Validity of Quasi-Linear Transport Model

C. Bourdelle 1), A. Casati 1), X. Garbet 1), F. Imbeaux 1),
J. Candy 2), F. Clairet 1), G. Dif-Pradalier 1), G. Falchetto 1), T. Gerbaud 1),
V. Grandgirard 1), P. Hennequin 3), R. Sabot 1), Y. Sarazin 1), L. Vermare 3), R. Waltz 2)

1) CEA, IRFM, F-13108 Saint-Paul-lez-Durance, France

2) General Atomics, P.O. Box 85608, San Diego, California 92186-5608, USA

3) Laboratoire de Physique et Technologie des Plasmas, CNRS-Ecole Polytechnique,
91128 Palaiseau Cedex, France

e-mail contact of main author: clarisse.bourdelle@cea.fr

Abstract. In order to gain reliable predictions on turbulent fluxes in tokamak plasmas, physics based transport models have to be improved. On a confinement time scale, nonlinear gyrokinetic electromagnetic simulations for all species are still too costly in terms of computing time. On the other hand, interestingly, quasi-linear approximation seems to retain the relevant physics for fairly reproducing both experimental results [1, 2] and nonlinear gyrokinetic simulations [3, 4, 5, 6]. Quasi-linear fluxes are made of two parts, the quasi-linear weight and the saturated squared electrostatic potential. The first one is shown to follow well nonlinear predictions; the second one is based on both nonlinear simulations and turbulence measurements. The resulting quasi-linear fluxes are shown to agree with the nonlinear ones when varying various dimensionless parameters such as the ion to electron temperature ratio, the collisionality ν* and the temperature gradients, ranging from Ion Temperature Gradient (ITG) to Trapped Electron Modes (TEM) turbulence.

1. Introduction

After more than 40 years from the first pioneering papers [7,8], quasi-linear theory remains still nowadays an open subject of research that can provide a powerful instrument for plasma physics understanding. Reviews can be found for example in [9,10,11,12]. Even if most part of the theoretical efforts has been applied to 1D plasma turbulence, several quasi-linear transport models have been proposed for the tokamak relevant 3D drift wave turbulence, providing feasible and commonly used predictive tools among which GLF23 [2], TGLF [13, 14], IFS-PPPL [1], MMM95 [15], Weiland model [16], QuaLiKiz [17]. Despite the apparently crude approximations adopted, the quasi-linear theory has revealed for a relevant number of cases an interestingly good agreement with both experimental results [1][2] and nonlinear gyrokinetic simulations [3][4][13][14][5][6].

Validating quasi-linear transport models requires studying carefully two distinct points. The first one is to improve and test the arbitrary choices made to characterize the fluctuating electrostatic potential in terms of wave number (k) and frequency spectra and saturation level. The second one consists in checking if approximating a linear response of the transported quantities (particles and energy) to the fluctuating potential is realistic. Clarifying these two aspects requires intensive comparisons with nonlinear simulations and fluctuation measurements. This is the core of the work presented here.

The validity of the quasi-linear approach is tested against nonlinear gyrokinetic simulations, using the Eulerian code GYRO [18] and the semi-Lagrangian code GYSELA [19]. The frequency and wave vector spectra obtained by reflectometry and laser backscattering measurements in Tore Supra are confronted to GYRO/GYSELA simulations: the choices for the saturated electrostatic potential are based on this confrontation. As in [6] for pure TEM turbulence, a good agreement of the de-phasing between the potential and the transported quantities, i.e. particles, ion and electron energy, is observed for coupled ITG-

TEM turbulence between quasi-linear (on the most unstable mode only) and nonlinear regimes. On the other hand, contrarily to the observation made in [5] for ETG turbulence, the quasi-linear weights (amplitude and phase) for various ITG-TEM cases are affected by a slight, but constant, over prediction with respect to the nonlinear values.

Finally, the quasi-linear fluxes (product of the weight with the fluctuating potential) given by QuaLikiz [17] are compared to the nonlinear GYRO fluxes while varying various dimensionless parameters such as R/L_T (temperature gradient length), T_i/T_e and collisionality ν^*. Interestingly, the ratios between ion energy flux, electron energy flux and particle flux for quasi-linear and nonlinear simulations are shown to agree well for coupled ITG-TEM turbulence.

2. Improved quasi-linear transport model

The general approach chosen for this new quasi-linear model, QuaLiKiz, is briefly recalled [17]. The hypotheses underlying quasi-linear theory are reviewed, pointing out the presence of two distinct ways of accounting for broadenings around the resonances. Also, the fluctuating electrostatic potential dependence in k has been modified in the light of nonlinear simulation results.

2.1 The model

The quasi-linear gyrokinetic expression of the turbulent fluxes results from the time average of the nonlinear Vlasov equation over a time τ larger than the characteristic time of the fluctuations $1/\gamma$ and smaller than the equilibrium evolution time T_0. Moreover, the fluctuating distribution function $\tilde{f}$ linearly responds to the fluctuating potentials $\tilde{H}$ through the Vlasov equation. In the model presented here the linearized Vlasov equation is computed by Kinezero [Bou02] using the ballooning representation. Kinezero accounts for electrostatic fluctuations only. Two ion species and electrons are taken into account in both their trapped and passing domains. It is an eigenvalue code that computes all the unstable modes.

Hence, the quasi-linear formulation leads to the following expressions for the particle and energy fluxes for each species s (resp. Γ_s and Q_{Es}) [17]:

(1)

$$\Gamma_s = \left\langle \tilde{n}_s \frac{ik_\theta \tilde{\phi}}{B} \right\rangle =$$

$$-\frac{n_s}{R}\left(\frac{q}{r}\right)^2 \frac{1}{B^2} \sum_{n,\omega_k} \int \frac{d\omega}{\pi} n^2 \left\langle \sqrt{\xi} e^{-\xi} \left(\frac{R\nabla n_s}{n_s} + \left(\xi - \frac{3}{2}\right) \frac{R\nabla T_s}{T_s} + \frac{\omega}{n\omega_{Ds}} \right) \mathrm{Im}\left(\frac{1}{\omega - n\Omega_s(\xi,\lambda) + i0^+} \right) \right\rangle_{\xi,\lambda} \left|\tilde{\phi}_{n\omega\omega_k}\right|^2$$

(2)

$$Q_{Es} = \left\langle \frac{3}{2} \tilde{P}_s \frac{ik_\theta \tilde{\phi}}{B} \right\rangle =$$

$$-\frac{n_s}{R}\left(\frac{q}{r}\right)^2 \frac{T_s}{B^2} \sum_{n,\omega_k} \int \frac{d\omega}{\pi} n^2 \left\langle \xi^{3/2} e^{-\xi} \left(\frac{R\nabla n_s}{n_s} + \left(\xi - \frac{3}{2}\right) \frac{R\nabla T_s}{T_s} + \frac{\omega}{n\omega_{Ds}} \right) \mathrm{Im}\left(\frac{1}{\omega - n\Omega_s(\xi,\lambda) + i0^+} \right) \right\rangle_{\xi,\lambda} \left|\tilde{\phi}_{n\omega\omega_k}\right|^2$$

The sum over ω_k (where $\omega_k + i\gamma_k$ are the complex eigenvalues of the linear dispersion relation) is the sum over all the unstable modes. The integrands are: $\xi = \left(\frac{1}{2} m_s V^2\right) / T_s$,

$\lambda = \mu B(r,\theta=0)/\left(\frac{1}{2}m_s V^2\right)$. $b = B(r,\theta)/B(r,0)$, with m_s the mass, V the velocity, μ the adiabatic invariant, B the magnetic field, (r,θ,φ) the radial, poloidal and toroidal coordinates. The frequencies are: $n\omega_{Ds} = -k_\theta \frac{T_s}{e_s BR}$, $n\Omega_s = -k_\theta \frac{T_s}{e_s BR}\xi(2-\lambda b)f(\lambda) + k_{//}V_{//}$, with $f(\lambda)$ a function of λ depending on the magnetic geometry. n_s is the density, T_s the temperature, P_s the pressure, V_{Ts} the thermal velocity, q the safety factor, s the magnetic shear, α the MHD parameter included in Kinezero [20] (which differs for trapped and passing particles), $k_{//}V_{//} \approx \pm k_\theta w \frac{s}{q}\frac{V_{Ts}}{R}\sqrt{\xi}$, n the toroidal wave number and $k_\theta = \frac{nq}{r}$ the poloidal wave vector.

The most delicate part in estimating the energy and particle fluxes using the quasi-linear theory is due to the fact that the model is not self-consistent (there is no back-reaction of the perturbed quantities on the fluctuating potential). The linearized gyrokinetic equation does not provide any information on the saturation of the fluctuating electrostatic potential in terms of its amplitude $\left|\tilde{\phi}_{n\omega\omega_k}\right|$ or on its spectral shape versus the wave number n and the frequency ω. Our choices on both spectra are discussed in the following subsections.

2.2 Resonance broadening and frequency spectra of the fluctuating potential

Adding a non negligible finite $+i0^+=+i\nu$ in the resonance terms of Eq. 1-2 does not simply fulfill causality; it is also linked to intrinsic nonlinear effects leading to irreversibility through mixing of the particles orbits in the phase space. In other words, this is the key point for passing from a resonance localized quasi-linear theory to a renormalized quasi-linear theory. Historically, this has been at the origin of the so called resonance broadening theory (RBT), firstly initiated by Dupree in [21] and followed by several other works, leading also to more elaborate theories like the direct interaction approximation (DIA) [22,23,24,25]. In the case of finite ν, the term $\mathrm{Im}\left(\frac{1}{\omega - n\Omega_s(\xi,\lambda) + i\nu}\right)$ is a Lorentzian of width ν, in contrast with the singular resonance localized expression found for ν→0. Also, it is to be noted that in the limit ν→0 the particle fluxes are not ambipolar, hence it is mandatory to introduce a finite ν value.

In principle, two kinds of broadening can exist in the quasi-linear fluxes expressed by Eq. 1-2. The first one actually coincides with the just mentioned RBT. The second one is instead related to an intrinsic ω-spectral shape of the squared fluctuating potential $\left|\tilde{\phi}_{n\omega\omega_k}\right|^2$. Here we refer to this second broadening mechanism as frequency broadening. Assuming for example that the frequency spectral shape is described by a function $S_{\omega_k}(\omega)$ centered in ω_k and with a non-zero width w, computing the quasi-linear particle flux according to Eq.1 should account for both broadenings, giving:

$$\text{(3)} \qquad \Gamma_s \propto \sum_{n,\omega_k}\int \frac{d\omega}{\pi} n^2 \left\langle \mathrm{Im}\left[\frac{\omega - n\omega_s^*}{\omega + i\nu - n\Omega_s}\right]\right\rangle \left|\phi_{n\omega\omega_k}\right|^2 = \sum_{n,\omega_k} n^2 \left\langle \mathrm{Im}\int_{-\infty}^{+\infty}\frac{d\omega}{\pi} S_{\omega_k}(\omega)\frac{\omega - n\omega_s^*}{\omega + i\nu - n\Omega_s}\right\rangle \left|\phi_n\right|^2$$

To our knowledge, most models assume more or less implicitly that for each wave number k a well defined frequency ω exists such that $\omega \to \omega_{\mathbf{k}}$. In other words this choice corresponds to:

$$\text{(4)} \qquad S_{\omega_k}(\omega) = \delta(\omega - \omega_k)$$

On the contrary, QuaLiKiz explicitly assumes a Lorentzian shape for the frequency broadening (Eq. 7 of [17]) in the following way:

(5) $$S_{\omega_k}(\omega)=\frac{w}{(\omega-\omega_k)^2+w^2}$$

with $w=\gamma_k$, the growth rate of the considered unstable mode. This choice is justified by several experimental measurements with light scattering diagnostics, showing that the frequency spectrum of the density fluctuations presents a non negligible broadening, either Lorentzian or Gaussian, around the frequency of the unstable mode [26, 27, 28, 29, 30].
Note that QuaLiKiz formulation considering no nonlinear resonance broadening ($\nu \to 0^+$) coupled to the choice (5) is completely equivalent to the more familiar quasi-linear theory based on RBT where $\nu=\gamma_k$ and $S_{\omega_k}(\omega)=\delta(\omega-\omega_k)$. Indeed:

(6)
$$\sum_{n,\omega_k}\int\frac{d\omega}{\pi}n^2\left\langle \mathrm{Im}\left[\frac{\omega-n\omega_s^*}{\omega-n\Omega_s+i\nu}\right]\right\rangle\left|\phi_{n\omega\omega_k}\right|^2=\sum_{n,\omega_k}n^2\left\langle \mathrm{Im}\int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\frac{\gamma_k}{(\omega-\omega_k)^2+\gamma_k{}^2}\frac{\omega-n\omega_s^*}{\omega-n\Omega_s+i0^+}\right\rangle\left|\phi_n\right|^2=$$
$$\sum_{n,\omega_k}n^2\left\langle\frac{\gamma_k}{(n\Omega_s-\omega_k)^2+\gamma_k{}^2}\left(n\Omega_s-n\omega_s^*\right)\right\rangle\left|\phi_n\right|^2=\sum_{n,\omega_k}n^2\left\langle\left(n\Omega_s-n\omega_s^*\right)\mathrm{Im}\left(\frac{1}{\omega_k+i\gamma_k-n\Omega_s}\right)\right\rangle\left|\phi_n\right|^2$$

In other words the QuaLiKiz transport model correctly accounts for both the resonant and non-resonant contributions to the quasi-linear fluxes, as done by the other models like GLF23, TGLF, Weiland model, IFS-PPPL, MMM95. Nevertheless the choices on the shape and the width of the broadening are still arbitrary. We have started comparing experimental turbulence measurements by Doppler reflectometer and nonlinear gyrokinetic simulations; indeed the frequency width increases as γ_k increases, but some extra dependence in k has to be introduced.

2.3 Saturation rule and k spectrum of the fluctuating potential

The saturation level of $\left|\tilde{\phi}_{n\omega\omega_k}\right|^2$ at k_{max} is chosen such that the effective diffusivity, D_{eff}, follows the mixing length rule:

(7) $$\max\left(D_{eff}\approx\frac{R\Gamma_s}{n_s}\right)\Bigg|_{k_{max}}=\frac{R}{n_s}\frac{k_\theta}{B}\frac{n_s e_s}{T_s}\left|\phi_{n\omega\omega_k}{}^2\right|\Bigg|_{k_{max}}=\frac{\gamma_k}{\left\langle k_\perp{}^2\right\rangle}\Bigg|_{k_{max}}$$

The fluxes are a sum over all the unstable modes; each mode is weighted by a saturation rule that uses its corresponding growth rate and mode structure. This is an arguable point; nevertheless QuaLiKiz fluxes computed in that way are shown to agree well with nonlinear GYRO simulations for mixed ITG-TEM turbulence for a large number of cases, as detailed in section 4.
The choice for $\left\langle k_\perp{}^2\right\rangle$ is based on both experimental observations and nonlinear simulation results. It should lead to a maximum $\left|\tilde{\phi}_{n\omega}\right|^2$ around $k_\theta\rho_i\approx 0.2$, i.e. lower than the linear stability prediction (typically $k_\theta\rho_i\approx 0.4$), as observed for example with BES [31] and in nonlinear simulations. It should also depend on q as observed in nonlinear simulations [32, 33]. A pertinent choice for $\left\langle k_\perp{}^2\right\rangle$ combining these 2 aspects has been proposed by [3, 32, 34] and recently discussed in [6]. Adding the impact of the MHD parameter α on the curvature drift to the expression proposed by [3, 32, 34], one obtains for strongly ballooned modes:

(8) $$\left\langle {k_\perp}^2 \right\rangle = {k_\theta}^2 \left(1 + (s-\alpha)^2 \left\langle \theta^2 \right\rangle\right)$$

with:

(9) $$\left\langle \theta^2 \right\rangle = \frac{\int \theta^2 \left|\phi_{n\omega}(\theta)\right|^2 d\theta}{\int \left|\phi_{n\omega}(\theta)\right|^2 d\theta}$$

Concerning the k spectral shape, in the previous version of QuaLiKiz we based our choice on turbulence measurements performed by light scattering experiments [35], showing that the density fluctuations $\left|\frac{\tilde{n}_k}{n}\right|^2 \approx \left|\tilde{\phi}_k\right|^2$ wave vector spectrum scales as $e^{-4k\rho_i}$ above $k\rho_i$=0.5. In order to increase the confidence in this critical choice, nonlinear GYRO simulation results are being compared with both Doppler and fast-sweeping reflectometers, the first results are encouraging [36]. Nonlinear local gyrokinetic simulations with kinetic electrons and collisions have been performed with GYRO, showing the maximum of the spectrum at $k_{max}\rho_s \approx 0.2$, down-shifted with respect to the maximum of the linear γ_k spectrum $k_{\theta,lin-max}\rho_s \approx 0.4$. A power law of the type $k_\theta \rho_s^{-x}$ is generally able to fit very well both the potential and the density fluctuations spectrum for $k_\theta > k_{\theta,nl-max}$. A slope 3<x<3.5 has been typically observed (see Fig. 1), reproducing reasonably well the experimental turbulence measurements in the medium-low k_θ range from Doppler reflectometry available on Tore-Supra [30]. On the other hand, the transition towards x≈6 observed by the measurements for smaller spatial scales corresponding to $k_\theta \rho_s > 1.0$, has not been reproduced by the GYRO simulations.

Hence, we now assume, from 0 to k_{max}: $\left|\tilde{\phi}_{n\omega}\right|^2 \propto k_\theta {\rho_s}^3$ and from k_{max} to infinity: $\left|\tilde{\phi}_{n\omega}\right|^2 \propto k_\theta {\rho_s}^{-3}$. Moreover, the experimentally observed asymmetry in k_θ and k_r spectra has been resolved as being due to Doppler reflectometer instrumental integration domain [36], thus supporting our implicit choice of isotropic k spectra.

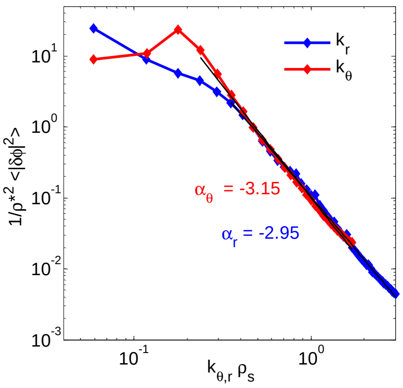

Fig. 1: Flux-surface averaged $\delta\phi$ power spectrum in k_r (blue) and k_θ (red), computed by GYRO simulation using Tore Supra parameters (r/a=0.5, R/L_{Ti}=8.0, R/L_{Te}=6.5, R/L_n=2.5)

3. Testing the quasi-linear weights versus nonlinear simulations

A rigorous validation of the quasi-linear approach has to be done apart from any hypothesis on the saturation spectrum. For this purpose, the following transport weight w_k has been defined, such that this quantity can be calculated in the full nonlinear as well in the quasi-linear regimes, for each wave number k and for each transport channel (particle and ion/electron energy):

$$(10) \qquad w_k = \left\langle \frac{\left\langle \Gamma_k(r,\theta,t),\ Q_k(r,\theta,t) \right\rangle_{r,\theta}}{\left\langle \left| \tilde{\phi}_k(r,\theta,t) \right|^2 \right\rangle_{r,\theta}} \right\rangle_t$$

These quasi-linear weights can be calculated from an initial value code, but in the case of an eigenvalue approach, the fluxes expressed in Eq. 1-2 can not be unequivocally divided by $\left|\tilde{\phi}_{n\omega\omega_k}\right|^2$. Therefore, the discussion on the transport weights is here limited to the most unstable mode; no simple tool allowing testing the validity of the quasi-linear approach for the subdominant modes has been yet developed.

The ratio between the quasi-linear and the nonlinear transport weights has been studied by means of both local (GYRO) and global (GYSELA) gyrokinetic simulations of pure ITG turbulence (i.e. with adiabatic electrons). Fig. 2 refers to the k_θ spectral structure of this ratio, where scales up to $k_\theta\rho_s$=1.48 have been resolved (results corresponding to $k_\theta\rho_s$>1.0 are omitted for GYSELA since a simplified gyro-averaging operator is applied in these range). Both the local and global simulations agree in identifying a systematic over-prediction of the linear transport with respect to the nonlinear regime, with a ratio around 1.5. Moreover, this linear/nonlinear ratio stays reasonably constant when changing the plasma parameters, especially at low k_θ scales, where it impacts most the transport level. The reason of this over prediction remains to be assessed.

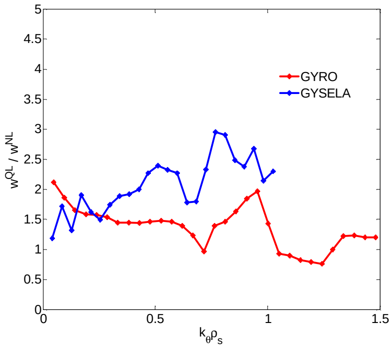

Fig. 2: Ratio of the quasi-linear and nonlinear transport weights defined in Eq. 10 versus $k_\theta\rho_s$ from local GYRO and global GYSELA simulations (adiabatic electrons, r/a=0.4, R/L_{Ti}=8.28, ρ=1/256)*

Since the transport weight can be calculated as the real part of a complex quantity, both amplitude and phase can be defined for nonlinear and quasi-linear regimes. Using the initial value code GYRO, the probability density function (PDF) of the de-phasing between the transported quantities (δn, δE_i, δE_e) and the fluctuating potential $\delta\phi^*$ from each k-mode has been calculated in the nonlinear saturation regime and compared to the linear de-phasing from the most unstable mode. Fig. 3 shows a very good agreement between the nonlinear and the linear phases in the plane θ=0, where the interchange instability is supposed to be dominant. This test of validity of the quasi-linear approach, introduced by [6, 32] for pure TEM turbulence, has been in this case successfully extended to coupled ITG-TEM turbulence. Nevertheless, when the plasma parameters are close to ITG/TEM transition (Fig. 3 d, e and f), the quasi-linear phase coming from the most unstable mode fails on predicting the particle transport (Fig 3d), whereas very interestingly the linear de-phasing for the energy fluxes remain reasonably close to the nonlinear values (Fig 3e and 3f).

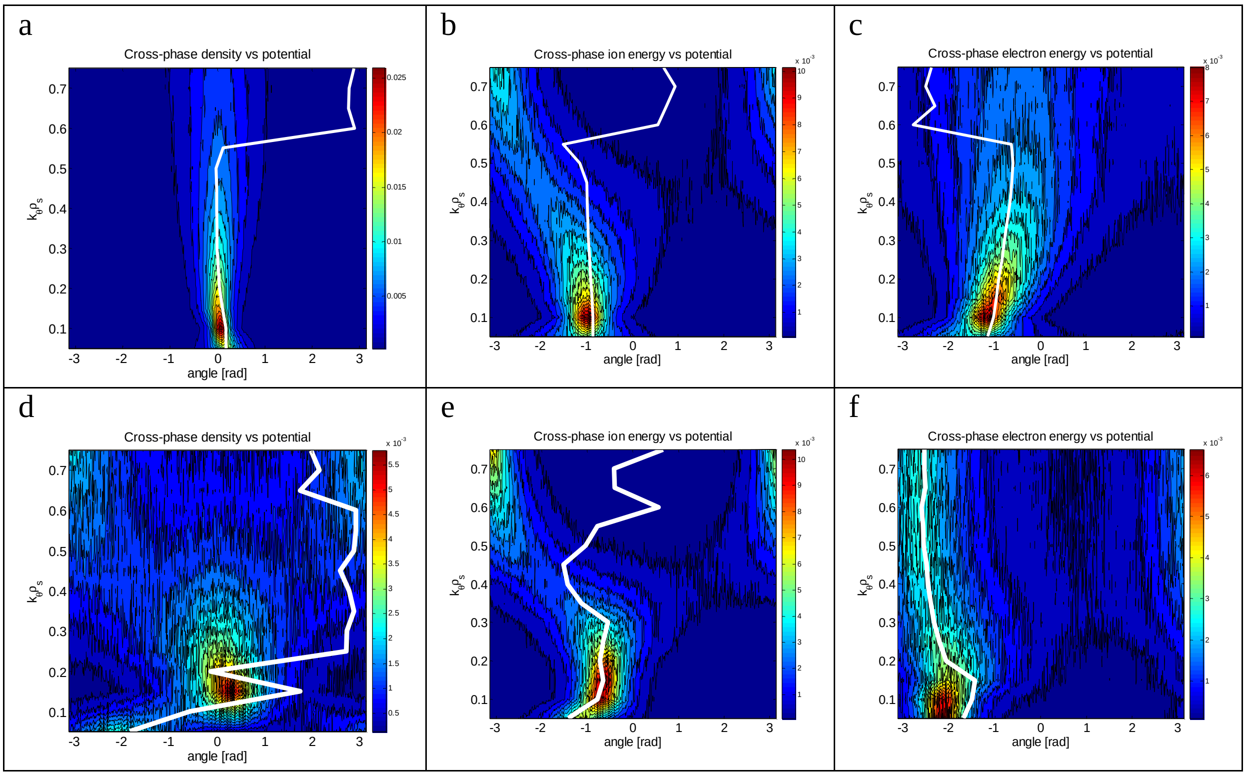

*Fig. 3***:** *PDF of the nonlinear cross-phases (contour plot) and the linear cross-phase for the most unstable mode (white line) from local GYRO simulation with kinetic electrons (r/a=0.5, $\rho^*=1/400$); the first row(a,b and c) refers to $R/L_{Ti}=R/L_{Te}=9.0$, $R/L_n=3.0$ (GA standard case), while the second one (d,e and f) assumes $R/L_{Ti}=6.0$, $R/L_{Te}=9.0$, $R/L_n=3.0$*

In the case of pure ITG turbulence, the nonlinear phase between δp_i and $\delta v_{\mathbf{ExB}}$ has been directly studied through global nonlinear gyrokinetic simulations with GYSELA, and compared to the phase between δp_i and $\delta\phi^*$ given by local GYRO simulations (Fig. 4). The two codes predict coherent total ion energy fluxes; nevertheless, phase shifts more peaked towards low k_θ scales are observed in the global GYSELA simulations with respect to the local ones by GYRO.

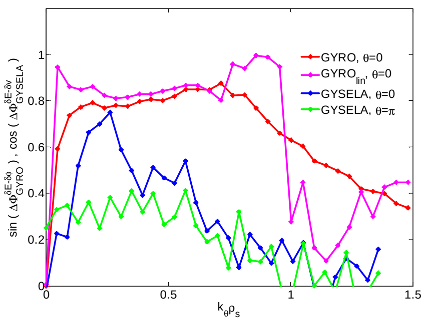

*Fig. 4***:** *Local GYRO versus global GYSELA simulation of pure ITG turbulence: $\sin(\langle\Delta\Phi(\delta p_i-\delta\phi)\rangle)$ for GYRO and $\cos(\langle\Delta\Phi(\delta p_i-\delta v_{ExB}\rangle)$ for GYSELA versus $k_\theta\rho_s$ are plotted*

4. Parametric impact on quasilinear fluxes, comparison with nonlinear predictions

In section 2, we have discussed the choices made for the saturated electrostatic potential in the present version of QuaLiKiz, based on nonlinear simulations and experimental measurements. In section 3, we have shown that the major approximation of the quasi-linear theory, namely assuming a linear response of the fluctuating transported quantities, is actually reasonable in a wide number of cases. In this fourth part, logically, we test the whole quasi-linear fluxes computed by the actual version of QuaLiKiz versus the nonlinear GYRO ion and electron energy fluxes and particle fluxes for various parameter scans.

In each case only one renormalisation factor, C_0, has been used in order to get the best fit to the nonlinear fluxes. In the first scan, both the ion and electron temperature gradients are simultaneously varied (Fig. 5). A second example is a direct application to an experimental collisionality (ν*) scan realised on Tore Supra plasmas [37]. The ν* scaling is particularly challenging for quasi-linear models, since the nonlinear collisional damping of zonal flows is not captured, while linearly a transport decrease is expected, driven by the collisional quenching of TEM. Fig. 6 demonstrates that, for experimental values of ν_{ei}, QuaLiKiz is able to well reproduce the nonlinear diffusivities predicted by comprehensive GYRO simulations, performed with pitch-angle scattering operators on both electrons and ions. The coupled dynamics between ion and electron non-adiabatic responses is crucial for both GYRO and QuaLiKiz, resulting in a slight decrease of transport on all the channels driven by higher collisionality. In particular, the particle flux reverses direction as ν* increases as already detailed in [34]. For the two finite ν* points, corresponding to two Tore Supra discharges, the decrease is within the experimental error bars of the power balance χ_{eff} [37]. The third scan (Fig. 7) illustrates a T_i/T_e scan, using DIII-D-like parameters as described in [38]. The ion heat flux decreases faster than the electron one when increasing T_i/T_e, as observed in [38]. Some discrepancies between the quasi-linear fluxes by QuaLiKiz and the nonlinear results by GYRO, mostly on the ion energy flux, could be ascribed to the role of zonal flows, whose amplitude variations are expected to strongly affect the transport level especially for this ITG dominated case. This issue is presently under investigation.

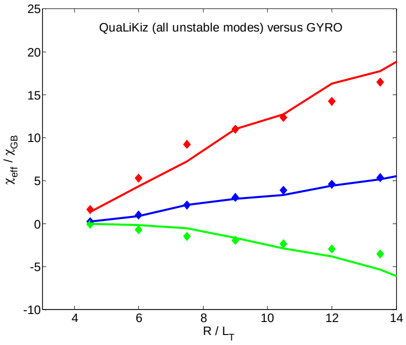

Fig. 5: Ion energy (red), electron energy (blue) and particle (green) effective diffusivities from GYRO (diamonds) and QuaLiKiz (lines) for R/L_T scan based on GA standard case

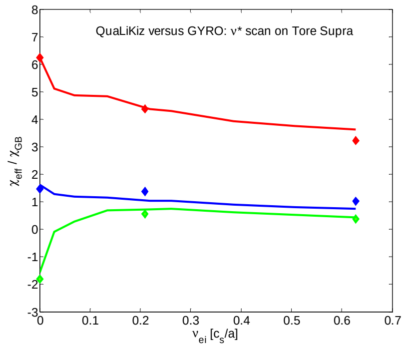

Fig. 6: Effective diffusivities from GYRO (diamonds) and QuaLiKiz (lines) for the ν scan on Tore Supra (r/a=0.5, R/L_{Ti}=8.0, R/L_{Te}=6.5, R/L_n=2.5)*

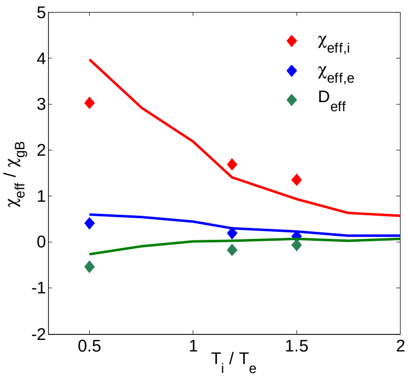

Fig. 7: Effective diffusivities from GYRO (diamonds) and QuaLiKiz (lines) on DIII-D T_i/T_e scan (r/a=0.3, R/L_{Ti}=6.5, R/L_{Te}=4.6, R/L_n=1.4)

5. Discussion

Despite the crude estimates needed on the saturated electrostatic potential, assuming a linear response of the transported quantities to the fluctuating potential has been proven to work rather well for a large number of cases. Moreover, interestingly, when coupling the choices for the saturated electrostatic potential with the quasi-linear response, we have shown to find quasi-linear fluxes agreeing well with nonlinear predictions for energy in the ion and electron channels, as well as for particle fluxes for a wide range of parameters. Nevertheless, a number of challenging issues remain to be tackled. i) The quasi-linear approach is known to fail in a number of cases: far from the threshold, onset of zonal flows, etc. Hence, the domain in which it can be applied should be better understood. ii) The choices for the saturated electrostatic potential deserve more comparisons with nonlinear simulations and experimental measurements. In Tore Supra, we are presently comparing density fluctuations k and frequency spectra from Doppler and fast-sweeping measurements versus GYRO and GYSELA simulations iii) Finally, only the integration of QuaLiKiz in a transport code such as CRONOS [39] will allow testing in situ the predictive capabilities of such an approach.

Acknowledgements

This work, supported by the European Communities under the contract of Association between EURATOM and CEA, was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] M. Kotschenreuther, W. Dorland, M. A. Beer and G. W. Hammett, Phys. Plasmas **2**, 2381 (1995)
[2] R. E. Waltz, G. M. Staebler, W. Dorland, G. W. Hammett, M. Kotschenreuther, and J. A. Konings, Phys. Plasmas **4**, 2482 (1997)
[3] F. Jenko, T. Dannert and C. Angioni, Plasma Phys. Control. Fusion **47**, B195 (2005)
[4] G. M. Staebler, J. E. Kinsey and R. E. Waltz, Phys. Plasmas **12**, 102508 (2005)
[5] Z. Lin, I. Holod, L. Chen, P. H. Diamond, T. S. Hahm and S. Ethier Phys. Rev. Lett. 99, 265003 (2007)
[6] T. Görler and F. Jenko Phys. Rev. Lett. 100, 185002 (2008)
[7] A. A. Vedenov, E. P. Velikov and R. Z. Sadgdeev, Nucl. Fusion, Suppl. Pt. **2**, 465 (1962)
[8] W. E. Drummond and D. Pines, Nucl. Fusion, Suppl. Pt. **3**, 1049 (1962)
[9] G. Laval and D. Pesme, Plasma Phys. and Control. Fusion, **41**, A239-A246 (1999)
[10] J. A. Krommes, report PPPL-3546 http://www.pppl.gov/pub_report (2001)
[11] Y. Elskens and D. Escande, *Microscopic Dynamics of Plasmas and Chaos* (Institute of Physics Publishing, Bristol, 2002)
[12] R. Balescu, *Aspects of Anomalous Transport in Plasmas* (Institute of Physics Publishing, Bristol, 2005)
[13] G.M. Staebler, J.E. Kinsey and R.E. Waltz, APS06 Issue Phys. Plasmas **14**, 0055909 (2007)
[14] J.E. Kinsey, G.M. Staebler, and R.E. Waltz, APS07 Issue Plasmas **15**, 0055908 (2008)
[15] G. Bateman, A. H. Kritz, J. E. Kinsey, A. J. Redd, and J. Weiland, Phys. Plasmas **5** 1793 (1998)
[16] J. Weiland, *Collective modes in inhomogenous plasma,* Institute of Physics Publishing, Bristol and Philadelphia (2000)

[17] C. Bourdelle, X. Garbet, F. Imbeaux, A. Casati, N. Dubuit, R. Guirlet and T. Parisot, Phys. Plasmas **14**, 112501 (2007)
[18] J. Candy and R. E. Waltz, Phys. Rev. Lett. **91**, 045001 (2003)
[19] V. Grandgirard et al., Commun. Nonlin. Sci. Numer. Simul., **13** 81–7 (2008)
[20] C. Bourdelle, X. Garbet, G.T. Hoang, J. Ongena and R.V. Budny, *Nucl. Fusion* **42** No 7 (July 2002) 892-902
[21] T. H. Dupree, Phys. Fluids **9**, 1773 (1966)
[22] T. H. Dupree, Phys. Fluids **11**, 2680 (1968)
[23] J. Weinstock, Phys. Fluids **12**, 1045 (1969)
[24] J. Weinstock Phys. Fluids **13**, 2308 (1970)
[25] S. A. Orszag and R. H. Kraichnan, Phys. Fluids **10**, 1720 (1967)
[26] Equipe T.F.R., Plasma Physics **25**, 641 (1983)
[27] A. J. Wootton, B. A. Carreras, H. Matsumoto, K. McGuire, W. A. Peebles, Ch. P. Ritz, P. W. Terry, and S. J. Zweben, Phys. Fluids B **2**, 2879 (1990)
[28] G. Antar, P. Devynck, C. Laviron et al, Plasma Phys. Control. Fusion **41**, 733 (1999)
[29] P. Hennequin, et al Proc. 26th EPS Conf. on Plasma Physics (1999)
[30] P. Hennequin, C. Honoré, A. Truc, A. Quéméneur, C. Fenzi-Bonizec, C. Bourdelle, X. Garbet, G.T. Hoang and the Tore Supra team, Nucl. Fusion **46,** S771 (2006)
[31] G.R. McKee, C.C. Petty, R.E.Waltz et al, Nucl. Fusion **41**, 1235 (2001)
[32] T. Dannert and F. Jenko, Phys. Plasmas **12**, 072309 (2005)
[33] A. Hirose, S. Livingstone and A.K. Singh, Nucl. Fusion **45**, 1628 (2005)
[34] C. Angioni, A. Peeters, F. Jenko and T. Dannert, Phys. Plasmas **12**, 112310 (2005)
[35] P. Hennequin, C. Honoré, A. Truc, A. Quéméneur, F. Gervais, R. Sabot, C. Fenzi, Plasma Phys. Control. Fusion **46**, B121 (2004)
[36] A. Casati, T. Gerbaud, P. Hennequin, et al, 13th EU-US Transport Task Force meeting, Copenhagen, (2008)
[37] T. Gerbaud, C. Bourdelle, et al Proc. 35th EPS Conf. on Plasma Physics (2008)
[38] C. C. Petty, M. R. Wade, J. E. Kinsey, R. J. Groebner, T. C. Luce, and G. M. Staebler Phys. Rev. Lett. **83**, 3661 (1999)
[39] V. Basiuk et al, Nucl. Fusion **43** (2003) 822