Limitations, Insights and Improvements to Gyrokinetics

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Abstract: For a tokamak, we consider gyrokinetic quasineutrality limitations when evaluating the axisymmetric radial electric field; an insight provided by considering the gyrokinetic entropy production restriction on an ion temperature pedestal like that of ITER; and an improved hybrid gyrokinetic-fluid treatment valid on slowly evolving transport time scales.

1. Introduction

We consider the limitations of gyrokinetic quasineutrality to evaluate the axisymmetric radial electric field; a gyrokinetic entropy production restriction on the ITER ion temperature pedestal and zonal flow behavior in the pedestal; and a hybrid gyrokinetic-fluid treatment valid on slowly evolving transport time scales.

Standard gyrokinetics incorrectly determines the axisymmetric, long wavelength electrostatic potential to leading order in gyroradius over major radius as demonstrated by considering a steady-state theta pinch with a distribution function correct to second order. Similarly, we argue gyrokinetic quasineutrality often improperly determines the potential in the long wavelength, axisymmetric limit for a tokamak.

Using canonical angular momentum as the radial variable allows strong gradients to be treated gyrokinetically. Entropy production is then found to require a physical lowest order banana regime ion distribution function to be nearly an isothermal Maxwellian with the ion temperature scale much greater than the poloidal ion gyroradius. Thus, the background ion temperature profile in ITER cannot have a pedestal like that of a density pedestal having a poloidal gyroradius width. Weak ion temperature variation with subsonic pedestal flow requires electrostatically restrained ions and magnetically confined electrons. These features are expected to result in finite orbit modifications to the zonal flow residual.

Simulating tokamaks on transport time scales requires evolving drift wave turbulence with axisymmetric neoclassical and zonal flow radial electric field effects retained. However, full electric field effects are more difficult to keep since they require evaluating the ion distribution function to higher order in the gyroradius expansion than standard gyrokinetics. An electrostatic hybrid gyrokinetic-fluid treatment using moments of the full Fokker-Planck equation removes the need to go to third order in the gyroradius expansion. This hybrid description evolves potential as well as density, temperatures and flows, and models all electrostatic turbulence effects with wavelengths much longer than an electron gyroradius.

2. Limitations of the Gyrokinetic Determination of the Radial Electric Field

A new recursive procedure is used to derive the electrostatic gyrokinetic equation for the full distribution function (a "full f" description) correct to first order in an expansion of gyroradius over magnetic field characteristic length [1]. The new, nonlinear gyrokinetic variables are constructed to higher order than is typically the case by generalizing the linear procedure of [2]. The higher order gyrokinetic variables are required for the hybrid description of section 4. The gyrokinetic procedure employed provides fresh insights into the limitations of the gyrokinetic quasineutrality equation that in the long wavelength limit must not determine the axisymmetric electrostatic potential to leading order in the gyroradius over scale length because of intrinsic ambipolarity [3,4].

The axisymmetric radial electric field in a tokamak is made up of two components that give rise to ExB drifts comparable to diamagnetic flows and magnetic drifts (this situation is normally referred to as the drift ordering). The relatively small amplitude, but rapidly radially varying zonal flow component of the electrostatic potential is generated by the turbulence associated with ion temperature gradient (ITG) modes, trapped electron modes (TEMs), and other tokamak instabilities. It is superimposed on a large amplitude component with a slow global structure on the scale of the minor radius. Gyrokinetic quasineutrality determines the short wavelength electrostatic potential, but it would violate intrinsic ambipolarity if it determined a global or steady state, axisymmetric, long wavelength radial electric field component that impacted the evolution of the turbulence.

In a steady state, axisymmetric tokamak, intrinsic ambipolarity [3,4] requires the heat and particle fluxes to be independent of electrostatic potential to second order in the expansion in ion gyroradius ρ_i divided by the local scale length L. This property is most easily seen to order ρ_i/L by considering the drift kinetic equation for the leading correction f_{1i} to the lowest order Maxwellian ions f_{0i} found by solving

$$v_{\|}\vec{n}\cdot\nabla f_{1i} - C_{1ii}\{f_{1i}\} = -\vec{v}_{di}\cdot\nabla\psi\frac{\partial f_{0i}}{\partial\psi} = -v_{\|}\vec{n}\cdot\nabla\left(\frac{Iv_{\|}}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\right), \tag{1}$$

where $\vec{v}_{di}$ is the magnetic plus electric drifts, C_{1ii} is the linearized ion-ion collision operator with $C_{1ii}\{v_{\|}f_0\} = 0$, and $\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi = B\vec{n}$ is the tokamak magnetic field with ζ the toroidal angle and ψ the poloidal flux function. Letting $g_i = f_{1i} + (Iv_{\|}/\Omega_i)(\partial f_{0i}/\partial\psi)$ gives

$$v_{\|}\vec{n}\cdot\nabla g_i = C_{1ii}\{g_i - \frac{Iv_{\|}}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\} = C_{1ii}\{g_i - \frac{If_{0i}v_{\|}}{\Omega_i T}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial T_i}{\partial\psi}\}, \tag{2}$$

showing the only drive for g_i is $\partial T_i/\partial\psi$, and giving a vanishing ion particle flux since $\langle n\vec{V}_i\cdot\nabla\psi\rangle_\psi = \langle\int d^3v f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = -\langle(I/\Omega_i)\int d^3v v_{\|}^2\vec{n}\cdot\nabla f_{1i}\rangle_\psi = 0$, where $\langle\ldots\rangle_\psi$ denotes flux surface average and (1) is employed. A moment procedure for the electron particle flux using $C_{1e}\{f_{1e}\} = C_{1ee}\{f_{1e}\} + C_{ei}\{f_{1e}\}$ with C_{1ee} the electron-electron operator and $C_{ei}\{f_{1e}\} = L_{ei}\{f_{1e} - (m/T_e)V_{\|i}v_{\|}f_{0e}\}$ the unlike operator, gives the electron particle flux as $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi = (mcI/e)\langle B^{-1}\int d^3v v_{\|}C_{1e}\{f_{1e} - (m/T_e)V_{\|i}v_{\|}f_{oe}\}\rangle_\psi$, with L_{ei} the Lorentz operator. The electron drift kinetic equation can be written as $v_{\|}\vec{n}\cdot\nabla g_e = C_{1e}\{g_e + (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\|i}v_{\|}f_{0e}\}$ with $g_e = f_{1e} - (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi)$. The $\partial\Phi/\partial\psi$ drives in the collision operator cancel, making g_e independent of the radial electric field so that $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi = (mcI/e)\langle B^{-1}\int d^3v v_{\|}C_{1e}\{g_e + (Iv_{\|}/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\|i}v_{\|}f_{oe}\}\rangle_\psi$ does not depend on the radial electric field to an order higher since $C_{ii}/C_{ee} \sim (m/M)^{1/2} \sim \rho_i/L$ is normally assumed, with L the radial scale length.

Alternately, a moment description can be used to further demonstrate that intrinsic ambipolarity must be satisfied to order ρ_i^2/L^2 since it is then the flux surface average of conservation of toroidal angular momentum that must give the radial electric field. To order ρ_i^2/L^2 the viscosity is diamagnetic (and so collisionless to lowest order) and the radial flux of toroidal angular momentum may be written in terms of the ion gyroviscosity $\vec{\pi}_{ig}$ within small up-down asymmetric contributions as [5] $\langle R^2\nabla\zeta\cdot\vec{\pi}_{ig}\cdot\nabla\psi\rangle_\psi = \langle (MI/B)\int d^3v v_{\|} f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = 0$. Inserting $f_{1i} = g_i - (Iv_{\|}/\Omega_i)(\partial f_{0i}/\partial\psi)$, using $\langle \int d^3v f_{0i}(v_{\|}/B)^2 v_{\|}\vec{n}\cdot\nabla(v_{\|}/B)\rangle_\psi = 0$, and recalling g_i depends only on $\partial T_i/\partial\psi$ gives a $\partial\Phi/\partial\psi$ independent result. Hence, the correct neoclassical radial electric field must be determined from toroidal angular momentum conservation in next order.

By considering a steady-state θ pinch using a model collision operator, we have explicitly shown gyrokinetics cannot determine its axisymmetric, long radial wavelength electrostatic potential to order ρ_i^2/L^2 [1]. In standard gyrokinetic treatments intrinsic ambipolarity is violated when the ion distribution function is retained to order ρ_i/L in the guiding center density and to order ρ_i^2/L^2 in the finite orbit polarization term. However, when f_i is kept to order ρ_i^2/L^2 in both places the radial electric cannot be determined and no inconsistency arises, as illustrated by the θ pinch case [1].

These results indicate that the gyrokinetic quasineutrality equation is not the most effective procedure for finding the electrostatic potential if the long wavelength components are to be properly retained in the analysis**.** In section 4 we discuss how second order accurate gyrokinetic variables can be employed [1] in a hybrid gyrokinetic-fluid moment description to insure third order accuracy in the gyroradius expansion.

3. Gyrokinetics in the Pedestal and Internal Barriers: A new gyrokinetic technique has been developed and applied to analyzing pedestal and internal transport barrier (ITB) regions in a tokamak [6]. In contrast to typical gyrokinetic treatments, canonical angular momentum $\psi_* \equiv \psi - (Mc/e)R^2\vec{v}\cdot\nabla\zeta = \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi - (Iv_{\|}/\Omega_i)$ is taken as the gyrokinetic radial variable rather than the radial guiding center location $\Psi \equiv \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi$. Such an approach allows strong radial plasma gradients to be treated, while retaining zonal flow and neoclassical behavior and the effects of turbulence. The nonlinear gyrokinetic equation obtained is capable of handling such problems as large poloidal ExB drift and orbit squeezing effects on zonal flow, collisional zonal flow damping, as well as neoclassical transport in the pedestal or ITB. This choice of gyrokinetic variables allows the toroidally rotating Maxwellian solution of the isothermal tokamak limit to be exactly recovered [7].

More importantly, we can prove that a physically acceptable solution for the lowest order ion distribution function in the banana regime anywhere in a tokamak must be nearly this same isothermal Maxwellian solution in the sense that the ion temperature variation scale must be much greater than poloidal ion gyroradius. Consequently, in the banana regime the background radial ion temperature profile in ITER cannot have a pedestal similar to that of plasma density or electron temperature if they vary on the scale of a poloidal ion gyroradius. To understand this insight first recall that the vanishing of the entropy production on a flux surface, $\langle \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\}\rangle_\psi = 0$, requires the lowest order axisymmetric ion distribution function f_{0i} to be a local Maxwellian, with f_{0i} independent of poloidal angle in the banana regime. However, in the pedestal or an internal barrier (or on axis), drift departures from flux surfaces can become comparable to the local scale length ($\rho_{pi}\nabla\ell n n \sim 1$ with n the plasma density) and the entropy production argument has to be modified to account for the loss of locality due to finite poloidal ion gyroradius ρ_{pi} effects requiring an equilibrium to be established over the entire pedestal (or barrier). Using the new gyrokinetic variables, we find that entropy production must vanish in the pedestal [6]:

$$\int_{\Delta V} d^3r \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\} = 0, \qquad (3)$$

where ΔV is the volume of the pedestal (between the top of the pedestal where $\rho_{pi}\nabla\ell nn << 1$ and the separatrix) or the internal transport barrier (between inner and outer bounding flux surfaces having $\rho_{pi}\nabla\ell nn << 1$). As a result, f_{0i} must be drifting Maxwellian at most, giving $C_{1ii}\{f_{0i}\} = 0$. In the banana regime f_{0i} is independent of poloidal angle as well. Consequently, to make the Vlasov operator vanish $f_{0i} = f_{0i}(\psi_*, E, \mu)$, where $E = v^2/2 + e\Phi/M$ is the total energy and μ the magnetic moment. It is only possible to make a drifting Maxwellian out of these variables by ignoring the μ dependence and assuming the drift is nearly a rigid toroidal rotation of frequency ω_i with the ion temperature variation slow compared to the poloidal ion gyroradius ($\rho_{pi}\nabla\ell nT_i << 1$, $\rho_{pi}\nabla\ell n\omega_i << 1$) as for an isothermal Maxwellian [6,7]:
$f_{0i}(\psi_*, E) = n(M/2\pi T_i)^{3/2}\exp[-M(\vec{v} - \omega_i R^2\nabla\zeta)^2/2T_i] = \eta(M/2\pi T_i)^{3/2}\exp(-ME/T_i - e\omega_i\psi_*/cT_i)$,
where $\eta = n\exp[(e\Phi/T_i) + (e\omega_i\psi/cT_i) - (M\omega_i^2 R^2/2T_i)]$ must also be nearly constant ($\rho_{pi}\nabla\ell n\eta << 1$). Thus, for a density pedestal having a scale length $L \sim \rho_{pi}$, the background ion temperature profile must have a much larger scale length than the pedestal - a restriction that will need to be satisfied by the ITER pedestal. As a result, the ion temperature pedestal must be somewhat broader than the poloidal ion gyroradius variation allowed for the density pedestal and the peak ion temperature in the core of ITER as set by ballooning-peeling calculations in the presence of bootstrap current may be reduced.

In addition, for a density scale length of ρ_{pi}, lowest order perpendicular momentum balance gives $\omega_i = -c[d\Phi/d\psi + (en)^{-1}d(nT_i)/d\psi]$ with $cR(en)^{-1}d(nT_i)/d\psi \sim v_i$ = ion thermal speed and $\Phi(\psi)$ the axisymmetric electrostatic potential. Consequently, in a subsonic pedestal in the banana regime it must be that to lowest order the ions are electrostatically confined [6] with $ed\Phi/d\psi \approx -(T_i/n)dn/d\psi$, as observed in the H mode pedestal of Alcator C-Mod [8]. Using total pressure balance we then see the electrons must be magnetically confined with a mean flow $\vec{V}_e$ comparable to the ion thermal speed ($\vec{V}_e \sim v_i$).

The strong localized axisymmetric radial electric field that arises under these circumstances modifies the collisionless zonal flow residual of Rosenbluth and Hinton [9] due to the strong poloidal ExB drift and its associated finite orbit effects as well as orbit squeezing [10]. For example, the axisymmetric radial electric field of the pedestal can be assumed to satisfy $ed\Phi_0/d\psi = -(T_i/n)dn/d\psi$. Then the residual associated with the small amplitude, shorter wavelength, axisymmetric zonal flow potential Φ_1 will differ substantially from reference [9].

Retaining the poloidal ExB drift as well as parallel streaming the ion poloidal drift frequency becomes

$$\dot{\theta} = [v_{\|} + cIB^{-1}\Phi'(\psi)]\vec{n}\cdot\nabla\theta \ . \tag{4}$$

In the tokamak core, the second term on the right side of (4) is much less than the first one, whereas in the pedestal these terms are comparable, thereby modifying the poloidal motion of particles. We remark that the ExB drift, $\vec{v}_E \sim v_i\rho_i/\rho_{pi}$, remains much less than the ion thermal speed as required by our gyrokinetic ordering. However, as sketched in figure 1, $\vec{v}_E$ is nearly parallel to the poloidal plane while $v_{\|}\vec{n}$ is almost perpendicular to it so with a small poloidal component of $v_{\|}\rho_i/\rho_{pi}$. As a result, these two velocities can compete in the poloidal cross-section of a tokamak.

In the conventional core case, a particle is trapped if its $v_{\parallel}$ is small enough. As equation (4) suggests, for a particle to be trapped in the pedestal its $v_{\parallel}$ should be rather close to $-cIB^{-1}\Phi'(\psi)$. Then, as a trapped particle in the pedestal undergoes its banana motion projected onto the poloidal cross-section, its parallel velocity oscillates around the value of $-cIB^{-1}\Phi'(\psi)$ (rather than zero) as illustrated in figure 2. Accordingly, as banana particles play a key role in neoclassical phenomena such as radial ion heat flux or polarization, the evaluation of these effects has to be revisited in the pedestal where it will differ from the core.

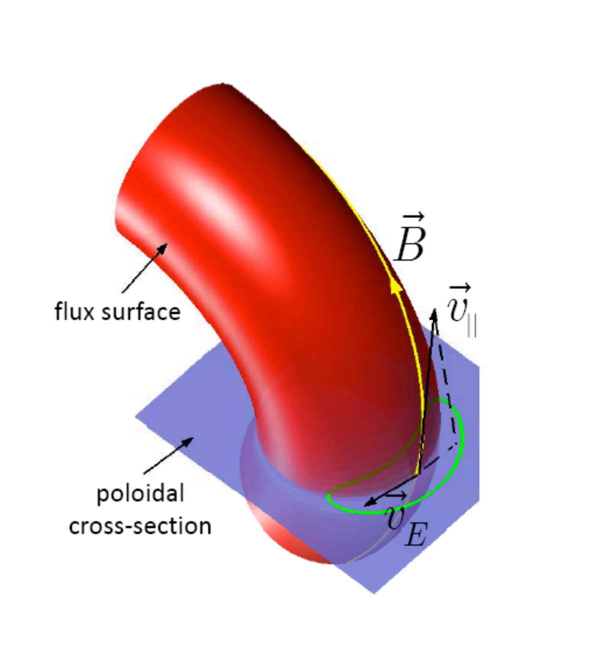

Figure 1: Projection of the parallel streaming into the poloidal plane.

It is interesting to notice that due to modifications of the trapping condition, banana particles acquire rather complicated toroidal behavior. Indeed, toroidal motion is still dominated by $v_{\parallel}$ but now it has a bounce average value of $-cIB^{-1}\Phi'(\psi)$. Of course, even in the conventional case banana particles are not perfectly confined toroidally because of toroidal components of the magnetic drift. However, these toroidal drifts are much less than v_i, while within the pedestal ordering $-cIB^{-1}\Phi'(\psi) \sim v_i$. As a result, the toroidal orbits of particles trapped poloidally in the pedestal dramatically differ from those in the core as shown in figure 2.

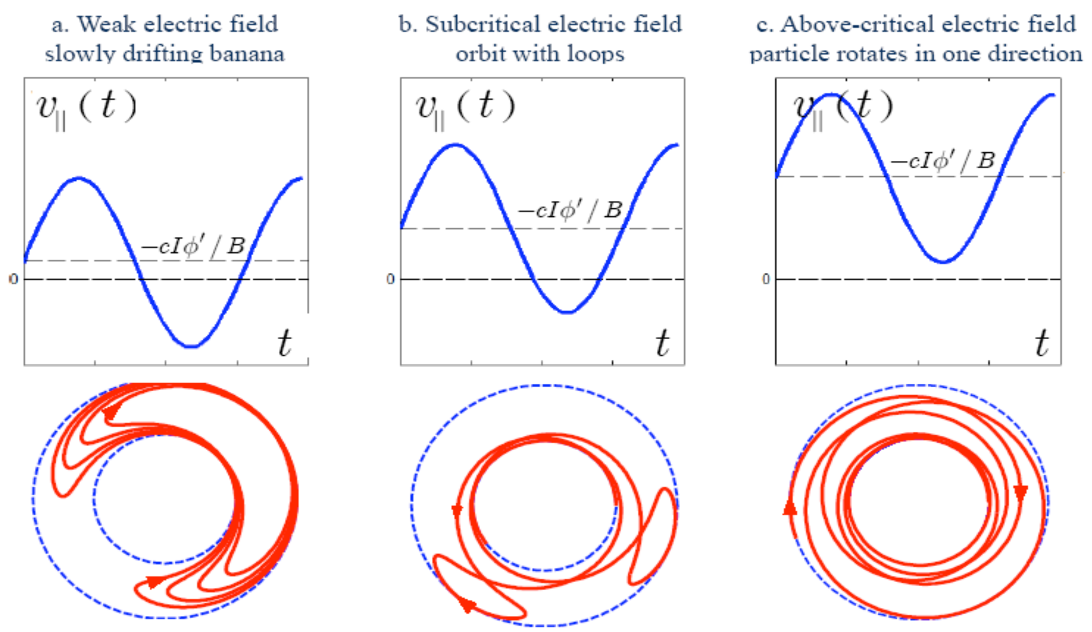

Figure 2: Toroidal projections of trapped particle orbits for weak, sub-critical, and above critical ExB drift.

4. Gyrokinetic Closure and Radial Electric Field on Transport Time Scales

Simulating electrostatic turbulence in tokamaks on transport time scales requires retaining and evolving a complete turbulence modified neoclassical transport description, including all the axisymmetric neoclassical and zonal flow radial electric field effects, as well as the turbulent transport normally associated with drift instabilities. Full electric field effects and their evolution are more difficult to retain than density and temperature evolution effects since the need to satisfy intrinsic ambipolarity in the axisymmetric, long wavelength limit requires evaluating the ion distribution function to higher order in gyroradius over background scale length than standard gyrokinetic treatments as already noted earlier. To avoid having to derive and solve a gyrokinetic equation good to third order in the gyroradius expansion, an alternate hybrid gyrokinetic-fluid treatment is formulated that employs moments of the full Fokker-Planck equation to remove the need for a very high order gyrokinetic distribution function. The description is an extension to gyrokinetics of drift kinetic treatments that yield expressions for the ion perpendicular viscosity as well as for the electron and ion parallel viscosities, gyroviscosities, and heat fluxes for arbitrary mean-free path plasmas, in which the lowest order distribution function is a Maxwellian [11].

An electrostatic hybrid gyrokinetic-fluid treatment using moments of the full Fokker-Planck equation removes the need to go to higher order. This hybrid description evolves electrostatic potential, plasma density, ion and electron temperatures, and ion and electron flows using conservation of charge, number, ion and electron energy, and total and electron momentum, respectively [12]. All electrostatic effects with wavelengths much longer than an electron gyroradius are retained so that ion temperature gradient (ITG) and trapped electron mode (TEM) turbulence and the associated zonal flow as well as all neoclassical behavior are treated. Closure for the electrons is obtained by solving the electron drift kinetic equation to find the leading order correction to the Maxwellian electrons f_{0e} needed to evaluate the parallel electron viscosity (or pressure anisotropy) as well as the momentum and energy exchange terms with the ions. In addition, the $\vec{v}v^2/2$ moment of the exact electron Fokker-Planck equation is used, along with this first order correction to f_{0e}, to evaluate the electron heat flux (collisional plus diamagnetic), thereby achieving closure for the electrons. Ion closure is achieved similarly by solving the ion gyrokinetic equation to leading order in ρ_i/L. However, ion closure is somewhat more complicated because the $\vec{v}\vec{v}$ as well as the $\vec{v}v^2/2$ moment of the ion Fokker-Planck equation must be used to evaluate the ion gyroviscosity and perpendicular viscosity, along with the ion heat flux. Moreover, to recover the correct results in the axisymmetric, long wavelength limit, the gyrokinetic variables must be determined to one order higher than normal [1,6]. Once this is done complete closure is obtained and a description valid on transport time scales is recovered that properly evolves the electrostatic potential and flows, as well as density and temperatures.

In this hybrid description distribution functions are only used to evaluate moments needed for closure and collisional exchange [12] - they are not used to evaluate density, temperature and flows. The results are given in terms of a few velocity space integrals of the gyrokinetic distribution function and make possible a hybrid fluid-gyrokinetic description that includes the neoclassical radial electric field as well as long wavelength turbulence and zonal flow effects. Moment equations evolve all other quantities such as density, temperatures, flows, and potential. As a result, either PIC or continuum, lowest order gyrokinetic and drift kinetic solutions, may be employed, and the kinetic equations need not be solved in conservative form. In addition, the flux surface average of conservation of toroidal angular momentum contains axisymmetric radial electric field terms from both the Reynolds stress and the collisional perpendicular ion viscosity whose respective coefficients compare as

$$\left\langle \frac{\tilde{n}}{n} \frac{\partial}{\partial \zeta} \left(\frac{e\tilde{\Phi}}{T_i} \right) \right\rangle_{\psi} \quad \text{vs} \quad \frac{q^2 R}{L_{\perp}} \frac{\nu_{ii}}{\Omega_i} \,, \tag{5}$$

with tildes denoting fluctuating quantities, $L_{\perp}$ the local perpendicular scale length, R the major radius, and q the safety factor. For $\tilde{n}/n \sim e\tilde{\Phi}/T_i \sim 10^{-2}$ with 0.1 de-phasing, both quantities are of order 10^{-5}, for $L_{\perp} \sim q^2\rho_{pi}$ and ITER like numbers of B = 5.3 T, T_i = 8 keV, n = 10^{19} m^{-3}, and R = 6 m. Consequently, even though momentum relaxation is expected to be anomalous, the axisymmetric steady state radial electric field may be determined by a competition between the turbulent Reynolds stress and collisional perpendicular ion viscosity for some parameters.

5. Discussion

In an axisymmetric, single ion species tokamak, intrinsic ambipolarity requires the distribution functions, and heat and particle fluxes be independent of electrostatic potential to leading order in gyroradius. Moreover, a moment description can be used to demonstrate that intrinsic ambipolarity must be satisfied to second order in gyroradius. We find that standard gyrokinetics incorrectly determines the axisymmetric, long wavelength electrostatic potential to leading order in gyroradius over major radius by considering a steady-state theta pinch with a distribution function correct to second order [1]. A similar problem arises in tokamaks. In both cases the correct radial electric field is determined from toroidal angular momentum conservation.

Using canonical angular momentum as the radial variable allows strong gradients to be treated gyrokinetically. Entropy production then requires a physical lowest order banana regime ion distribution function to be nearly an isothermal Maxwellian with the ion temperature scale much greater than the poloidal ion gyroradius [6]. Thus, the background ion temperature profile must have a pedestal with a scale much larger than that of any density pedestal with an ion poloidal gyroradius scale. In addition, weak ion temperature variation with subsonic flow in such a pedestal requires electrostatically restrained ions and magnetically confined electrons thereby impacting zonal flow behavior.

Simulating tokamaks on transport time scales requires evolving drift turbulence with axisymmetric neoclassical and zonal flow radial electric field effects retained. Full electric field effects are more difficult to retain than density and temperature effects since they require evaluating the ion distribution function to higher order than standard gyrokinetics. An electrostatic hybrid gyrokinetic-fluid treatment using moments of the full Fokker-Planck equation removes the need to go to higher order. This hybrid description evolves potential, density, temperatures, and flows, and models all electrostatic turbulence effects with wavelengths much longer than an electron gyroradius.

Acknowledgements: Work supported by U. S. Department of Energy grants at DE-FG02-91ER-54109 at MIT and DE-AC52-06NA-25396 at LANL, and the Center for Multiscale Plasma Dynamics at the University of Maryland.

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