Quasilinear Transport Fluxes Driven by Electrostatic Microinstabilities in Tokamaks

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Abstract Quasilinear transport fluxes driven by microinstabilites in weakly-collisional tokamak plasmas are calculated by a semi-analytical approach based on a solution of the gyrokinetic equation, where collisions are modelled by a Lorentz operator. Scalings with collisionality, magnetic drift frequency, diamagnetic frequency and ratio of the density and temperature scale lengths have been determined. The particle transport due to ion temperature gradient (ITG) modes can be reversed from inward to outward as electron collisions are introduced, if the plasma is far from marginal stability. However, if the plasma is close to marginal stability, collisions may even enhance the inward transport. Comparison with transport fluxes calculated when collisions are modelled by an energy-dependent Krook operator shows that the sign and the magnitude of the fluxes are very sensitive to the form of the collision operator.

1. Introduction

Turbulent transport in tokamak plasmas is considered to be mainly caused by drift waves destabilized by trapped electrons and ion temperature gradients. These microinstabilities and their effect on the transport can be studied by complex nonlinear gyrokinetic codes, for example GYRO [1]. To ease the interpretation of the results of these codes and experimental results it is useful to construct simpler models that can, after careful benchmarking with codes, give various parametric scalings. In particular, the collisionality dependence of the microinstabilities is interesting from both experimental and theoretical point of view. On the experimental side, the evolution of the density profile has been shown to depend on the collisionality. On the theoretical side, it has been shown that the transport fluxes are dependent on the choice of the collision operator.

Numerical simulations of ion temperature gradient (ITG) and trapped electron (TE) modes have shown that collisions may influence the sign and magnitude of the quasilinear fluxes driven by these instabilities [2]. In the collisionless limit, numerical simulations of ITG-mode driven turbulence give an inward particle flux, both in fluid, gyrofluid and gyrokinetic descriptions. However, nonlinear gyrokinetic calculations show that even a small value of the collisionality affects strongly the magnitude and sign of the anomalous particle flows. The inward particle flow obtained in the collisionless limit is rapidly converted to outward flow as electron-ion collisions are introduced. The particle flux is expected to change sign for very small collisionalities, much smaller than the collisionality achievable in current tokamak experiments. The choice of the model collision operator affects the collisionality threshold for the reversal of the particle flux [3]. This means that collisionless models or models using the Krook model operator are not adequate to calculate the quasilinear transport fluxes for typical experimental parameters. Also for TE-modes, the quasilinear fluxes depend on collisions, and the effect of collisions is different if they are modeled by the Lorentz operator than if they are modeled with the Krook operator.

The aim of the present work is to develop a collisional model for electrostatic turbulence that makes it possible to derive analytical expressions for the quasilinear flux to show explicitly the dependence on collisionality, density and temperature gradients, so that the sign and magnitude of the flux can easily be estimated. We give approximate analytical expressions for weakly collisional plasmas with large aspect ratio and circular cross section. The quasilinear fluxes driven by microinstabilities has previously been studied in [4] by approximating the collision operator with an energy-dependent Krook-operator. Here we use a Lorentz operator, but include the results for the Krook operator for comparison and completeness. The collisionality dependence of the quasilinear flux due to the TE-instability has been studied in [5], using a Lorentz collision operator, and here we generalize the expression presented there by including the magnetic drift unperturbatively.

2. Perturbed density response

The perturbed electron and ion responses are obtained from the linearized gyrokinetic (GK) equation, [4]

$$\frac{v_\parallel}{qR}\frac{\partial g_a}{\partial \theta} - i(\omega - \omega_{Da})g_a - C_a(g_a) = -i\frac{e_a f_{a0}}{T_a}\left(\omega - \omega_{*a}^T\right)\phi J_0(z_a), \tag{1}$$

where θ is the extended poloidal angle, ϕ is the perturbed electrostatic potential, $f_{a0} = n_a/\left(\sqrt{\pi}v_{Ta}\right)^3 \exp\left(-x_a^2\right)$ is the equilibrium Maxwellian distribution function, $x_a = v_a/v_{Ta}$ is the velocity normalized to the thermal speed $v_{Ta} = (2T_a/m_a)^{1/2}$, n_a, T_a and e_a are the density, temperature and charge of species a, $\omega_{*a} = -k_\theta T_a/e_a B L_{na}$ is the diamagnetic frequency, $\omega_{*a}^T = \omega_{*a}\left[1 + \left(x_a^2 - 3/2\right)\eta_a\right]$, $\eta_a = L_{na}/L_{Ta}$, $L_{na} = -[\partial(\ln n_a)/\partial r]^{-1}$, $L_{Ta} = -[\partial(\ln T_a)/\partial r]^{-1}$, are the density and temperature scale lengths, k_θ is the poloidal wavenumber, $\omega_{Da} = -k_\theta\left(v_\perp^2/2 + v_\parallel^2\right)(\cos\theta + s\theta\sin\theta)/\omega_{ca}R$ is the magnetic drift frequency, $\omega_{ca} = e_a B/m_a$ is the cyclotron frequency, q is the safety factor, $s = (r/q)(dq/dr)$ is the magnetic shear, r and R are the minor and major radii, J_0 is the Bessel function of order zero and $z_a = k_\perp v_\perp/\omega_{ca}$. We consider an axisymmetric, large aspect ratio torus with circular magnetic surfaces. We use the usual ordering for the relation of the electron/ion bounce frequencies and the eigenfrequency of the mode $\omega_{bi} \ll \omega \ll \omega_{be}$ and we consider weakly-collisional plasmas so that $\nu_{\star e} = \nu_e/\epsilon\omega_{be} \ll 1$, where ν_e is the electron-ion collision frequency and $\epsilon = r/R$ is the inverse aspect ratio. The perturbed electrostatic potential is approximated by $\phi(\theta) = \phi_0(1+\cos\theta)/2\left[H(\theta+\pi) - H(\theta-\pi)\right]$, where H is the Heaviside function and then $\langle\phi\rangle = \phi_0 E(\kappa)/K(\kappa)$. This approximation for the perturbed electrostatic potential breaks down for low shear or near marginal instability but as we will show, the qualitative features of the transport are captured by our calculations, although for quantitatively accurate results one of course has to resort to numerical simulations.

The ion self-collisions and ion-electron collisions are neglected, while the electron-ion collisions are modeled by a Lorentz operator $C_e = \nu_e(v)\frac{2\xi}{B}\frac{\partial}{\partial\lambda}\xi\lambda\frac{\partial}{\partial\lambda} \equiv \nu_e(v)\mathcal{L}$, where $\nu_e(v) = \nu_T/x_e^3$, $\xi = v_\parallel/v$ and $\lambda = 2\mu/(m_a v^2)$ with $\mu = m_a v_\perp^2/2B$. The circulating electrons are assumed to be adiabatic. The non-adiabatic part of the trapped electron distribution can be expanded $g_e = g_{e0} + g_{e1} + ...$ in the smallness of ω/ω_{be} and the normalized collisionality $\nu_{\star e}$, which gives $\partial g_{e0}/\partial\theta = 0$ in lowest order. The orbit averaged GK equation gives the constraint for g_{e0} as $i(\omega - \langle\omega_{De}\rangle)g_{e0} + \langle C_e(g_{e0})\rangle = (ie\langle\phi\rangle/T_e)(\omega_{*e}^T - \omega)f_{e0}$. Using WKB-analysis to solve the homogeneous equation and then the method of variation of parameters to determine the solution of the inhomogeneous equation it is possible to construct an approximate solution to the orbit averaged GK equation and the perturbed electron response is proportional to [3]

$$\left\langle \int g_{e0} d^3v \right\rangle = -\frac{4i\pi\sqrt{2\epsilon}}{\omega_0}\int_0^\infty v^2 dv \hat{S}\left(\frac{1}{u} - \frac{\sqrt{\hat{\nu}}}{u^{3/2}}\right), \tag{2}$$

where $\hat{S} = -(e\phi_0/T_e)\left(\omega - \omega_{*e}^T\right) f_{e0}$, $\hat{\nu} \equiv \nu_e/\omega_0\epsilon$, $u = -i(2y - \hat{\omega}_D)$, $\omega_0 = \omega/y$, $y = \sigma + i\hat{\gamma}$, $\sigma = \text{sign}(\Re\{\omega\})$, $\hat{\omega}_D = \omega_{D0}/\omega_0$ with $\omega_{D0} = -k_\theta v^2/\omega_{ce}R$. The above analysis is not valid close to the trapped-passing boundary. A boundary layer analysis in [3] has shown that the effect of the boundary layer reduces the collisional term, with a factor $\pi/2\sqrt{\log\hat{\nu}^{-1/2}}$. The reduction is less than 20% in the experimentally relevant collisionality regime, and in the following analysis this will be neglected.

The velocity integral in (2) can be evaluated in terms of ${}_2F_0$ generalized hypergeometric functions and the perturbed electron density response becomes

$$\begin{aligned}\frac{\hat{n}_e}{n_e}/\frac{e\phi_0}{2T_e} = 1 - \sqrt{2\epsilon}&\left[\hat{\omega}_{\eta*e} - \frac{3}{2}\left(\frac{\eta_e\hat{\omega}_{*e}}{y} - \frac{\hat{\omega}_{Dt}}{2y}\hat{\omega}_{\eta*e}\right)\mathcal{F}^1_{5/2}\left(\frac{\hat{\omega}_{Dt}}{2}\right)\right] \\ &+\frac{\Gamma(\frac{3}{4})\sqrt{\epsilon\hat{\nu}_t}}{\sqrt{-i\pi y}}\left[2\hat{\omega}_{\eta*e}\mathcal{F}^{3/2}_{3/4}\left(\frac{\hat{\omega}_{Dt}}{2}\right) - \frac{3\eta\hat{\omega}_{*e}}{2y}\mathcal{F}^{3/2}_{7/4}\left(\frac{\hat{\omega}_{Dt}}{2}\right)\right],\end{aligned} \tag{3}$$

where $\mathcal{F}^a_b(z) = {}_2F_0\left(a, b; ; z/y\right)$, $\hat{\omega}_{Dt} = \omega_{D0}/(\omega_0 x_e^2)$, $\hat{\nu}_t = \hat{\nu}x^3$, $\hat{\omega}_{*e} = \omega_{*e}/\omega_0$ and $\hat{\omega}_{\eta*a} = 1 - (1 - 3\eta_a/2)\hat{\omega}_{*a}/y$. Note that the expression for the perturbed electron density in (4) is exact in ω_{De}, no approximation regarding the relative magnitude of ω_{De} and ω has been made. In the limit of low normalized magnetic drift frequencies, expanding (4) around $\hat{\omega}_{Dt} = 0$ and keeping only the first order terms, the perturbed electron density reduces to the following expression (in agreement with Eq. (17) of [3])

$$\begin{aligned}\frac{\hat{n}_e}{n_e}/\frac{e\phi_0}{2T_e} = 1 - \sqrt{2\epsilon}&\left\{\left(1 - \frac{\hat{\omega}_{*e}}{y}\right) + \frac{3\hat{\omega}_{Dt}}{4y}\left[1 - (1+\eta_e)\frac{\hat{\omega}_{*e}}{y}\right]\right\} \\ +\frac{\Gamma(\frac{3}{4})i\sqrt{-iy\epsilon\hat{\nu}_t}}{\sqrt{\pi}y}&\left\{1 - \left(1 - \frac{3\eta_e}{4}\right)\frac{\hat{\omega}_{*e}}{y} + \frac{9\hat{\omega}_{Dt}}{16y}\left[1 - \left(1 + \frac{\eta_e}{4}\right)\frac{\hat{\omega}_{*e}}{y}\right]\right\}.\end{aligned} \tag{4}$$

For the ions we neglect the parallel dynamics, by assuming $k_\parallel v_{Ti} \ll \omega$. In this limit (1) can be solved by neglecting the parallel derivative and replacing ω_{Di} with its flux-surface averaged value, so the perturbed ion response becomes

$$\frac{\hat{n}_i}{n_i} = \frac{e\phi_0}{2T_i}\left[-1 + \int d^3v \frac{f_{i0}J_0^2(z_i)\left(\omega - \omega_{*i}^T\right)}{\omega - \hat{\omega}_{Ds}}\right] \tag{5}$$

where $\hat{\omega}_{Ds} = (2+3s)\omega_{Di0}/4\omega_0$, and $\omega_{Di0} = -2k_\theta v_{Ti}^2/3\omega_{ci}R$ and we used the constant energy resonance approximation for the ion resonance $[v_\perp^2 + 2v_\parallel^2 \rightarrow 4(v_\perp^2 + v_\parallel^2)/3]$ [4]. In order to make further progress analytically, we restrict our analysis to long wavelength perturbations and keep only the linear terms in $b = (k_\theta \rho_s)^2$, with $\rho_s = c_s/\omega_{ci}$ the ion Larmor radius and $c_s = \sqrt{T_e/m_i}$ is the ion sound speed. This approximation is valid for e.g. the fastest growing ITG modes ($k_\theta \rho_s \sim 0.3$). Then the perturbed ion response becomes

$$\frac{\hat{n}_i}{n_i} / \frac{e\phi_0}{2T_i} = -\frac{\hat{\omega}_{*i}}{y} + \left(\frac{3\hat{\omega}_{Ds}}{2y} - b\right)\left[\hat{\omega}_{\eta * i} - \frac{5}{2}\left(\frac{\eta_i \hat{\omega}_{*i}}{y} - \frac{\hat{\omega}_{Ds}}{y}\hat{\omega}_{\eta * i}\right)\mathcal{F}^1_{7/2}\left(\hat{\omega}_{Ds}\right)\right]. \quad (6)$$

Similarly to (4) it is instructive to give a simple expression of the ion response for small normalized magnetic drift frequencies. Expanding in $\hat{\omega}_{Ds}$ we obtain

$$\frac{\hat{n}_i}{n_i} / \frac{e\phi_0}{2T_i} = -b - [1 - b(1+\eta_i)]\frac{\hat{\omega}_{*i}}{y} + \left\{(3-5b) - [3(1+\eta_i) - 5b(1+2\eta_i)]\frac{\hat{\omega}_{*i}}{y}\right\}\frac{\hat{\omega}_{Ds}}{2y}. \quad (7)$$

The dispersion relation follows from the quasi-neutrality condition $\hat{n}_i = \hat{n}_e$ where the perturbed electron and ion densities are given by (4) and (6), respectively.

Figure 1 shows the growth rate and real frequency of the ITG and TE-modes as functions of normalized collisionality for different FLR-parameters and L_n/R. Clearly the effect of collisionality is not very large for ITG, but considerable for TE for very small collisionalities. For ITG the growth rates are sensitive to the FLR-parameter b, but less sensitive to L_n/R. The real frequencies are almost independent on the FLR-parameter, but they are sensitive to L_n/R. For TE, both the growth rates and eigenfrequencies are insensitive to the FLR-parameter but they are sensitive to L_n/R.

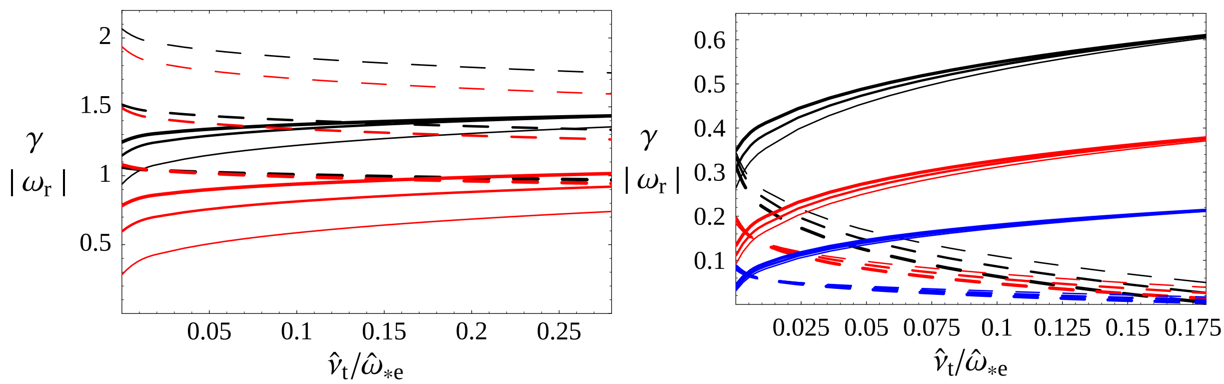

Figure 1: *Growth rate and real frequency of the ITG (left) and TE-mode (right). The solid lines are the real frequencies of the mode and the dashed lines are the growth rate, both normalized to ω_{*e}. The parameters are: $\eta_e = 3$, $s = 1$, $q = 2$, $\epsilon = 1/6$, $L_n/R = 1/3$. The FLR-parameter b increases from the thin to the thick lines: $b = 0, 0.1, 0.2$. The red lines are for $L_n/R = 1/5$ and the blue lines are for $L_n/R = 1/10$.*

3. Quasilinear fluxes

The collisionality dependence of the quasilinear fluxes has been studied previously in [3], but without solving the dispersion relation, and neglecting the effect of the collisionality on the growth rates and eigenfrequencies of the modes. Here we study the collisionality dependence of the transport fluxes including the effect of collisions on the eigenfrequency and we benchmark the results with linear calculations with GYRO. Since the eigenfrequency of the ITG-mode is insensitive to the collisionality, the quasilinear particle flux driven by ITG is almost identical to the one calculated in [3]. In contrast, the particle flux driven by TE is different, and neglecting the collisionality dependence of the eigenfrequency is not appropriate in weakly collisional plasmas. The quasilinear particle flux is ambipolar and is given by

$$\Gamma_e = \frac{k_\theta p_e}{2eB}\left|\frac{e\phi_0}{T_e}\right|^2 \Im\left(\frac{\hat{n}_e/n_e}{e\phi_0/T_e}\right). \tag{8}$$

ITG It is instructive to expand (4) for small $\hat{\gamma}$, and show explicitly the sign of the different terms in the expression for the flux. If $y = \omega/\omega_0 = -1 + i\hat{\gamma}$, then to lowest order in $\hat{\gamma}$ we have

$$\begin{aligned}\Gamma_e^{\rm ITG} = \frac{k_\theta p_e}{2eB}\left|\frac{e\phi_0}{T_e}\right|^2 \Bigg\{ & \sqrt{\frac{\epsilon}{2}}\left[1 - \frac{3\hat{\omega}_{Dt}}{2}(1+\eta_e)\right]\hat{\gamma}\hat{\omega}_{*e} - \sqrt{\frac{\epsilon}{2}}\frac{3\hat{\omega}_{Dt}\hat{\gamma}}{4} \\ & - \frac{\Gamma(3/4)\sqrt{\epsilon\hat{\nu}_t}}{\sqrt{2\pi}}\Bigg\{-1 + \frac{\hat{\gamma}}{2} + \left(\frac{3}{4}\eta_e - 1\right)\left(1 - \frac{3\hat{\gamma}}{2}\right)\hat{\omega}_{*e} \\ & + \frac{9\hat{\omega}_{Dt}}{16}\left[1 - \frac{3\hat{\gamma}}{2} + \left(1 + \frac{\eta_e}{4}\right)\left(1 - \frac{5\hat{\gamma}}{2}\right)\hat{\omega}_{*e}\right]\Bigg\}\Bigg\}.\end{aligned}$$

If the plasma is close to marginal instability, $\hat{\gamma} \simeq 0$, collisions (represented by the term proportional to $\sqrt{\hat{\nu}_t}$) lead to an inward flux if $\eta_e > 4(16(1+\hat{\omega}_{*e}) - 9\hat{\omega}_{Dt}(1+\hat{\omega}_{*e}))/[3(16 + 3\hat{\omega}_{Dt})\hat{\omega}_{*e}]$. For typical experimental parameters the inequality for η_e given above is expected to be satisfied and therefore the total flux is expected to be inwards. However, if the plasma is further away from marginal instability, so that $\hat{\gamma} > 2/3$, the terms to $1 - 3\hat{\gamma}/2$ and $1 - 5\hat{\gamma}/2$ change sign, and then collisions will lead to an outward flux.

Figure 2ab show the quasilinear electron flux calculated from the unexpanded solution (valid even $\hat{\gamma} \simeq 1$) normalized to $p_e k_\theta/(2eB)|e\phi_0/T_e|^2\sqrt{\epsilon}$ as function of normalized collisionality $\hat{\nu}_t = \nu_e x^3/\omega_0\epsilon$ for various values of $\hat{\omega}_{Dt}$ and $\eta_e = 3$. In these figures the collisionality dependence of the eigenfrequency and growth rate is neglected. Figure 2a is for a case where the plasma is far from marginal stability: $\hat{\gamma} = 0.7$. In the absence of collisions, the flux is inwards if the curvature and thermodiffusive fluxes (the terms proportional to $\hat{\omega}_{Dt}$ and η_e in the first row of (4)) dominate over diffusion. If collisions are included, the particle flux may be reversed, if the part of the flux that is dependent on the collisionality is positive. This reversal happens for instance for $\hat{\omega}_{Dt} = 0.6$, see Fig. 2a. However, if the ITG-instability growth rate is weak ($\hat{\gamma} \ll 1$) and η_e is large, the situation is completely different. Figure 2b shows the normalized quasilinear electron flux for the same parameters as in Fig. 2a, but for $\hat{\gamma} = 0.1$, representing a case close to marginal stability. The term proportional to the $\sqrt{\hat{\nu}_t}$ will change sign and now this will also lead to an inward flux. If the magnetic drift is high enough to give an inward flux for zero collisionality, then collisions will enhance this and the flux will therefore never be reversed. If the magnetic drift is very small, the flux is outwards for $\hat{\nu}_t = 0$. Then collisions may reverse the sign of the flux, but now from outwards to inwards.

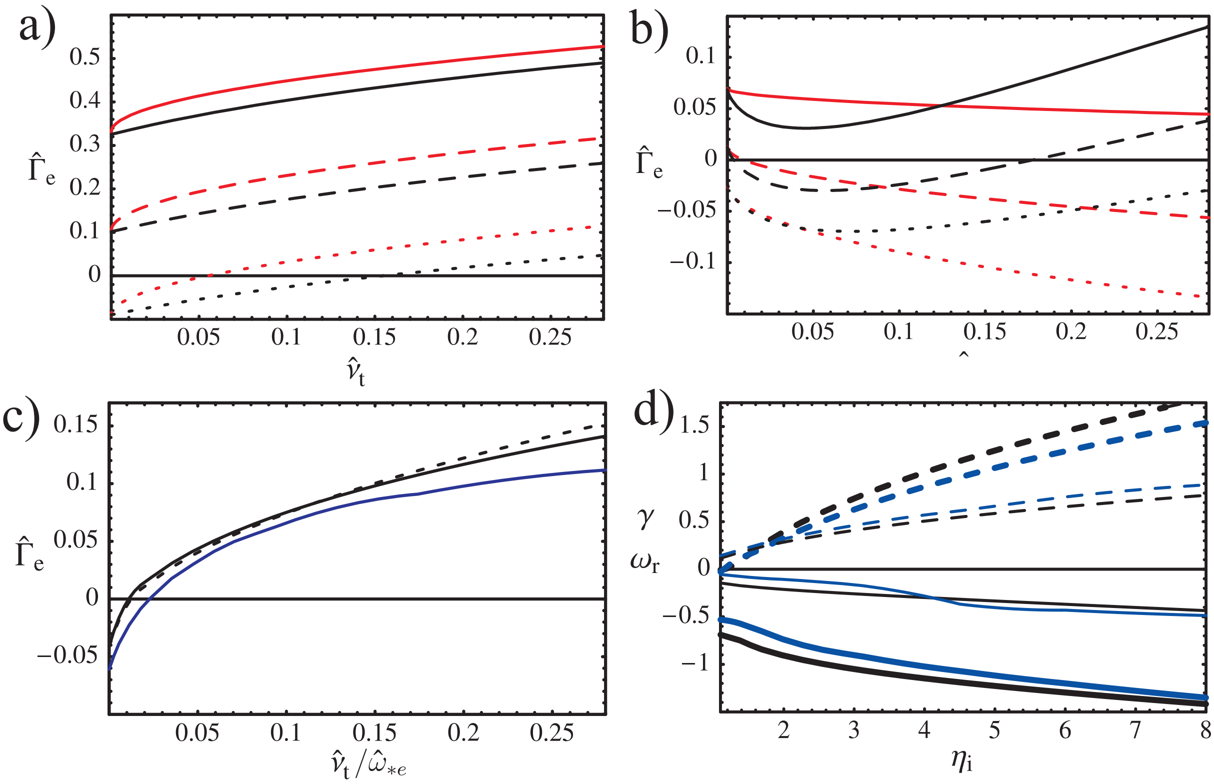

Figure 2: *Quasilinear electron flux (normalized to $k_\theta p_e/(2eB)|e\phi_0/T_e|^2\sqrt{\epsilon}$) as function of normalized collisionality. [a, b] $\hat{\omega}_{*e} = 1$, $\eta_e = 3$, red: Lorentz operator, black: Krook operator. $\hat{\omega}_{Dt}$ is 0 (solid), 0.2 (dashed), 0.6 (dotted). [a] $\hat{\gamma} = 0.7$ [b], $\hat{\gamma} = 0.1$. [c] Quasilinear electron flux for $\omega(\hat{\nu}_t)$ (black solid), $\omega = \mathrm{const.}$ (dotted) compared with the linear GYRO result without parallel dynamics (blue). The other parameters are $\eta_e = 3$, $s = 1$, $q = 2$, $\epsilon = 1/6$, $L_n/R = 1/3$. [d] The eigenfrequency $|\Re(\omega)|$ (solid) and growth rate $\Im(\omega)$ (dotted), both normalized to ω_{*e} as functions of η_i for the parameters given in [c] and for different $\tau = T_e/T_i$ (thin $\tau = 5$, thick $\tau = 1$); present model (black), GYRO (blue).*

Figure 2c shows the normalized quasilinear flux driven by ITG as a function of collisionality together with the linear GYRO result (blue line). The dotted line is the case when the collisionality dependence of the eigenfrequency and growth rate is neglected, and it is clear that the agreement between the three curves is very good. Figure 2d shows the growth rate and real frequency of the ITG-mode as function of η_i in the collisionless case for different values of τ, compared with results of linear GYRO simulations. The blue lines are the results of linear GYRO simulations. The results of the semi-analytical model reproduce well the numerical simulations and the effect of τ decreases the growth rates and frequencies.

TE The real part of the eigenfrequency is positive, and this means that $y = \omega/\omega_0 = 1+i\hat{\gamma}$ and the electron flux to lowest order is

$$\Gamma_e^{\text{TEM}} = \frac{k_\theta \rho_e}{2eB}\left|\frac{e\phi_0}{T_e}\right|^2 \left\{ \sqrt{\frac{\epsilon}{2}}\left[1+\frac{3\hat{\omega}_{Dt}}{2}(1+\eta_e)\right]\hat{\gamma}\hat{\omega}_{*e} - \sqrt{\frac{\epsilon}{2}}\frac{3\hat{\omega}_{Dt}\hat{\gamma}}{4} - \frac{\Gamma(3/4)\sqrt{\epsilon\hat{\nu}_t}}{\sqrt{2\pi}}\left\{1-\frac{\hat{\gamma}}{2}+\left(\frac{3}{4}\eta_e-1\right)\left(1-\frac{3\hat{\gamma}}{2}\right)\hat{\omega}_{*e} + \frac{9\hat{\omega}_{Dt}}{16}\left[1-\left(1+\frac{\eta_e}{4}\right)\left(1-\frac{5\hat{\gamma}}{2}\right)\hat{\omega}_{*e}\right]\right\}\right\}.$$

There are two main differences compared with the ITG driven flux. First, the part of the flux that is driven by the curvature has opposite sign compared with ITG, and therefore contributes to the outward flux instead of driving an inward pinch. Second, the part of the flux that arises due to collisions is different and may have opposite sign compared with the ITG case, depending on the parameters.

Collisions modeled by a Krook operator Starting from the gyrokinetic equation for the electrons but modeling the collision operator with an energy-dependent Krook operator we have $i(\omega - \langle\omega_{De}\rangle)g_{e0} - \nu_{\text{eff}}g_{e0} = -(e\phi_0/T_e)(\omega - \omega_{*e}^T)f_{e0}$ so that $g_{e0} = -(e\phi_0/T_e)(\omega - \omega_{*e}^T)/(\omega - \langle\omega_{De}\rangle + i\nu_{\text{eff}})f_{e0}$, where $\nu_{\text{eff}} = \nu_T/\epsilon x^3$. The velocity-space integral of the perturbed electron distribution can be used to determine the imaginary part of the perturbed electron density, and that gives the quasilinear particle flux from (8). If the plasma is far from marginal stability, the results for the Lorentz and Krook operator are qualitatively same, as shown in Fig. 2a. However, as Fig. 2b shows, as we approach marginal stability, the form of the collision operator matters more and more, and both the sign and the magnitude of the flux may be very different.

Figure 3 shows the threshold in collisionality for which the flux reverses for different values of the normalized magnetic drift frequency. The red curves correspond to the Lorentz model-operator and the black curves correspond to the Krook-model. The different lines correspond different values of $\hat{\omega}_{Dt}$. It is interesting to see that the Lorentz operator gives lower threshold for flux reversal. Above the lines the transport is outwards and below it is inwards.

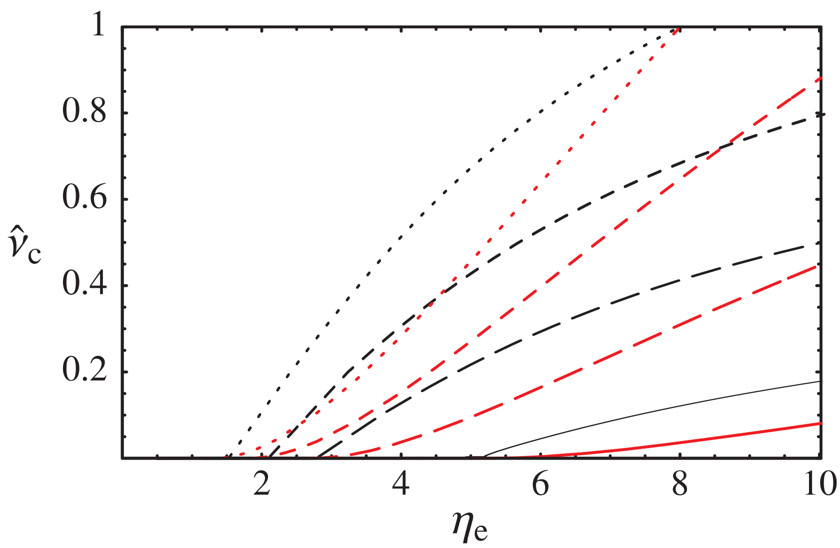

Figure 3: *Collisionality threshold as function of η_e for $\hat{\gamma} = 0.7$ and $\hat{\omega}_{*e} = 1$. From below $\hat{\omega}_{Dt}$ is* 0.2 *(solid),* 0.4 *(long-dashed),* 0.6 *(short-dashed),* 0.8 *(dotted).*

4. Conclusions

In this paper we presented a semi-analytical collisional model for electrostatic turbulence and the quasilinear transport fluxes driven by them. The semi-analytical model presented here includes electron-ion collisions modelled by a Lorentz operator and does not rely on expansion in the smallness of magnetic drift frequencies. By assuming large-aspect ratio, low-beta, toroidally symmetric, circular cross section and weakly-collisional plasmas, and assuming a ballooning eigenfunction for the electrostatic potential, analytical expressions can be derived for the ion and electron perturbed densities and the quasilinear fluxes. The model is semi-analytical because the roots of the dispersion relation are obtained numerically. The results agree well with linear gyrokinetic calculations with GYRO.

The collisionality dependence of the quasilinear particle flux due to microinstabilities has been studied and it has been shown that if the plasma is far from marginal stability, the inward transport due to ITG-modes is reversed as electron collisions are introduced, in agreement with nonlinear gyrokinetic simulations. However, if the plasma is close to marginal stability, collisions will lead to an additional inward flux, and therefore the total flux is expected to be inwards. The transport is therefore affected significantly by the parameter η_e, both directly via the terms proportional to η_e in the expression for the flux, but also indirectly, via the ITG growth rate that is important to determine the sign of the flux. If the electron collisions are modeled with a Lorentz collision operator, the particle flux is proportional to the square-root of the collisionality. The choice of the model collision operator affects the collisionality threshold for the reversal of the particle flux $\hat{\nu}_c$. This is especially important when the plasma is close to marginal stability. The collisionality threshold $\hat{\nu}_c$ depends on the magnitude of the normalized magnetic drift $\hat{\omega}_{Dt}$ and the ratio of density and temperature scale lengths, η_e. For higher η_e and higher $\hat{\omega}_{Dt}$, higher collisionality is needed to reverse the particle flux.

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