Study of MHD stability beta limit in LHD by hierarchy integrated simulation code

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Abstract.
The beta limit by the ideal MHD instabilities (so-called “MHD stability beta limit”) for helical plasmas is studied by a hierarchy integrated simulation code. A numerical model for the effect of the MHD instabilities is introduced such that the pressure profile is flattened around the rational surface due to the MHD instabilities. The width of the flattening of the pressure gradient is determined from the width of the eigenmode structure of the MHD instabilities. It is assumed that there is the upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient. The upper limit of the mode number is determined using a recent high beta experiment in the Large Helical Device (LHD). The flattening of the pressure gradient is calculated by the transport module in a hierarchy integrated code. The achievable volume averaged beta value in the LHD is expected to be beyond 6%.

1. Introduction

The LHD configuration is most promising for a fusion reactor in helical devices since the highest electron and ion temperature and electron density are obtained among the present and past helical devices, and 5% of the volume averaged beta value is achieved which is required for an economic fusion reactor [1]. In the helical plasmas, stability against pressure driven magnetohydrodynamics (MHD) instabilities is theoretically predicted to be the crucial problem. To date, the beta value was considered to be limited by the "conventional" MHD stability criteria that $D_I = 0$ corresponding to the stability condition of the Mercier mode and/or $D_I = 0.2$ at a low order rational surface where the low-n interchange mode becomes linearly unstable (the linear growth rate $\gamma{\sim}10^2\omega_A$, where ω_A is Alfvén frequency) [2]. According to the recent LHD experimental analysis, the quasi-stationary pressure gradient is achieved beyond the limit predicted from the "conventional" MHD stability criteria [3,4]. Since plasma discharges with such a pressure gradient that γ and/or D_I are beyond a finite value ($\gamma=1.5\times10^2\omega_A$, $D_I = 0.3$), however, have not been observed, it is envisaged that the MHD instabilities still limit the pressure gradient. So far the analysis of the so-called “MHD stability beta limit” was studied by using the “conventional” MHD stability criteria. However, the “MHD stability beta limit” for helical plasmas based on such experimental analysis of the MHD stability has not been predicted yet. In this paper, the “MHD stability beta limit” is studied by simulation based on the criteria consistent to the experimental results, which would lead to explore the capability of the LHD configuration from the perspective of the MHD stability and a guideline for the design of an LHD type fusion reactor.

For analyzing the beta profiles of the “MHD stability beta limit”, a hierarchy integrated simulation code TASK3D [5,6] is extended to include the MHD dynamics. The hierarchy-integrated simulation is mainly based on a transport simulation combining various simplified models describing physical processes in different hierarchies. This is suitable for investigating whole temporal behavior of experimentally observed macroscopic physics quantities. The TASK3D is a hierarchy integrated simulation code to be applicable for three dimensional configurations, which is being developed based on the integrated modeling code for tokamak plasmas, TASK (Transport Analyzing System for tokamaK) [7], being developed in Kyoto University. The simplified models adopted in the hierarchy integrated simulation code should be confirmed by the hierarchy-extended simulation approach or it should be constructed by the experimental results. For the hierarchy-extended simulation approach, there are nonlinear analysis of the MHD stability for high beta plasmas in the LHD [8,9]. Their works assumed that the resistivity is relatively large and found that the MHD instabilities affect the pressure profile in relatively low beta region. In this paper, since the main concern is the exploration of the capability of the LHD configuration, the effect of the ideal interchange mode on the MHD stability beta limit is mainly studied.

This paper is organized as follows. In section 2, the simulation scheme using the extended TASK3D is introduced. How to include the effect of the limitation on the pressure gradient due to MHD instabilities into the transport coefficient is also explained. In section 3, the extended TASK3D is applied to high beta plasma in the LHD and the mode number of MHD instabilities which directly influence the pressure gradient is investigated. Next, numerical results of the achievable beta profile are presented for two types of the profile. Finally, conclusions are given in section 4.

2. Numerical Model

Since MHD stability strongly depends on the MHD equilibrium, it is necessary to evaluate the MHD equilibrium and its stability characteristic by changing the pressure profile recurrently for obtaining the pressure profile limited by MHD instabilities. In this paper, the change of pressure due to the MHD dynamics is modeled through the transport process. Then, transport simulation should be done together with the calculation of the MHD equilibrium and its stability with the beta value near the MHD stability limit in the hierarchy integrated code, TASK3D. The main modules of the TASK3D are the one-dimensional transport module (TR module), the rotational transport module (EI module) and the radial electric field module (ER module). The simulation by using the combination of the transport TR module and the MHD equilibrium module, EQ-VMEC (VMEC [10] + interface programs) can also be done.

In this paper, ideal interchange modes, which are most important instabilities in helical plasmas, are considered to give the MHD stability beta limit. In general, the linear growth rate of the interchange mode becomes larger as the poloidal mode number increases. On the other hand, the width of the mode structure becomes narrower. The higher modes do not dynamically influence the pressure gradient since the width of the mode structure of the higher modes is narrow. The higher modes generate a turbulent state and affect the pressure gradient through the enhanced transport [11]. Hence, in the extended TASK3D code, a numerical model for MHD instabilities is introduced such that the linearly unstable MHD instabilities flatten the pressure gradient around the rational surface. And the flattening width is assumed to be determined from the width of the mode structure since the saturation level of the interchange mode has strong correlation with the width of the linear mode structure. It is also assumed that there is an upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient.

The numerical scheme of the extended TASK 3D code including the MHD dynamics is as follows. First, a pressure profile is given and the equilibrium quantities such as the rotational transform are calculated by the VMEC module. Next, the linear stability calculation is done and the transport coefficient is determined from the eigenmode structure. In the transport module TR, time evolution of the temperature profile is calculated. The density profile is fixed in this simulation. When the interchange mode becomes unstable, the effect of the MHD instability reflects on the transport coefficient by changing the transport coefficient to a larger value. That is, the transport coefficient χ is assumed as $\chi = \chi_0 + \chi_M$, where χ_0 is the transport coefficient for the case without the MHD instability and χ_M is the enhanced transport coefficient due to the MHD instabilities. Here χ_0 =1 m^2/s is assumed. For the numerical model introduced here, when the mode becomes unstable, the flattening of the pressure occurs and the pressure gradient is limited. With the newly obtained temperature (pressure) profile, the equilibrium quantities are calculated again by the VMEC module and then the MHD stability for the new equilibrium profile is evaluated. By repeating the procedure the steady pressure profile is obtained.

Since the interchange mode is considered in this paper, the MHD stability is evaluated by using the following normalized reduced equations for a straight helical plasma,

$$\left(\frac{\partial}{\partial t} + \mathbf{e}_z \times \nabla\phi \cdot \nabla\right)\nabla_\perp^2 \phi = -\nabla_\parallel(\nabla_\perp^2 A) + \nabla P \times \nabla\Omega \cdot \mathbf{e}_z,$$

$$\left(\frac{\partial A}{\partial t}\right) = -\nabla_\parallel \phi,$$

$$\left(\frac{\partial}{\partial t} + \mathbf{e}_z \times \nabla\phi \cdot \nabla\right) P = 0,$$

$$\nabla_\perp^2 \equiv \nabla^2 - \frac{\partial}{\partial z}, \nabla_\parallel \equiv \frac{\partial}{\partial z} + \nabla\psi \times \mathbf{e}_z \cdot \nabla,$$

where time t is normalized to $a/\varepsilon v_a$, r to a, z to R_0, $\hat{\phi}$ to $\varepsilon a v_a B_0$, A to $\varepsilon a B_0$ and P to B_0^2/μ_0, $\varepsilon = a/R_0$ is the inverse aspect ratio, a is the minor radius, R_0 is the major radius, v_A is the Alfvén velocity and B_0 is the magnitude of the magnetic field at the magnetic axis. To linearlize the above equations, a perturbation described by $f(r)\exp(im\theta + in\varsigma)$ in the cylindrical plasma limit. Then the following eigenmode equation for $\hat{\phi}$ can be derived:

$$\gamma^2\left(\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - k_\theta^2\right)\hat{\phi} = k_\parallel^2\left(\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - k_\theta^2\right)\hat{\phi} + k_\theta^2 P_0'\Omega'\hat{\phi},$$

where $k_\parallel = m\iota - n$, $k_\theta = m/r$ and the primes denote the derivative with respect to r. The above eigenmode equation is solved by the shooting method. In the EQ-VMEC module, the Mercier parameter and the MHD equilibrium are obtained at the same time. However, for the straight helical plasma model the toroidal effect is neglected so that the stability condition for the straight helical plasma model does not agree with the Mercier criteria calculated from the EQ-VMEC module. For eliminating the problem, the averaged magnetic curvature is determined from the Mercier parameter D_I calculated by the VMEC module,

$$\Omega' P' = -(\iota')^2 r^2\left(\frac{1}{4} + D_I\right).$$

Then the enhanced transport coefficient χ_M is determined from the eigenmode function for the unstable modes.

$$\chi_M = Max[C, C\sum(1 + \tanh((\phi_{m,n} - 1/e)/0.01)/2],$$

where C=100 m^2/s is chosen. Here, it is assumed that the width of flattening of the pressure gradient is the width of 1/e of the peak amplitude of the eigenmode structure.

3. Numerical Results

As described in section 2, we assumed that the interchange modes give the MHD stability limit beta value. In order to include the effect of MHD instabilities, the numerical model is introduced such that the linearly unstable MHD modes flatten the pressure profile around the rational surface with the width of the mode structure. It is also assumed that there is the upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient. In this study, the upper limit of the mode number, m_c, is evaluated from a high beta experimental data. Experimentally, modes with m=2 can be usually observed by a magnetic measurement. However, modes with m=3 rarely are observed and modes with $m \geq 4$ are not observed [12]. The large flattening of the pressure which strongly influences the plasma confinement is not also observed. However, since plasma discharges with such a pressure gradient that the low-n linear growth rate and/or the Mercier parameter are beyond a finite value have not been observed, it is envisaged that the MHD instabilities still limit the pressure gradient. The upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient is evaluated by using the experimental result [13] where the volume averaged beta value is 4.8% shown by black line in Fig.1. As shown in Fig2(a) , when the beta value becomes high, the magnetic well is generated in the core region and its depth becomes deep due to large Shafranov shift in the LHD configuration so that the interchange modes are stable in the core region. However, there is the Mercier unstable region in the plasma edge and the pressure gradient may be limited by MHD instabilities. For the experimental beta profile, the eigenmode functions for $m \leq 10$ obtained from the linear stability calculation are shown in Figure 2(b). When the upper limit of the mode number is chosen as m=7, the overlapping of modes occurs and the pressure gradient is seemed to be further limited.

Next, the effect of the number of the ideal MHD modes on the pressure profile is investigated. By repeating the MHD equilibrium calculation, the linear stability calculation and the transport calculation alternately, the steady pressure profile close to the experimental one is obtained. Here, the profile of the heating power is determined such that the experimental profile is reproduced by the simulation when the effect of the MHD instabilities is not included. When the mode with $m \leq 4$ is assumed to directly affect the pressure gradient, the calculated stationary beta profile shown by the red curve in Fig.1 is almost same as the experimental profile. On the other hand, when the effect of the modes with $m \leq 7$ or $m \leq 10$ are included, the pressure profile close to the experimental profile cannot be maintained. Under

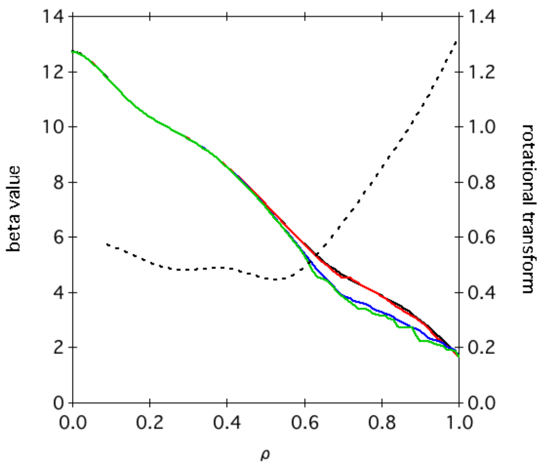

FIG. 1. Beta profiles of an experimental result for $<\beta>$=4.8%(black curve) and numerical results for m_c=4(red curve), m_c=7(blue curve) and m_c=10(green line). The profile of the rotational transform for the experimental beta profile is shown by the dashed curve.

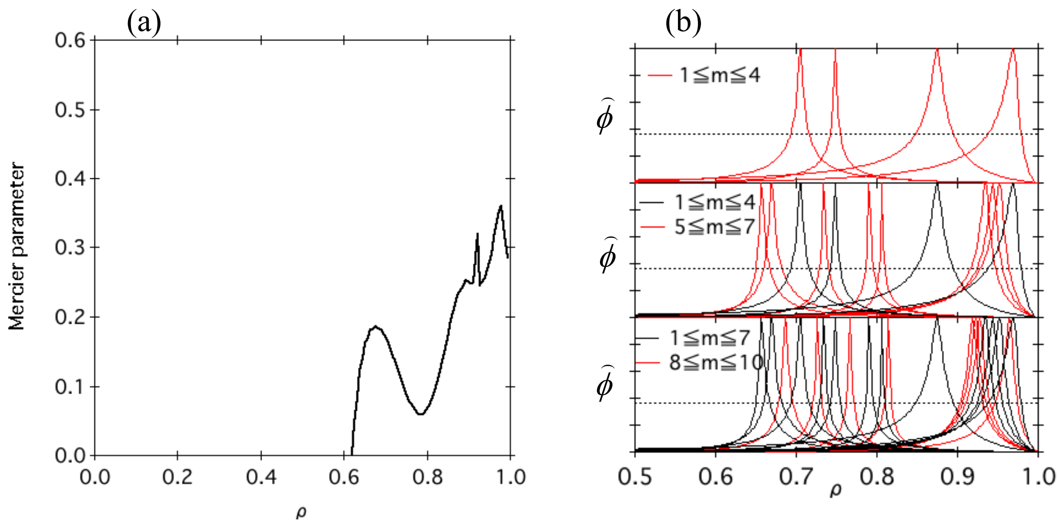

FIG. 2. (a)The Mercier parameter profile and (b) the eigenmode structure for the beta profile of $<\beta>$=4.8% shown by black curve in Fig.1.

these conditions, the calculated steady profile shown by green line in Fig.1 is almost same as the experimental profile. These simulation results agree with the prediction from the stability analysis of the fixed pressure profile. For m_c=7, the beta profile close to the experimental profile cannot be steadily maintained. Hence, m_c = 4 is suitable for the upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient and consistent the experimental results. In the following simulation, m_c=4 is the standard model for the calculation of the achievable beta value. For reference, the calculation for m_c=7 is also done.

By changing the heating profile in the TR module, the achievable beta values for two types of the pressure profile are investigated. Figure 3 shows the dependence of the achievable volume averaged beta value and the Shafranov shift of the magnetic axis $\delta=(R_{00}-R_{\mathrm{eff}})/a_p$ on the peaking factor $P_0/<P>$, where P_0 is the pressure at the center and $<P>$ is the averaged

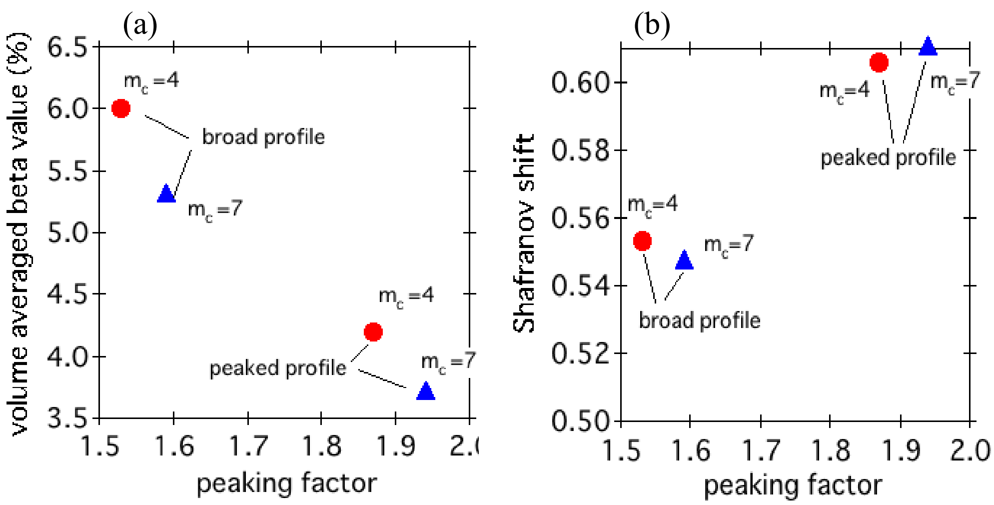

FIG. 3. The achievable beta value and the Shafranov shift for a peaked and a broad profile.

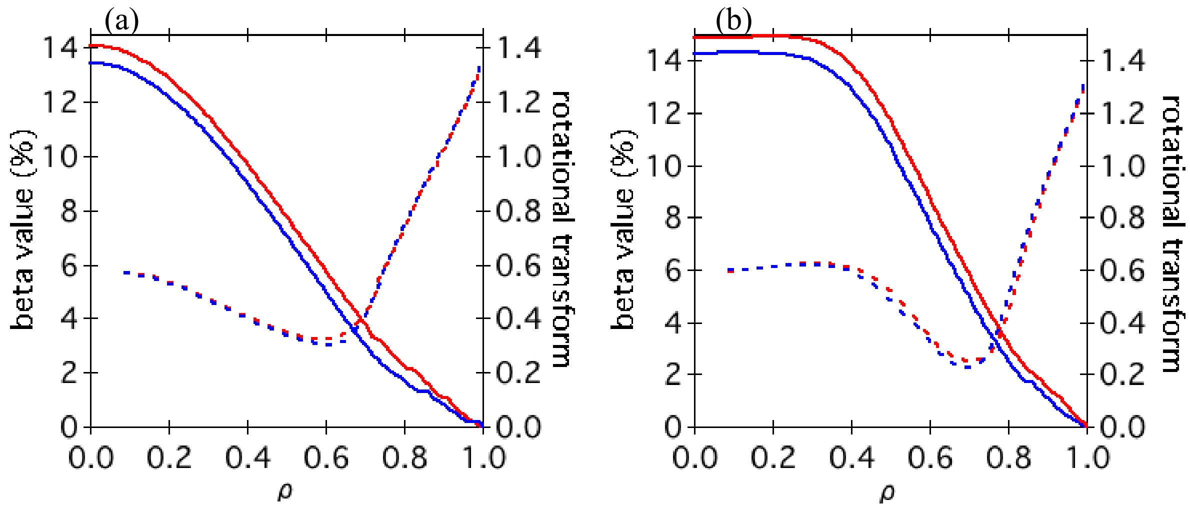

FIG. 4. The beta profile and the rotational transform profile at the achievable beta value for (a) the peaked profile and (b) the broad profile corresponding to Fig. 3. The blue lines correspond to the results for m_c=4 and the red lines to m_c=7.

pressure defined as $<P>=\int_0^1 P d\rho$. In this study, the equilibrium limit is defined as δ =0.6. Under the equilibrium limit model, the achievable beta for the peaked profile is limited by the equilibrium limit. The achievable beta value of 4.2% and 3.7% are obtained for m_c=4 and m_c=7, respectively. The pressure gradient for m_c=7 is further limited than for m_c=4 since there is a larger number of the rational surfaces in the edge region (0.7<ρ<1). Therefore the peaking factor for m_c=7 is larger than for m_c=4 under the same beta value at the center and the achievable beta value for m_c=7 is lower than for m_c=4.

For the broad pressure profile, the volume averaged beta values of 6.0% and 5.3% are obtained for m_c = 4 and m_c=7, respectively. Both values of Shafranov shift δ are less than 0.6 so that the achievable beta value is not limited by the equilibrium limit. For the broad beta profile calculated for m_c=4 shown by the red curve in Fig. 4(b), it is found that all modes with m≤4 are stable from the analysis of the MHD stability. For the beta profile which has same shape of the profile calculated for m_c=4 as shown in Fig.4(b) but the beta value is as much as 1.05 times, the eigenmode of the interchange modes with m ≤4 are shown in Fig.5. As the beta value increases, the (m,n)=(4,1) mode is destabilized near the minimum of the rotational

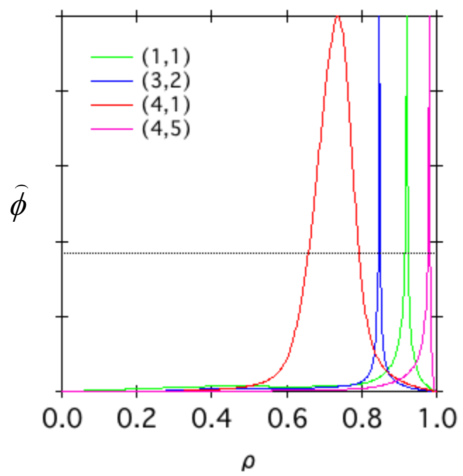

FIG. 5. The eigenmode structure of unstable interchange modes with m<=4 for the broad beta profile of $<\beta>$=6.2%.

transform. The (m,n)=(4,1) mode causes the large flattening of the pressure gradient so that the high beta value cannot be steadily maintained.

4. Conclusion

In this study, the achievable volume averaged beta limited by MHD instabilities is investigated by using the hierarchy integrated code TASK3D. Ideal interchange modes are considered to give the MHD stability beta limit. In order to include the effect of the MHD instabilities into the TASK3D, a numerical model for the MHD instabilities is introduced such that the linearly unstable MHD instabilities flatten the pressure gradient. The width of the flattening of the pressure gradient is determined from the width of the linear mode structure since he saturation level of the interchange modes has strong correlation with the width of the linear mode structure. It is also assumed that there is an upper limit of the mode number of the MHD instabilities which directly affect the pressure gradient. The upper limit of the mode number is identified from a recent experimental result of high beta plasma in the LHD. From the analysis, it is found that $m \leq 4$ is the mode number of the MHD instabilities which dynamically affect the pressure profile. Under the condition, the achievable beta of 4.2% and 6.0% are obtained for a peaked profile and a broad profile, respectively. When the mode with $m \leq 7$ are considered to affect the pressure profile, the pressure gradient is further limited in the edge region. The achievable beta value for the peaked profile decreases to 3.7% and the broad profile to 5.3%, respectively. For high beta plasmas, the interchange mode is stable in the core region since the magnetic well is generated and its depth becomes deep due to large Shafranov shift in the LHD configuration. Also the ballooning mode is considered to be stable [14,15]. However, the interchange mode limits the pressure gradient in the periphery region ($0.7 < \rho < 1$). The achievable beta value is limited by the equilibrium limit for the peaked profile. For the broad profile, the achievable beta value is not limited by the equilibrium limit. It is limited by (m,n)=(4,1) mode for which the rational surface is located near the minimum of the rotational transform.

The achievable volume averaged beta value is expected to be beyond 6% from our calculations. In this paper, the achievable beta value is investigated for only two types of the beta profile. From the results, there is a possibility that higher beta value is achieved for broader beta profile. In this study, the beta value is assumed to be zero in the region where the magnetic surface is predicted to be stochastic. However, a finite pressure gradient is experimentally observed in the stochastic region. When the beta value is finite at the boundary of the well-defined magnetic surface, the higher beta value than the achievable beta value calculated here may be obtained.

In order to explore the capability of the LHD configuration, the analysis for various types of the beta profile should be necessary. In this paper, the upper limit of the mode number of the MHD instabilities which dynamically affect the pressure profile is determined from a high beta experimental result in the LHD. It is also necessary to check systematically whether the assumption is appropriate for various experimental conditions in the LHD. Concurrently with the analysis, the validity should be confirmed by the hierarchy-extended simulation approach [16]. To study the beta profile on the MHD stability limit including the effect of the bootstrap current and draw up a plasma discharge scenario for obtaining the achievable highest beta value by taking account of temporal change of MHD equilibrium are near future works. For the MHD equilibrium limit, a simple model is used here. Detailed analysis of the MHD equilibrium limit is studied by using a three dimensional MHD equilibrium code, HINT, where the existence of the magnetic surface is not assumed a priori [17]. The numerical modeling of the analysis is also one of the future works.

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