Stabilization of the Vertical Mode in Tokamaks by Localized Nonaxisymmetric Fields

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Abstract. Vertical instability of a tokamak plasma can be controlled by nonaxisymmetric magnetic fields localized near the plasma edge at the bottom and top of the torus. The required magnetic fields can be produced by a relatively simple set of parallelogram-shaped coils.

The tokamak is the most widely used device in large scale experimental facilities for magnetic confinement fusion research. In this paper we show that vertical instability of the tokamak plasma, which imposes an important constraint on tokamak design, can be controlled by nonaxisymmetric magnetic fields localized near the plasma edge at the bottom and top of the torus. The required magnetic fields can be produced by a relatively simple set of parallelogram-shaped coils (Fig. 1) placed near the bottom and top of the torus.

Since the late 1960's, the tokamak has evolved from an axisymmetric device with circular cross-section to an axisymmetric device with strongly shaped cross-section. It is natural to ask whether it would be advantageous to add some nonaxisymmetric shaping. There are already some experimental studies in that direction showing that edge localized modes (ELMs) can be stabilized using non-axisymmetric fields.[1] This was also the line of reasoning, in part, that motivated the physics design studies for the US National Compact Stellarator Experiment (NCSX), presently under construction.(Ref. [2] and references therein.)

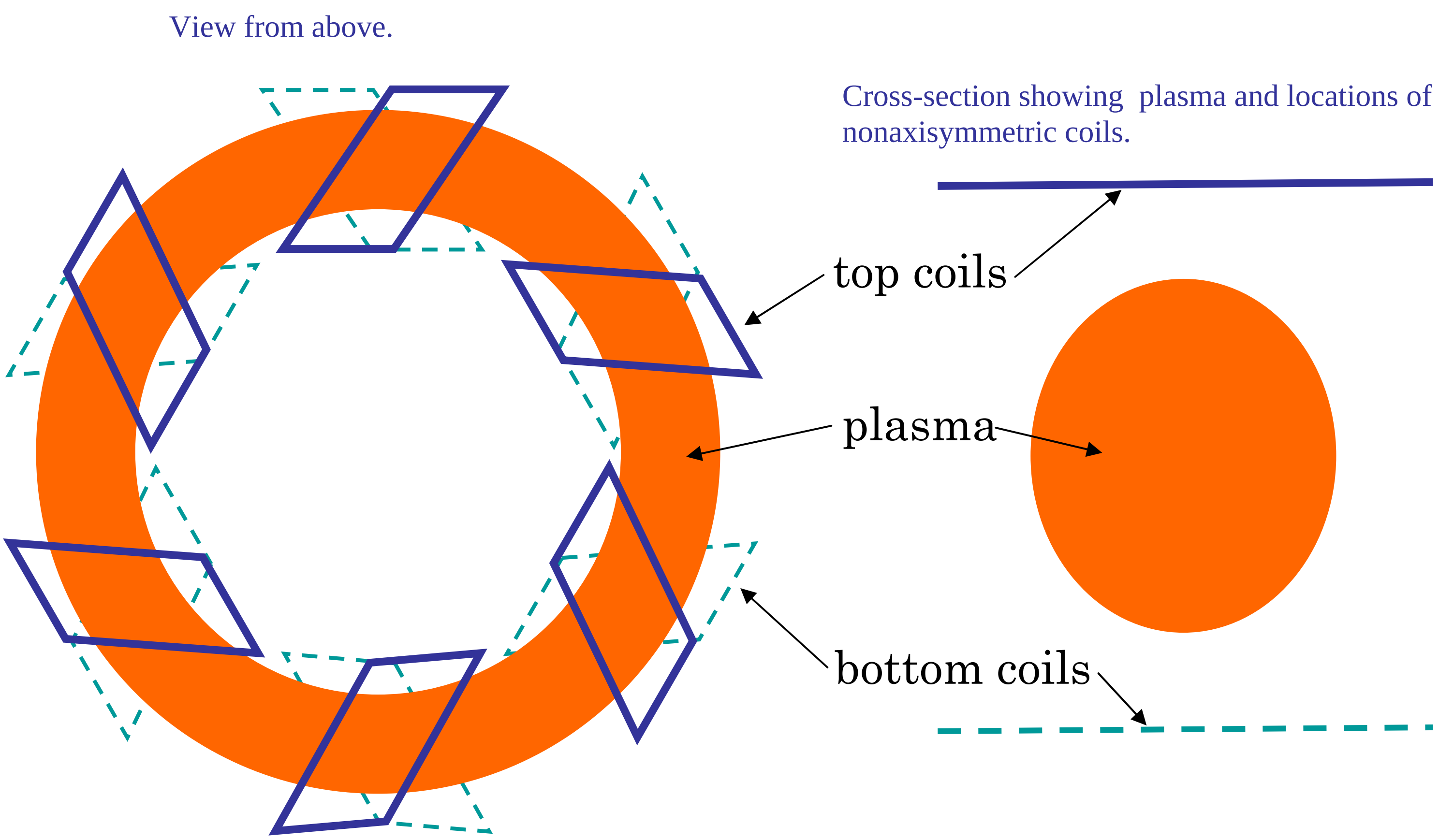

Fig. 1. *Parallelogram-shaped coils for stabilizing the vertical mode in a tokamak.*

This paper focuses on the stabilization of the vertical mode in tokamaks. As we will discuss below, the free energy driving the vertical instability in tokamaks increases with increasing vertical elongation of the plasma cross-section. On the other hand, empirically derived global confinement scaling laws for tokamaks find that confinement improves with increasing vertical elongation. Similarly, the Troyon scaling law for plasma stability predicts an increase in the β limit for ballooning and kink modes with increasing elongation. There is evidence that the benefits of increasing elongation diminish and perhaps disappear altogether at sufficiently high elongation[3], but the elongation at which this occurs is well above that at which the largest present day tokamaks can routinely operate. Contemporary tokamaks typically operate in a regime where a conducting wall surrounding the plasma stabilizes the vertical mode, with the mode remaining unstable on the resistive time scale of the wall. Feedback stabilization is used to suppress the resistive mode. Disruptions due to vertical instability are sufficiently common that they have been given a name and an acronym: vertical displacement events (VDEs). These may be caused by accidental crossing of the ideal instability threshold during a shot, failure of the feedback control system, etc. Disruptions initiated by other instabilities often culminate in a vertical mode. In these cases, a sudden loss of thermal energy and a redistribution of the current destabilize the vertical mode. (See Ref. [4] and references therein).

The nonaxisymmetric fields discussed in this paper are not stellarator fields, in the sense that they do not have closed vacuum flux surfaces (they do not produce closed flux surfaces in the absence of a plasma current), and they do not generate vacuum rotational transform. Three-dimensional magnetic fields provide control over field line properties not available in axisymmetric configurations such as tokamaks. In an axisymmetric configuration, field lines cannot circle the magnetic axis in the absence of a net toroidal plasma current. The absence of vacuum flux surfaces is closely related to the absence of vacuum rotational transform. In order to trace out vacuum flux surfaces, it is necessary for the magnetic field lines to wind around in the poloidal direction. The generation of vacuum flux surfaces and rotational transform requires global three-dimensional magnetic fields, which employ the control provided by nonaxisymmetry along the entire field line trajectory. In this paper, we take advantage of the control provided by three-dimensional fields to target the physics of the vertical instability more directly, providing stabilization by a localized three-dimensional magnetic field that does not produce vacuum flux surfaces.

Stellarator fields are generally produced either by helical coils that wind around the plasma, or by modular coils, which have the appearance of three-dimensionally deformed toroidal field coils. The coils that we describe here for generating our localized three-dimensional field are simpler, and they do not link the plasma. They could potentially be installed on existing tokamaks.

The physical mechanism driving the vertical instability can be understood by treating the plasma as a large aspect-ratio, current-carrying conductor in a vertical magnetic field. (See Ref. [7] and references therein.) Adopting the conventional cylindrical coordinates (R,ϕ, z), ϕ is the toroidal angle, and R is the radial coordinate. Take the vertical field to be approximately uniform, with a small quadrupole component added to control the plasma ellipticity. If the conductor is displaced slightly in the vertical direction, the sign of the resulting force is determined by the sign of $\partial B_R / \partial z$. For an axisymmetric externally generated vacuum field, $\nabla \times \mathbf{B} = 0$ relates this to the sign of $\partial B_z / \partial R$, which is in turn determined by the sign of the quadrupole field. A quadrupole field that increases the vertical elongation produces a destabilizing change in $\partial B_R / \partial z$.

Allowing the magnetic field to be nonaxisymmetric decouples $\partial B_R / \partial z$ from $\partial B_z / \partial R$. To stabilize the vertical mode, we add a nonaxisymmetric field whose appropriately averaged value of $\partial B_R / \partial z$ in the plasma is stabilizing.

Fu has analytically calculated the stabilization of the vertical mode by a stellarator field for a large aspect-ratio, low β, elliptically shaped plasma using the stellarator expansion[5]. The analytical stability criterion was found to agree well with numerical calculations[6]. The calculation described in this paper differs from that of Ref. [5] in using a localized nonaxisymmetric magnetic field produced by a relatively simple set of coils. As already mentioned, the nonaxisymmetric field does not produce vacuum rotational transform. As in Ref [5] (and as in much of the analytical work on tokamak vertical instabilities (Ref. [7,8] and references therein.)) we assume a large aspect ratio plasma that is well approximated by a cylindrical plasma with periodic boundary conditions at z=0 and $z = L = 2\pi R$, we take $\beta = 0$, and we assume a uniform equilibrium current density in the plasma. In the following, when we use the term “nonaxisymmetry” in the context of the large aspect ratio limit it should be taken to mean “z-dependent”. We pursue the analytical calculation with these simplifying assumptions for the purpose of demonstrating the physics of the stabilization, and to obtain an estimate of the required magnitude of the nonaxisymmetric field for stabilization, indicating that application of the effect appears to be reasonable. Numerical calculations will be required for more detailed evaluation and design optimization.

To construct coils, we first consider surface currents on two ribbons defined by $y = \pm y_c$, $-w/2 \le x \le w/2$, where we take the y axis to be in the vertical direction, the x axis in the horizontal direction, and the z axis to be parallel to the cylindrical plasma. To simplify the calculations, we will assume that w is sufficiently large that edge effects can be neglected in calculating the field in the plasma.

Letting $\boldsymbol{K}$ denote the surface current, it follows from $\nabla \cdot \boldsymbol{K} = 0$ that we can write $\boldsymbol{K}$ in terms of a current potential, $\boldsymbol{K} = \nabla \times (u\hat{y})$, where u is taken to vanish everywhere except on the two ribbons. Specified in this form, the current is explicitly divergence free. We will focus primarily on the surface current on the upper ribbon and the field produced by that current. The surface current on the lower ribbon and the associated field will follow by imposing stellarator symmetry. This symmetry property, which is generally satisfied by stellarators, dictates that $u_-(x,z) = u_+(x,-z)$, where the “+” and “-“ subscripts denote the current potentials on the upper and lower ribbon respectively. We will suppress the “+” subscript in the following when we are focusing on the upper ribbon. We will be interested in relatively localized fields, whose magnitude dies off rapidly as a function of distance from the coil, so that the nonlinear effects of the magnetic fields produced by the two sets of coils will be important in nonoverlapping regions, allowing us to calculate the nonlinear effects due to the two sets of coils separately.

In the interior of the upper ribbon we take u to depend on x and z only through $x - \alpha z$, $u(x,z) = u(x - \alpha z)$, where α is a constant. Let N be the number of periods in the toroidal direction. That is, u is periodic in z with periodicity length $2\pi R / N$, $N \ge 1$. Fourier decomposing with respect to z, and taking u to be even in z, we get

$u(x,z)=\sum_{n=1}^{\infty}\cos[n(k_x x+k_z z)]$ where $k_z = N/R$, $k_x=-k_z/\alpha$. Each harmonic corresponds to a surface current in the ribbon interior that is in the $\pm(\hat{z}+\alpha\hat{x})$ direction, with the amplitude varying sinusoidally as a function of x - αz. There is a delta function current along the edges of the ribbon at $x=\pm \mathrm{w}/2$ that connects the current in the alternating directions and preserves $\nabla\cdot \boldsymbol{K} = 0$. If $u_0 \neq 0$, the delta function current on each edge has a nonzero axisymmetric component. We assume that the contribution of this axisymmetric current to the field is canceled by nearby axisymmetric poloidal field coils.

The vacuum field produced by the surface current can be expressed in terms of a scalar potential, $\mathbf{B}=\nabla\chi$. From $\nabla\cdot\mathbf{B}=0$ it follows that $\nabla^2\chi=0$. The jump conditions across the ribbon at $y=y_c$ are $[[\hat{y}\cdot\mathbf{B}]]=0$ and $[[\hat{y}\times\mathbf{B}]]=\mu_0\mathbf{K}$, where [[ ]] denotes the difference between the value just above y_c and just below y_c. For $y < y_c$, and w/2 - $|x|$ sufficiently large relative to $|y_c - y|$ we get

$$\chi(x,y,z)=\sum_{n=1}^{\infty}\chi_n \exp[nk(y-y_c)]\cos[n(k_x x+k_z z)],$$

where k_x and k_z are as defined above, $k=(k_x^2+k_z^2)^{1/2}$, and $\chi_n=\mu_0 u_n/2$. We will make the simplifying assumption that w is sufficiently large that we can use this expression throughout the region of interest in the plasma.

For values of k that are of interest, at most a few of the low order Fourier modes in Eq. (2) have a significant effect on the field in the plasma. The magnitudes of the higher harmonics decrease rapidly as a function of distance from the coils. This gives us some freedom in our choice of the u_n for designing a set of coils. A particularly simple set of coils can be obtained if u is taken to have the form of a square wave,

$$u(x,z)=u_1\left\{\sum_{n=1}^{\infty}\left(-1\right)^{n+1}\frac{\cos\left[(2n-1)(k_x x+k_z z)\right]}{(2n-1)}-\frac{\pi}{4}\right\}.$$

This gives a set of filamentary coils in the shape of parallelograms.

For the plasma equilibrium, we use the stellarator expansion.[11] Let $\mathbf{B}_c$ be the field produced by the nonaxisymmetric coils, $\mathbf{B}_p$ the field produced by the plasma current in the absence of $\mathbf{B}_c$, and $B_t\hat{z}$ the field produced by the toroidal field coils. The stellarator expansion assumes $B_t >> B_c >> B_p$. To zeroth order, the magnetic field line trajectories are straight lines in the z direction. To first order, each field line sees a sinusoidally varying B_{cx} and B_{cy} along its path, causing the field lines to spiral helically about the unperturbed orbit. The order B_c^2 effect arises from the fact that the helical field line trajectories see a larger $\mathbf{B}_c$ perturbation when they are at larger values of y. This produces a net drift of the field lines in the $\pm\hat{\mathbf{x}}$ direction, analogous to the well known grad-B drift of particle trajectories in a magnetic field. A simple perturbation analysis of the field line trajectories breaks down in second order, because the drift is a secular contribution that becomes larger than the first order terms. The secularity can be handled by standard multiple scale methods, which incorporate it into the zeroth order term. The method of averaging[12] is one such multiple scale method. It averages the second order effects over a period and constructs an effective axisymmetric field that includes these effects. We will denote this averaged field by $\overline{\mathbf{B}}_c$. The validity of this treatment requires that Δx, the drift of the field line over one period, satisfy $k_x \Delta x \ll 1$. This gives an additional condition for the validity of the stellarator expansion: $(k_x / k_z)(B_c / B) \ll 1$.

To simplify the stability analysis, we take $k(y_c - a) >> 1$, where a is the minor radius of the plasma, so that only the lowest harmonic of Eq. (2) needs to be retained in the plasma. The averaged nonaxisymmetric field is then calculated to be $\bar{\mathbf{B}}_c = \nabla \psi_c \times \hat{\mathbf{z}}$, with $\psi_c = -\exp(2ky)\chi_1^2 k_x k / (2k_z B_t)$. The constant ψ_c surfaces are surfaces of constant y. As already mentioned, the nonaxisymmetric magnetic field does not produce closed vacuum flux surfaces.

Taking $B_t > 0$, the averaged nonaxisymmetric field is in the $\mp\hat{\mathbf{x}}$ direction, depending on the sign of k_x / k_z, and its magnitude increases with increasing y. From the discussion of the physics of the vertical instability, above, we expect that this will be stabilizing for the vertical mode if $j_z k_x / k_z > 0$, where j_z is the z component of the current density. To provide a more rigorous and quantitative evaluation of the vertical stability, we use the energy principle[13] in the form $\delta W = \delta W_p + \delta W_v$, where $\delta W_p = (1/2)\int_p \left(|Q|^2 / \mu_0 - \boldsymbol{\xi} \cdot \mathbf{j} \times \mathbf{Q} \right) d^3x$ is an integral over the plasma volume, $\delta W_v = (1/2)\int_v ((B_V^{(1)})^2 / \mu_0) d^3x$ is an integral over the vacuum region, $\mathbf{Q} \equiv \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$, $\boldsymbol{\xi}$ is the plasma displacement, and $\mathbf{B}_V^{(1)}$ is the perturbed field in the vacuum region. The pressure has been taken to be negligible In evaluating δW_v, we will take the boundary conditions at infinity, with no stabilization due to conductors outside the plasma.

It follows from the work of Johnson and Greene [14] that, under the assumptions of the stellarator approximation, the equilibrium field **B** can be replaced by $\mathbf{B}_p + \mathbf{B}_c$ in the above expression for δW_p. The perturbed vacuum field is determined by the perturbed plasma boundary, which in turn is determined by $\mathbf{B}_p + \mathbf{B}_c$ and ξ.

We will assume that the plasma cross-section is approximately circular, with a small elliptical perturbation of the boundary shape and a small perturbation of the boundary shape due to the nonaxisymmetric field. It will be convenient to use both cylindrical coordinates (r, θ, z) and Cartesian coordinates (x, y, z) in the stability calculation. The algebra will be simplified somewhat by taking $\theta = 0$ to lie along the y axis, so that $y = r\cos(\theta)$, $x = -r\sin(\theta)$.

Stability is determined by the sign of δW, with the equilibrium unstable if $\delta W < 0$. For the zeroth order equilibrium, a cylindrical plasma with circular cross-section, we Fourier transform $\boldsymbol{\xi}$ with respect to z and θ. The cross terms in δW between the $\boldsymbol{\xi}_{nm}$ with different m and/or n vanish, and δW can be evaluated independently for each $\boldsymbol{\xi}_{nm}$, where m is the poloidal mode number and n is the toroidal mode number. For an equilibrium with uniform current density, $\delta W > 0$ for n = 0, m > 1. For n = 0, m = 1, the $\xi_{0,1}(r)$ which minimizes δW corresponds to a rigid shift of the plasma, and gives $\delta W = 0$.

Introducing a small nonaxisymmetric field, as described above, plus a small axisymmetric elliptical distortion of the plasma boundary, the averaged equilibrium field is $\mathbf{B}_p + \mathbf{B}_c = \nabla\psi \times \hat{\mathbf{z}} + B_0\hat{\mathbf{z}}$, where

$$\psi = \psi_0 \left\{ r^2 \left[1 - 2\varepsilon_{\mathrm{e}} \cos(2\theta) \right] + \varepsilon_{\mathrm{c}} a^2 \exp[2k(y-a)] \right\},$$

ψ_0 is a constant, and

$$\psi_0 \varepsilon_{\mathrm{c}} = -\frac{1}{2} \frac{\chi_1^{\ 2}}{B_t} \frac{k_x k}{a^2 k_z} e^{2ka}.$$

For $\varepsilon_{\mathrm{c}} = 0$, the equilibrium is vertically unstable when $\varepsilon_{\mathrm{e}} < 0$, corresponding to vertical elongation of the plasma.[7] To calculate the stability for $\varepsilon_{\mathrm{c}} \neq 0$ we evaluate δW_p and δW_{v}. The averaged equilibrium field that goes into these expressions is axisymmetric, so we can Fourier transfer $\boldsymbol{\xi}$ as a function of $\phi = z/R$ and analyze the stability independently for each $\boldsymbol{\xi}_n$. We are interested in $n = 0$.

It can be expected that the displacement $\boldsymbol{\xi}$ that minimizes δW will depend on ε_{c}. Denote the field for the circular cylinder by $\mathbf{B}_0$. For $\mathbf{B} = \mathbf{B}_0$, restricting consideration to $n = 0$ displacements, δW has a local (and global) minimum at $\boldsymbol{\xi} = \xi_0 \hat{\mathbf{y}}$. It follows that if we add an $\mathrm{O}(\varepsilon)$ perturbation to $\boldsymbol{\xi} = \xi_0 \hat{\mathbf{y}}$, the resulting change in δW is $\mathrm{O}(\varepsilon^2)$. On the other hand, we will see that an $\mathrm{O}(\varepsilon)$ perturbation of $\mathbf{B}_0$ produces an $\mathrm{O}(\varepsilon)$ change in δW. It follows that we can evaluate the $\mathrm{O}(\varepsilon)$ change in δW using $\boldsymbol{\xi} = \xi_0 \hat{\mathbf{y}}$. This result is closely related to the well known property of the variational formulation of eigenvalue equations that an $\mathrm{O}(\varepsilon)$ error in the eigenfunction gives an $\mathrm{O}(\varepsilon^2)$ error in the eigenvalue.

The perturbed field in the plasma produced by the displacement $\boldsymbol{\xi} = \xi_0 \hat{\mathbf{y}}$ is $\mathbf{B}_{\mathrm{p}}^{(1)} = \mathbf{Q} = \nabla \times \psi_{\mathrm{p}}^{(1)}$, where $\psi_{\mathrm{p}}^{(1)} = -\xi_0 \partial\psi/\partial y = -2\xi_0\psi_0 \left\{ \left[1 - 2\varepsilon_e \right] r\cos\theta + \varepsilon_c k a^2 \exp[2k(y-a)] \right\}$.
The continuity of $\hat{\mathbf{n}} \cdot \mathbf{B}$ at the plasma-vacuum interface (where $\hat{\mathbf{n}}$ is the unit normal to the interface) is equivalent to requiring $\psi_{\mathrm{V}}^{(1)} = \psi_{\mathrm{p}}^{(1)}$ at the interface, where $\mathbf{B}_{\mathrm{V}}^{(1)} = \nabla \times \psi_{\mathrm{V}}^{(1)}$ in the vacuum region. To O(ε), the plasma boundary is given by $r = a\left\{ 1 + \varepsilon_e \cos(2\theta) - (1/2)\varepsilon_c \exp[2ka(\cos\theta - 1)] \right\}$. Note that the nonaxisymmetric field decreases the plasma width in the vertical direction. For $ka \gg 1$ the effect is localized near x=0, and we will see that for $ka \gg 1$ the reduction in width is small compare to the effect of the ε_e term.

In the following we make repeated use of the identity

$$\exp(2ky) = \exp(2kr\cos\theta) = I_0(2kr) + 2\sum_{j=1}^{\infty} I_j(2kr)\cos(j\theta).$$

In the vacuum region, $\nabla \times \mathbf{B}_{\mathrm{V}}^{(1)} = 0$ gives $\nabla^2 \psi_{\mathrm{V}}^{(1)} = 0$, so that

$$\psi_{\mathrm{V}}^{(1)} = \sum_{m=0}^{\infty} \psi_{\mathrm{V}m}^{(1)} (r/a)^{-m} \cos(m\theta).$$

To lowest order, matching $\psi_{\mathrm{V}}^{(1)} = \psi_p^{(1)}$ at the plasma boundary gives $\psi_{\mathrm{V}}^{(1)} = -2\xi_0\psi_0 a\cos\theta$. The $m \neq 1$ terms in $\psi_{\mathrm{V}}^{(1)}$ are O(ε). It is straightforward to verify that the contributions of the $m \neq 1$

terms to δW_{V} are O(ε^2). It follows that we only need concern ourselves with the $m=1$ term in $\psi_{\mathrm{V}}^{(1)}$. Matching at the boundary gives

$$\psi_{\mathrm{V}1}^{(1)} = 2\xi_0\psi_0\left\{\varepsilon_c \exp(-2ka)[2kaI_1\left(2ka\right) - I_0(2ka) - I_2(2ka)] - a^2(1-\varepsilon_e)]\right\},$$

where I_j is the modified Bessel function of the jth kind.

Having calculated the perturbed field in the plasma and in the vacuum region, we can now evaluate δW. Taking into account the nonaxisymmetric coils at the bottom of the torus as well as those at the top we pick up an extra factor of 2 in front of ε_c. We find

$$\delta W = 4V_p(\xi_0\psi_0)^2\left\{\varepsilon_c \exp(-2ka)[2kaI_1\left(2ka\right) - I_0(2ka) - 2I_2(2ka)] - \varepsilon_e\right\}.$$

For large ka, $I_j(2ka) \approx \exp(2ka) / (4\pi ka)^{1/2}$, giving $\delta W \approx 4V_p(\xi_0\psi_0)^{1/2}\left[\varepsilon_c(ka/\pi)^{1/2} - \varepsilon_e\right]$. Note that the sign of ε_c matters. Depending on the sign, the nonaxisymmetric field can either stabilize or destabilize the vertical mode. The sign is determined by the sign of the pitch of the coil filaments relative to the sign of the pitch of the magnetic field lines, which in turn determines the sign of $\partial\bar{B}_x/\partial z$ relative to that of j_z.

Expressing the stability condition in terms of the maximum value of B_c/B in the plasma ≡ $\max_p(B_c/B)$ and in terms of κ, we get

$$\max{}_p(B_c/B)^2 > (a/R)(k_z/k_x)(\pi ka)^{1/2}(\kappa-1)/(2q)$$

for stability. With the condition on the validity of the stellarator expansion that $(k_x/k_z)(B_c/B) \ll 1$, we get $\max{}_p(B_c/B) \gg (a/R)(\pi ka)^{1/2}(\kappa-1)/(2q)$. This suggests that for $R/a \approx 3$ and $q \approx 3$ we need $\max{}_p(B_c/B) > .1$ to see a substantial stabilization effect.

Finally, we can conjecture about the nonlinear behavior of the vertical instability in the presence of the nonaxisymmetric field. The δW analysis calculates the response to an infinitesimal perturbation, and it depends on $\partial\bar{B}_{cx}/\partial y$. A finite vertical excursion of the plasma sees an exponential increase in $\bar{B}_{cx}$, and this suggests that the nonaxisymmetric field can prevent large vertical excursions of the plasma even for equilibria that are linearly unstable to the vertical mode. This also suggests that, although the linear stabilization described here can be obtained with only a single set of nonaxisymmetric coils either at the top or bottom of the plasma, it is likely desirable to have both sets of coils for suppression of finite vertical excursions.

In conclusion, the analysis of this paper finds that the addition of a relatively simple set of parallelogram-shaped nonaxisymmetric coils can improve the stability of tokamaks to vertical modes, providing stable equilibria with more highly elongated cross-sections and potentially leading to devices with improved performance in terms of beta limits and/or confinement. Furth-Hartman coils are calculated to have essentially the same vertical stabilization effect as the simple parallelogram-shaped coils described here, so that the vertical stabilization demonstrated experimentally by Furth-Hartman coils supports the feasibility of stabilizing vertical modes by the simpler coil set. The physical picture that we have presented for the stabilization suggests that the stability properties will not depend on the precise shape of the coils, so that the coil winding surface can be curved to conform to the local shape of the plasma, if desired, or curvature of the coils can be introduced to optimize relative to other considerations. It can also be argued[16] that the simple parallelogram coils we propose should have the same effect on vertical modes as Furth-Hartman coils[9], whose effect on the vertical instability has been demonstrated experimentally.[10]

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