Critical Toroidal Rotation Profile for Resistive Wall Modes and Control of Magnetic Islands in Tokamaks

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Abstract A three-mode model for the resistive wall modes based on the toroidally coupled resistive wall tearing modes is developed. The linear neoclassical tearing mode response is used at the singular surfaces to enhance plasma inertia and reduce the toroidal rotation speed for stability. The dispersion relation is obtained from the determinant of a 6×6 matrix. The toroidal rotation profile for the stability of the coupled mode can be determined by coupling the dispersion relation to the toroidal momentum balance equation. It is illustrated that toroidal rotation profile is more relevant to the stability of the resistive wall modes than the toroidal rotation speed at a given radius. The appearance of the magnetic islands degrades plasma confinement. It is shown that the width of the magnetic island can be controlled by the bootstrap current density induced by the ablation of the injected pellet at the island *O*-point. The magnetic island can be healed if the pellet induced bootstrap current density is large enough. It is also demonstrated that tailoring plasma profiles can control the stability of the island.

1. Introduction

The toroidal rotation speed that is required to stabilize the resistive wall modes in tokamaks is important to the successful operation of advanced tokamaks such as International Thermonuclear Experimental Reactor (ITER) [1]. We develop a three-mode model including the effects of the toroidal coupling [*e.g.*, toroidally coupled (m,1) and (m ± 1,1) mode, with poloidal mode number m=3] to describe resistive wall modes in tokamaks. The dispersion relation for the mode is derived from the determinant of a 6×6 matrix. The enhancement of plasma inertia, that leads to a reduced toroidal rotation speed for stability, and the dissipation resulting from the neoclassical effects are included in the layer physics. The toroidal plasma rotation speed at each rational surface and the mode frequency appear in the dispersion relation. In regions between the magnetic axis and a rational surface, between two rational surfaces (there are two such regions in our model), and between a rational surface and the plasma boundary, the toroidal plasma rotation profile is calculated by solving toroidal momentum diffusion equation including a momentum source with plasma rotation speed at the magnetic axis, at the rational surfaces, and at the plasma boundary as the boundary conditions. These coupled equations uniquely determine a toroidal plasma rotation profile for the stability of the resistive wall modes described by the model. The model is especially useful in determining the toroidal rotation profile for the stability when the toroidal rotation speed is externally controlled as is frequently done in the experiments. From the results of this model, it is noted that the toroidal rotation profile is more important than the rotation speed at a given radius in determining the stability of the resistive wall modes in tokamaks.

The appearance of magnetic islands in tokamaks degrades plasma confinement time for tokamaks that are intended for producing fusion power. It has been known that using the momentum of the radio frequency waves to drive a current inside the island can heal the island. However, to provide design flexibility, it is highly desirable to have alternative methods to control the magnetic island besides using the current driven by radio frequency waves. Here, we propose to inject pellets at the island *O*-point to heal the island. We derive the island evolution equation to include the bootstrap current density that is driven by the peaked plasma density inside the island resulting from the vaporization of the pellet. The effects of the asymmetric island shape have to be taken into account to obtain non-trivial results. We show either that the saturated island width can be reduced or that the island can be healed depending on the magnitude of the pellet driven bootstrap current. We also note that by controlling the plasma profiles one can also control magnetic islands. For a flat density profile, a flat electron temperature profile, and a steep ion temperature profile, magnetic island formation can be prohibited. The results of the theory can be tested in existing and future tokamak experiments such as ITER.

2. A Three-Mode Model for Resistive Wall Modes

The three-mode model for resistive wall modes developed here is based on the toroidally coupled resistive wall tearing mode [2-7]. To obtain an analytic expression for the dispersion relation, we solve Ampere's law by assuming a parabolic equilibrium current density profile and adopting the large aspect ratio expansion procedure in treating toroidal coupling [6,8]. The model consists of plasmas confined in the region $0 \le r \le a$, a resistive wall located at $r = b > a$, and vacuum in the region $a \le r \le b$ and $r \ge b$. Here, r is the local minor radius. For an (m,n) mode with the singular surface located at $r = r_{sm}$, we characterize it in terms of the perturbed poloidal flux $\psi_m(r)$ that is parameterized by its value at the singular surface ψ_{sm} and at the wall ψ_{wm}, where m is the poloidal mode number and n is the toroidal mode number. The boundary conditions are that $\psi_m(r)$ vanishes at $r = 0$ and ∞, and both $\psi_m(r)$ and $d\psi_m/dr$ are continuous at $r = a$. At $r = r_{sm}$, and $r = b$, $\psi_m(r)$ is continuous. Matching the discontinuity of $d\psi_m/dr$ to that of the linear neoclassical tearing mode at $r = r_{sm}$, and to that of the resistive wall at $r = b$ yield a 6×6 matrix

$$\begin{pmatrix} D_{ssm+1} & D_{swm+1} & D^{m}_{ssm+1} & D^{m}_{swm+1} & 0 & 0 \\ D_{wsm+1} & D_{wwm+1} & D^{m}_{wsm+1} & D^{m}_{wwm+1} & 0 & 0 \\ D^{m+1}_{ssm} & D^{m+1}_{swm} & D_{ssm} & D_{swm} & D^{m-1}_{ssm} & D^{m-1}_{swm} \\ D^{m+1}_{wsm} & D^{m+1}_{wwm} & D_{wsm} & D_{wwm} & D^{m-1}_{wsm} & D^{m-1}_{wwm} \\ 0 & 0 & D^{m}_{ssm-1} & D^{m}_{swm-1} & D_{ssm-1} & D_{swm-1} \\ 0 & 0 & D^{m}_{wsm-1} & D^{m}_{wwm-1} & D_{wsm-1} & D_{wwm-1} \end{pmatrix} \begin{pmatrix} \psi_{sm+1} \\ \psi_{wm+1} \\ \psi_{sm} \\ \psi_{wm} \\ \psi_{sm-1} \\ \psi_{wm-1} \end{pmatrix} = 0. \tag{1}$$

For simplicity, we will not display the expressions of the matrix elements and refer the readers to Ref.[9] for detailed calculations. The dispersion relation for the coupled modes is

$$\det \mathbf{D} = 0, \tag{2}$$

where $\mathbf{D}$ denotes the matrix in Eq.(1).

The '*ss*' elements in Eq.(1) contain $\Delta'(\gamma - in\Omega_m)$, the dimensionless tearing mode stability parameter. Here, γ is the mode frequency, and Ω_m is the toroidal rotation frequency of the plasma at $r = r_{sm}$. We adopt the linear neoclassical tearing mode response for Δ' to enhance plasma inertia [6,10-13]. The expression for $\Delta'(\gamma)$ is

$$\Delta'\delta = -\pi \frac{Q}{8} \frac{\Gamma(\frac{Q-1}{4})}{\Gamma(\frac{Q+5}{4})}, \tag{3}$$

where $\delta^2 = \delta_\eta \delta_{in}$, $Q = \delta_{in}/\delta_\eta$, $\delta_\eta^2 = 1/(\gamma\tau_R)$, $\delta_{in}^2 = \gamma^2 \tau_H^2 \mathcal{N}$, $\tau_R = r_s^2/\eta$, $\tau_H = R/(nsV_A)$, $s = r_s q_s'/q_s$, $V_A = B/\sqrt{\rho}$, η is plasma resistivity, ρ is mass density, R is the major radius, q is the safety factor, the subscript s denotes the quantity is evaluated at the singular surface, B is the magnetic field strength, prime denotes d/dr, and the inertia enhancement factor $\mathcal{N}$ is

$$\mathcal{N} = (\frac{B}{B_p})^2 \frac{\mu_1}{\mu_1 + \gamma} + (1 + 2q^2) \frac{\gamma}{\mu_1 + \gamma}. \tag{4}$$

The viscous coefficient μ_1 in the banana regime for large aspect ratio tokamaks can be approximated as $\mu_1 \approx 0.778\, \varepsilon^{1/2} \nu_{ii} + 1.6\varepsilon^{3/2} \gamma$, where ν_{ii} is the ion-ion collision frequency [14]. For toroidally rotating tokamak plasmas, we substitute γ in Eqs.(3), and (4) with $(\gamma - in\Omega_m)$ at each singular surface.

In between singular surfaces, plasma boundary, and the magnetic axis, we solve a simple toroidal momentum equation for toroidal angular frequency Ω

$$\frac{1}{r}\frac{\partial}{\partial r}(r D_\Omega \frac{\partial \Omega}{\partial r}) = S, \tag{5}$$

where D_Ω is the diffusion coefficient, most likely to be anomalous, and S is the momentum source.

It is obvious that when the toroidal angular frequency at one singular surface Ω_m is modified by the external toroidal momentum source, the angular frequencies at two other singular surfaces can respond to maintain the stability of the coupled mode. This qualitative conclusion seems to be compatible to the experimental observations [15,16]Thus, we conclude that the toroidal rotation profile is more important than the toroidal rotation speed at a given radius. The quantitative analysis of the dispersion relation together with the toroidal momentum balance equation will be presented separately [9].

3. Control of Magnetic Islands by Pellet Injection

When a pellet is ablated in the vicinity of a rational surface inside the plasma, the local plasma density increases. Because plasma confinement improves in the vicinity of the island [17], the peaked local plasma density lasts a long time. This phenomenon is observed in experiments and the helical variation of the peaked density following that of the island structure is called a snake [18,19]. The peaked plasma density inside the island modifies the local bootstrap current density and thus the stability property of the island.

In the vicinity of an island, it is convenient to label the magnetic surface in terms of the normalized helical flux function $\overline{\Psi}$. The dominant part of $\overline{\Psi}$ is symmetric relative to the singular surface and has the form of [20]

$$\overline{\Psi} = X^2 - \cos\xi , \qquad (6)$$

where $\overline{\Psi} = -\Psi/\tilde{\psi}_0$, $\tilde{\psi}_0$ is $\tilde{\psi}$ evaluated at r_s, $\delta\tilde{\psi} = \tilde{\psi}\cos\xi$ is the perturbed poloidal magnetic flux due to the presence of the island, $\tilde{\psi}$ is the amplitude of the perturbation, $\xi = m\theta - n\zeta$, θ is the poloidal angle, ζ is the toroidal angle, $X = x/\bar{r}_w$, $x = (r - r_s)/r_s$, $\bar{r}_w = r_w/r_s$, $r_w = \sqrt{2q_s^2\tilde{\psi}_0/(q_s' B_0 r_s)}$ is a measure of the width of the island, and B_0 is the magnetic field strength at the magnetic axis. The full width of the island $w = 2\sqrt{2}\ r_w$. The region inside the separatrix of the island is defined by $-1 < \overline{\Psi} < 1$. The island O-point is located at $\overline{\Psi} = -1$, and the separatrix is at $\overline{\Psi} = 1$. Because the width of the island is finite, solving Ampere's law to higher order results an asymmetric modification to Eq.(6) [21-23]

$$\overline{\Psi}_1 = -\frac{1}{3}\bar{r}_w X^3 + X\, C(\xi), \qquad (7)$$

where $C(\xi)$ is a function of ξ. The detailed functional dependence of $C(\xi)$ is not important to our theory because it cancels out in our calculations. To investigate the stability property of the island in the presence of the pellet induced bootstrap current density, we need to know the asymmetric part of $\overline{\Psi}$.

The pellet induced bootstrap current density J_b is

$$J_b = -2.4\ \frac{c}{r_w}\frac{dp}{d\overline{\Psi}}\left\langle\frac{\sqrt{\varepsilon}}{B_p}\ \frac{\partial\overline{\Psi}}{\partial X}\right\rangle, \qquad (8)$$

where the angular brackets denote the average over the constant $\overline{\Psi}$ surface [23]: $\langle A\rangle = (\oint d\xi(1+\bar{r}_w X)A/|\partial\overline{\Psi}/\partial X|)/\oint d\xi(1+\bar{r}_w X)/|\partial\overline{\Psi}/\partial X|$. Note that $\sqrt{\varepsilon}$ and B_p are inside the angular brackets in Eq.(5). This indicates that the r_w modifications to those quantities are important. After performing the averaging processes using Eqs.(6) and (7) yields,

$$J_b = -4.8\ \sqrt{\varepsilon_s}\ \frac{cp}{r_s B_p}\frac{1}{p}\frac{dp}{d\overline{\Psi}}\left\{[3+2(s-1)]\left[\frac{E(1/k)}{K(1/k)} - \frac{k^2-1}{k^2}\right] - \frac{8}{3}\left[\frac{\sin^{-1}(1/k)}{K(1/k)}\right]^2\right\}, \qquad (9)$$

where $k = \sqrt{2/(1+\overline{\Psi})}$, K, and E are complete elliptic integral of the first and the second kind respectively.

Substituting Eq.(9) into Ampere's law, integrating it radially, and matching the jump in the radial component of the perturbed magnetic field to that of the tearing mode stability parameter yield an evolution equation for the width of the island

$$\frac{\partial r_w}{\partial t} = 0.43\frac{\eta c^2}{4\pi}\left\{\Delta' + 0.503[1+2.646(s-1)]\ \frac{\sqrt{\varepsilon_s}}{r_w}\beta_p\frac{q_s}{r_s q_s'}\frac{1}{p(-1)}\frac{dp}{d\overline{\Psi}}\right\}, \qquad (10)$$

where $\beta_p = 8\pi p(-1)/B_p^2$ is the ratio of the peaked plasma pressure inside the island to the pressure of the poloidal magnetic field, and $p(-1)$ is the plasma pressure at the island O-point. We have assumed that $dp/d\overline{\Psi}$ is a constant in deriving Eq.(10) for simplicity. From Eq.(9), we see that pellet induced bootstrap current density indeed affect the island evolution, and its saturation. If we include the effects of the flattening of the equilibrium bootstrap current density into Eq.(10), we obtain

$$\frac{\partial r_w}{\partial t}=0.43\frac{\eta c^2}{4\pi}\{\Delta'+0.503[1+2.646(s\text{-}1)]\frac{\sqrt{\varepsilon_s}}{r_w}\beta_p\frac{q_s}{r_s q_s'}\frac{1}{p}\frac{dp}{d\overline{\Psi}}\text{-}2.58\frac{\sqrt{\varepsilon_s}}{r_w}\beta_{p0}\frac{q_s}{r_s q_s'}\frac{r_s}{p_0}\frac{dp_0}{dr}\},\quad(11)$$

where the subscript 0 indicates the equilibrium quantity is evaluated at r_s prior to the appearance of the magnetic island.

For a negative value of Δ' and a negative value of $r_s dp_0/dr/p_0$, the saturated island width is

$$r_w=\frac{(\sqrt{\varepsilon_s}q_s/r_s q_s')\{2.58\beta_{p0}r_s dp_0/(p_0 dr)-0.503[1+2.646(s-1)]\beta_p dp/(pd\overline{\Psi})\}}{\Delta'}.\qquad(12)$$

The width of the saturated island width is reduced if $\beta_p|(dp/d\overline{\Psi})/p|$ is comparable to $\beta_{p0}|(r_s dp_0/dr)/p_0|$. Indeed, a peaked plasma pressure that is about a factor of two larger than the ambient pressure has been observed in snakes [19]. Thus, β_p can be about a factor two larger than β_{p0}, and if $|(dp/d\overline{\Psi})/p|$ is comparable to $|(r_s dp_0/dr)/p_0|$ the saturated island width is reduced by about a factor of 40% if $s=1$. Of course, if the terms in the numerator cancel each other, the neoclassical island is eliminated completely. To achieve such precision, the size of the pellets has to be controlled, and plasma parameters around r_s prior to the appearance of the magnetic islands measured accurately. Likely, one needs to perform many experiments of this kind to gain the necessary experience to perfect the control scheme.

The results shown in Eqs.(10)-(12) can be further refined by including the electron and the ion temperature gradients in the bootstrap current density expression. For example, Eq.(10) is modified to [24,25]

$$\frac{\partial r_w}{\partial t}=0.43\frac{\eta c^2}{4\pi}[\Delta'+0.503[1+2.646(s-1)]\frac{\sqrt{\varepsilon_s}}{r_w}\beta_p\frac{q_s}{r_s q_s'}(\frac{1}{p(-1)}\frac{dp}{d\overline{\Psi}}\text{-}0.67\frac{p_e}{p(-1)T_e}\frac{dT_e}{d\overline{\Psi}}\text{-}1.17\frac{p_i}{p(-1)T_i}\frac{dT_i}{d\overline{\Psi}})],\qquad(13)$$

where p_e is the electron pressure, p_i is the ion pressure, and T_i is the ion temperature. All these quantities are functions of $\overline{\Psi}$. Of course, $p(-1)$ is the total pressure as indicated previously. To obtain Eq.(13), we have assumed that the ratios of p_e/T_e and p_i/T_i are not a function of $\overline{\Psi}$, and that $dT_e/d\overline{\Psi}$ and $dT_i/d\overline{\Psi}$ are constants for simplicity. Examining Eq.(13), it is obvious that it is less effective to use either the electron temperature gradient term or the ion temperature gradient term to reduce the original drive for the neoclassical island because the numerical numbers in front these terms are only a fraction of unity.

We also like to point out an interesting possibility to remove neoclassical island by controlling plasma profiles. As noted in Eq.(13), the contributions of the temperature gradients to the island evolution equation can be very different from that of the density gradient. If we include the terms from the temperature gradients in the island evolution equation, it becomes

$$\frac{\partial r_w}{\partial t} = 0.43\frac{\eta c^2}{4\pi}[\Delta' - 2.58\frac{\sqrt{\varepsilon_s}}{r_w}\beta_{p0}\frac{q_s}{r_s q_s'}(\frac{r_s}{p_0}\frac{dp_0}{dr} - 0.67\frac{r_s p_{e0}}{T_{e0}p_0}\frac{dT_{e0}}{dr} - 1.17\frac{r_s p_{i0}}{T_{i0}p_0}\frac{dT_{i0}}{dr})]. \quad (14)$$

Thus, for a flat density profile, and a flat electron temperature profile, neoclassical island can be healed if ion temperature profile is steep enough to change the overall sign of the terms inside the parenthesis in Eq.(14).

A similar theory using neutral particle beam injection instead of pellet injection to control magnetic island has also been proposed with a different method in obtaining the asymmetric part of the island magnetic surface [26].

4. Conclusions

We have illustrated using a three-model based on the three toroidally coupled resistive wall tearing modes that the toroidal rotation profile is more relevant to the stability of the resistive wall modes than the toroidal rotation speed at a given radius.

We have also demonstrated that nonlinear island evolution and the saturation of the island can be controlled by the bootstrap current density that results from the peaked local density of ablated pellets at the island *O*-point. It is possible that the magnetic island can be healed if the pellet induced bootstrap current density cancels that from the equilibrium in the island evolution equation. It is also noted that tailoring plasma profiles may heal the island as well. For a flat density profile, and a flat electron temperature profile, neoclassical island can be healed if ion temperature profile is steep enough to change the overall sign of the terms inside the parenthesis in Eq.(14).

5. References

[1] BONDESON, A., and WARD, D., Phys. Rev. Lett. **72** (1994) 2709.
[2] FINN, J. M., Phys. Plasmas **2** (1995) 198.
[3] GIMBLETT, C. G., and HASTIE, R. J., Phys. Plasmas **7** (2000) 258.
[4] FITZPATRICK, R., and AYDEMIR, A. Y., Nucl. Fusion **36** (1996) 11.
[5] CHU, M. S., et al., Phys. Plasmas **2** (1995) 2236.
[6] MIKHAILOVSKII, A. B., and KUVSHINOV, B. N., Plasma Phys. Rep. **21** (1995) 789.
[7] MANICKAM, J, et al., Phys. Plasmas **2** (1994) 1601.
[8] CONNOR, J. W., et al., Phys. Fluids **31** (1988) 577.
[9] SHAING, K. C., et al., (in preparation, 2008).
[10] BONDESON, A., and CHU, M. S., Phys. Plasmas **3** (1996) 3013.
[11] MIKHAILOVSKII, A. B., and TSYPIN, V. S., Sov. J. Plasma Phys. Rep. **9** (1983) 91.
[12] SHAING, K. C., Phys. Plasmas **14** (2007) 052511.
[13] PORCELLI, F., Phys. Fluids **30** (1987) 1734.
[14] HSU, C. T., et al., Phys. Plasmas **1** (1994) 132.
[15] SABBAGH, S. A., et al., Phys. Rev. Lett. **97** (2006) 045004.
[16] SABBAGH, S. A., et al., Nucl. Fusion **46** (2006) 635.
[17] SHAING, K. C., Phys. Plasmas **9** (2002) 3470.
[18] WELLER, A., et al., Phys. Rev. Lett. **59** (1987) 2303.
[19] GILL, R. D., et al., Nucl. Fusion **32** (1992) 723.

[20] RUTHERFORD, P. H., Phys. Fluids **16** (1973) 1903.
[21] HASTIE, R. J., et al., Phys. Rev. Lett. **95** (2005) 065001.
[22] MILITELLO, F., et al., Phys. Plasmas **13** (2006) 112512.
[23] ARCIS, N., et al., Phys. Plasmas **13** (2006) 052305.
[24] HINTON, F. L., and HAZELTINE, R. D., Rev. Mod. Phys. **48** (1976) 239.
[25] HIRSHMAN, S. P., and SIGMAR, D. J., Nucl. Fusion **21** (1981) 1079.
[26] SEN, A., KAW, P. K., and CHANDRA, D., Nucl. Fusion **40** (2000) 707.