{nd} order in time) constructed using PETSc (Portable Extendible Toolkit for Scientific Computation). The size of the numerical box is $[-l_x : l_x, -\pi : \pi]$ with $l_x = 1$ and the grid resolution is 100×112 points. In all our simulation we set $C = 1$, $\rho = 0.15$, $J_{eq} = 1$ and $\omega_* = 0.01$, while $D = \mu$, $\widetilde{\psi}$ and u vary. Figure 2 shows $\widehat{F}_y$ and $\Delta_{\rm pol}$ as a function of the velocity, $\widehat{u} = u/v_*$, for two different island widths $\widehat{w} = 0.5$ and $\widehat{w} = 2.9$, and for $\widehat{D} = \widehat{\mu} = 0.1$ and $C = 1$ [here $\widehat{w} = w/\rho$, $\widehat{D} = D/(\omega_*\rho^2)$ and $\widehat{\mu} = \mu/(\omega_*\rho^2)$]. The solid lines represent cases where the odd parity of the fields is enforced, so that turbulence is artificially suppressed, in agreement with the linear analysis presented above. The dashed lines represent time-averaged values of $\widehat{F}_y$ and $\Delta_{\rm pol}$ in simulations where both parities are allowed to evolve. The free rotation solutions correspond to the intersection of the force curves with the x-axis ($\widehat{F}_y = 0$) and are marked by vertical dashed lines. Turbulence is seen to eliminate the free rotation solution at the electron diamagnetic frequency (black vertical dashed line on the right), pushing the solution towards a more stable root (from $\widehat{u} = 1$ to approximately $\widehat{u} = 0.75$ for the island with $\widehat{w} = 2.9$). Note, however, that at fixed frequency the turbulence is destabilizing, as indicated by the fact that the dashed lines for Δ_{pol} lie above the solid lines for the quiescent plasma. By virtue of the frozen-in property of the electrons, the reduction in rotation frequency corresponds to a reduction in the local diamagnetic frequency, i.e. to a density flattening that stabilizes the turbulence. Indeed, the turbulent fluctuations are quenched when u is reduced, and the turbulent solution ceases to exist (at $\widehat{u} \cong 0.4$ for $\widehat{w} = 2.9$). To summarize, we have observed that the turbulence

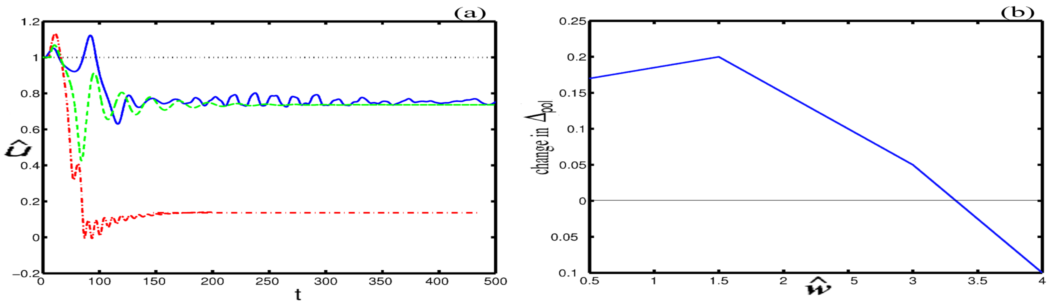

Figure 3: (a) Time evolution of $\widehat{u}$ for $\widehat{w} = 0.5$ (solid line), $\widehat{w} = 2.9$ (dashed line), and $\widehat{w} = 4.1$ (dash-dotted line). (b) Change in Δ_{pol} after a turbulence-induced transition versus the imposed island width. The change occurs between the initial state with $\widehat{u} = 1$ and the final state. A positive (negative) value indicates that the transition is destabilizing (stabilizing).

induces transitions that slow down the "poloidal" rotation of the magnetic island and, for sufficiently large islands, lead to quiescent states. This picture is confirmed by simulations where $\widehat{F}_y = 0$ and the island rotation frequency is allowed to evolve accordingly to Eq.4 [see Fig.3(a)]. The net effect of these transition is stabilizing for sufficiently large islands ($\widehat{w} > 3.2$ for the case treated here). By contrast, our results show that islands of width comparable with the turbulent eddies can be destabilized by the turbulence [see Fig.3(b)].

4. Diamagnetic effects on error field penetration

In this section we discuss the penetration in the plasma of an external magnetic field, generated by coil misalignment or imposed, in presence of diamagnetic effects (i.e. pressure gradients). The plasma response to a static external magnetic perturbation is governed by ideal MHD everywhere but in a narrow layer around the resonant surface where the helical component of the equilibrium magnetic field vanishes. The calculation is thus split between an ideal "outer" and a resistive "inner" solution, where the more dissipative effects are investigated in a simplified geometry. The problem is completed by matching the two solutions over the region where they overlap. When the flow shields the penetration of the wall perturbation, the inner response can be described by linear, constant-ψ layer physics, and is characterized by a complex quantity used in the matching, $\Delta = -(2\pi\widetilde{\psi}_0)^{-1}\int_{\infty}^{\infty} dx \oint d(ky) J e^{-iky}$, where $\widetilde{\psi}_0$ is the value of the fundamental eigenfunction of the magnetic flux at the reconnecting surface and J is calculated from the linear layer theory [3] and depends on the physical model used. In the standard theory, the quantity Δ depends on the rotation velocity of the plasma at the reconnecting surface relative to the external magnetic perturbation, v. More precisely, when diamagnetic and Finite Larmor Radius (FLR) effects are considered, it can be shown that Δ is a function of the quantities: $Q = v/\delta$, $Q_* = v_*/\delta$, $P = \mu/\eta$, $R = \rho/\delta$, where $\delta = (\eta/k)^{1/3}$ [14].

Conveniently, Δ can be used to determine the effective penetration: $(w/w_{vac})^2 = \Delta'_{coil}/|\Delta - \Delta'_{mode}|$, where w is the penetrated island width, w_{vac} is the island width in the absence of the plasma, Δ'_{mode} and Δ'_{coil} are real numbers that depend on the equilibrium configuration (see Ref.[3]). In order to determine the amount of penetrated flux, for a given set of physical parameters, it is therefore necessary to evaluate the velocity Q. This can be done by applying the "poloidal" momentum balance equation Eq.4 (with $\widehat{F}_y = 0$) to the inner region. In the standard theory, the balance between the electromagnetic force

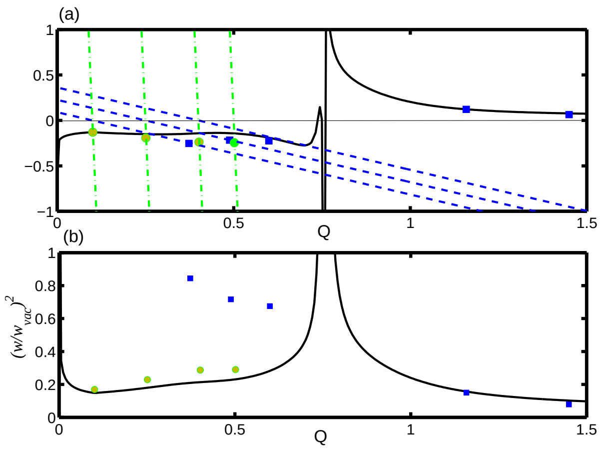

Figure 4: (a) The solid line shows the theoretical electromagnetic force and the dashed (dash-dot) straight lines the viscous force for a case with $w_{vac} = 0.4$ ($w_{vac} = 0.1265$) and different wall velocities: $Q_0 = \{0.1, 0.25, 0.4, 0.5\}$. Squares and circles are the result of the numerical simulations for $Q_0 = \{0.1, 0.25, 0.4, 0.5\}$ and $w_{vac} = 0.4$ and $w_{vac} = 0.1265$, respectively. (b) Fraction of penetrated magnetic flux for the same cases described above.

(which is proportional to the imaginary part of Δ and therefore depends on Q), and the viscous force (dependent on the difference between Q and the velocity at the wall, Q_0) provides a nonlinear implicit relation for v. This relation assumes that the contribution of the $(\mathbf{v} \cdot \nabla)v_y$ term is negligible. However, when diamagnetic effects are included in the theory and v is between the ion and the electron diamagnetic frequency, the magnetic island can excite drift waves, which can contribute to the momentum balance, thus modifying the standard picture. In particular, the waves act as a drag on the island and take the velocity at the reconnecting surface toward the electron diamagnetic velocity. This quasilinear effect is more important for larger islands, and it facilitates the penetration of the wall perturbation.

To verify this picture, we performed fully nonlinear electromagnetic simulations with the same code described in the previous section (with $l_x = 2$ and a grid of 128×64 points). The physical parameters are fixed and set to: $P = 0.2$, $R = 1.2$, $Q_* = 0.75$ and $\delta = 0.1$. Our results are shown in Fig. 4(a), which is a graphic representation of the "poloidal" momentum balance. Here the solid line describes a quantity proportional to the electromagnetic force, and the dashed (dash-dot) straight lines the viscous force for cases with $w_{vac} = 0.4$ ($w_{vac} = 0.1265$) and different wall velocities: $Q_0 = \{0.1, 0.25, 0.4, 0.5\}$. The shape of these curves is obtained using theoretical models which include FLR and diamagnetic effects [14, 15]. If the electromagnetic and viscous forces are the only relevant forces, the intersection of their characteristic curves gives the velocity of the plasma at the reconnection surface. However, when the plasma velocity is in the diamagnetic band ($0 \leq Q \leq Q_*$), our simulations show excitation of non localized waves. Furthermore, the characteristic points obtained from the simulations (represented by squares and circles for $w_{vac} = 0.4$ and $w_{vac} = 0.1265$, respectively) agree with the theoretical predictions only when the emitted waves are small (i.e. when the wall perturbation is small and Q is far from Q_*), or when they are not present at all ($Q > 0.75$). Figure 4(b) shows the theoretical (solid line) and numerical penetration fraction. In the presence of drift waves, the flux penetrates even if the standard theory predicts shielding.

5. Conclusions

We have investigated the interaction between drift waves and magnetic reconnection in two contexts: the dynamics of the NTMs and the penetration in the plasma of an external magnetic perturbation. In the first case, we have found that turbulent electrostatic fluctuations induce transitions in the rotation frequency of the NTM. The consequent slowing down of the magnetic island has a net destabilizing effect on small islands ($w \cong \rho$), while it becomes stabilizing for larger islands ($w \gg \rho$). In the second case, we show with numerical simulations that the standard theory of error field penetration fails to capture the penetration threshold when drift waves are present in the system. Indeed, the waves act as a drag on the magnetic island and, through a quasilinear mechanism, can facilitate the penetration of the wall magnetic perturbation.

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