Nonlinear Dynamics of Impurities in Turbulent Tokamak Plasmas

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1 Introduction

Impurity behavior in tokamak plasmas is a complex problem related to confinement and transport of the bulk ions and electrons and to plasma-wall interaction. This is a very important issue for the development of fusion reactors. Besides of a strong theoretical and experimental effort, this topic is not completely understood. We discuss here several aspects of impurity transport in turbulent plasmas.

We have shown [1] that the gradient of the confining magnetic field generates a pinch (average velocity) in turbulent plasmas. It is a ratchet type process that appears in test particle approach due to the modification of guiding center trajectories. It determines the contamination of the plasma from the source of impurities localized at the border. Particle collisions and plasma poloidal rotation are included in the test particle model. We show that strong nonlinear effects appear when trajectory trapping or eddying is effective. An additional effect of the magnetic field gradient appears when particle density is considered: the divergence of the $\mathbf{E}\times\mathbf{B}$ drift produces a pinch velocity, the curvature or turbulent equipartition pinch [3], [4]. We show that the density pinch is not equal to the curvature pinch and that it is strongly influenced by the ratchet effect. Impurity accumulation (density peaking) is studied as function of the characteristics of the turbulence.

The evolution and the statistical characteristics of the impurity density passively advected by the drift turbulence modeled by the Hasegawa-Wakatami (HW) equation is numerically investigated. We have shown [9] that the impurity density and the vorticity of the $\mathbf{E}\times\mathbf{B}$ drift exhibit similar multifractal behavior. A good agreement is found between the impurity density relative exponent and the She-Leveque model, which shows that intense vortex filaments are responsible for impurity transport. The numerical simulation of the impurity density in HV turbulence is performed and the impurity pinch is analyzed.

2 Impurity pinch produced by the inhomogeneity of the magnetic field

We consider in slab geometry an electrostatic turbulence represented by an electrostatic potential $\phi^e(\mathbf{x},t)$, where $\mathbf{x} \equiv (x_1,x_2)$ are the Cartesian coordinates in the plane perpendicular to the confining magnetic field directed along z axis, $\mathbf{B} = B\mathbf{e}_z$. The magnetic field depends on the distance from the main symmetry axis as $B = B_0 \exp(-x_1/L_B)$, where B_0 is the value of the magnetic field in the origin of the coordinates that is at $\mathbf{x} = \mathbf{0}$ and L_B is its characteristic decay distance. The electrostatic potential is considered to be a stationary and homogeneous Gaussian stochastic field with known two-point Eulerian correlation function (the Fourier transform of the spectrum). We study the transport of test particles (section 2.1) and of passive fields (section 2.2) advected by such stochastic field.

The aim of this study is to determine the transport of impurities as function of the characteristics of the turbulence. It provides the transport coefficient scaling in different regimes and the understanding of the nonlinear effects. In particular, the conditions that correspond to impurity accumulation can be identified. A self-consistent numerical study of impurity dynamics is presented in section 3.

2.1 Test particle pinch

The test particle motion in the guiding center approximation is modeled by

$$\frac{d\mathbf{x}(t)}{dt} = -\exp(x_1/L_B)\nabla\phi(\mathbf{x},t) \times \mathbf{e}_z + \mathbf{V}_p + \eta(t), \tag{1}$$

where $\mathbf{x}(t)$ is the trajectory of the particle guiding center, ∇ is the gradient in the (x_1,x_2) plane, $\phi(\mathbf{x},t) = \phi^e(\mathbf{x},t)/B_0$, $\mathbf{V}_p$ is the average velocity and $\eta(t)$ is the collisional velocity. The turbulence is characterized by three parameters: the amplitude V of the stochastic $\mathbf{E}\times\mathbf{B}$ drift, the correlation time τ_c, which is the decay time of the Eulerian correlation and the correlation length λ_c, which is the characteristic decay distance, which combine in the Kubo number

$$K = \frac{\tau_c}{\tau_{fl}} = \frac{V\tau_c}{\lambda_c} \tag{2}$$

where $\tau_{fl} = \lambda_c/V$ is the time of flight of the particles over the correlation length. The shape of the Eulerian correlation does not influence the general behavior of the transport, but only the strength of the trapping effect [**?**]. The collisional velocity η is modeled by a zero average Gaussian white noise with the collisional diffusion coefficient $\chi = \rho^2\nu/2$, where $\rho = V_{th}/\Omega$ is the Larmor radius, V_{th} is the thermal velocity, $\Omega = qB/m$ is the cyclotron frequency and ν is the collision frequency. The collisional diffusion coefficient depends on space through the Larmor radius due to the magnetic field inhomogeneity as $\chi = \chi_0 \exp(2x_1/L_B)$, where χ_0 corresponds to the reference magnetic field B_0. The average velocity $\mathbf{V}_p$ is taken along x_2 axis that represents the poloidal direction. This velocity is the difference between particle average velocity and the poloidal rotation velocity of the stochastic potential. The collisions and the poloidal rotation introduce two dimensionless parameters

$$\overline{\chi_0} = \frac{\tau_{fl}}{\tau_{coll}} = \frac{\chi_0}{\Phi}, \quad \overline{V}_p = \frac{\tau_{fl}}{\tau_p} = \frac{V_p}{V} \tag{3}$$

where the collisional time $\tau_{coll} = \lambda_c^2/\chi_0$ is the time during which the collisional mean square displacement attains λ_c^2, $\tau_p = \lambda_c/V_p$ is the time of decorrelation by the average velocity and $V_p = |\mathbf{V}_p|$. One can note that Kubo number, $\overline{\chi_0}$ and $\overline{V}_p$ are similar in the sense that all describe physical effects (time variation of the potential, collisions and average velocity respectively) which perturb the motion along the potential contour lines. Small $\overline{\chi_0}$ and $\overline{V}_p$ and large K correspond to nonlinear regimes strongly influenced by the structure of the stochastic potential.

We use the decorrelation trajectory method [6], [7] for the calculation of the average velocity and of the diffusion coefficient for given Eulerian correlation of the potential and for arbitrary values of the parameters of this model (K, $\overline{\chi_0}$, $\overline{V}_p$ and $\overline{R} = L_B/\lambda_c$). This is a semi-analytical approach based on the study of the stochastic equation (1) in subensembles of realizations of the stochastic field, which are determined by given values of the potential and of the velocity in the starting point of the trajectories.

An average asymptotic velocity of the $\mathbf{E}\times\mathbf{B}$ stochastic drift (the ratchet effect) is obtained for $\overline{\chi_0}, \overline{V}_p = 0$ provided that there is a gradient of the magnetic field (finite $\overline{R}$) and time variation of the stochastic potential [1]. For a static potential the average velocity is transitory and it has a finite asymptotic value $\mathbf{V}^R$ only for finite correlation time (i.e. finite K) of the potential. This average velocity is along the gradient of the magnetic field (along x_1 axis) and is given by $V^R = (V/\overline{R})f(K)$ where $f(K)$ is a dimensionless function. This function is positive for small Kubo numbers (corresponding to V^R directed against the gradient of the magnetic field), at a value K_{inv} that is of the order 1, the ratchet velocity becomes negative (parallel with ∇B) and it decays to zero for $K \to \infty$. The absolute value of this function is represented in Fig. 1 by the blue line. The physical explanation for this behavior of the average velocity is the following. For fast variation of the stochastic field ($K < 1$), the displacements during the correlation time are much smaller than λ_c and they are along the initial velocities. The latter decrease in the direction of the gradient of the magnetic field ∇B producing displacements that are smaller than in the opposite direction. An average displacement appears in the direction $-\nabla B$ (positive x_1). At large Kubo numbers a part of particles are trapped and perform during τ_c almost periodic motion on the corresponding contour line of the potential. The space dependence of the magnetic field does not change the paths of the guiding centers in static potential, which are the contour lines of the

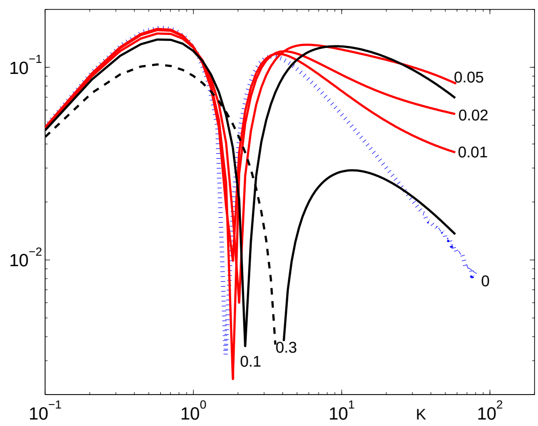

Figure 1: *The ratchet pinch normalized with $V/\overline{R}$ as function of the Kubo number for the values of $\overline{\chi_0}$ that label the curves*

potential, but it only influences the velocity along the paths. It is larger in the low magnetic field side and thus the particles stay there shorter time than in the high field side. Then, the average displacement is negative and corresponds to a pinch velocity in the direction of ∇B.

The influence of collisions on the motion in the stochastic potential is obtained as an effective Eulerian correlation [2]. The spreading of the trajectories due to collisions produces the decay of this function. Namely, the amplitude of the collision averaged potential decays in time with the factor $1+2\overline{\chi_0}t$ and the square of the correlation length increases with the same factor. This behavior is obtained also for constant magnetic field and determines an effective Kubo number that decays in time. The specific effect for inhomogeneous magnetic fields consists in the drift of the effective Eulerian correlation with the velocity $-3\overline{\chi_0}/\overline{R}$ along x_1 axis. This drift is produced by the correlation that appears between the inhomogeneity of the magnetic field and the stochastic potential through the collisional displacements that are inhomogeneous as well. This is a nontrivial nonlinear effect. Collisions also determine a direct contribution to the pinch due to the space dependence of the diffusivity χ induced by the magnetic field.

A very strong influence of collisions on the turbulent pinch is seen to appear from very small collisional diffusivity. The pinch velocity is much modified for $K>1$ (in the nonlinear regime of the turbulence), but very weakly influenced for $K<1$ (in the quasilinear regime). This regime is represented in Fig. 1 by the red lines, which correspond to very small collisional diffusivity ($\overline{\chi_0}=0.01$, and 0.05). These curves are superposed for $K<1$ and show significative increase of the pinch velocity at large K. At larger values of $\overline{\chi_0}$ the dependence is reversed (see the black lines in Fig. 1). But at these values of $\overline{\chi_0}$ the average velocity is collision dominated and the direct contribution of collisions is much larger than the turbulence effect.

The poloidal velocity determines the decrease of the pinch velocity [2]. The decrease appears only at large Kubo numbers if $\overline{V}_p < 1$ and for all values of K if $\overline{V}_p > 1$. The dependence on the Kubo number at large K is practically unchanged: it remains approximately as K^{-1}.

In the presence of poloidal rotation and collisions, the pinch velocity becomes a complicated function of the three parameters $\overline{\chi_0}$, $\overline{V}_p$ and K. In the weekly collisional nonlinear regime characterized by existence of trapped trajectories, the collisions produce the increase of the pinch velocity. They also can produce a second inversion of the sense of the average velocity at large values of K. The increase of the pinch velocity by collisions roughly compensates the decay due to $\overline{V}_p$ reaching at large K typical values that are of the order of those obtained in the unperturbed $\mathbf{E}\times\mathbf{B}$ drift $(\overline{\chi_0}, \overline{V}_p = 0)$.

There is however an important difference between the perturbed and unperturbed case. The ratio V^R/D is much larger than in the unperturbed case due to the strong decrease of the radial diffusion coefficient produced by the poloidal rotation. This ratio is the measure of the effect of the direct transport. The latter is dominant for large values of this parameters and leads to peaked probability profiles. The values of this parameter for the unperturbed $\mathbf{E}\times\mathbf{B}$ transport are small, of the order $1/2R$ for both quasilinear and nonlinear conditions. Much larger values are obtained in the nonlinear case for $K > 1$ in the presence of a weak poloidal rotation. Collisions can also contribute to the increase of the ratio of direct to diffusive transport but the main contribution comes from the poloidal rotation, which strongly increase this ratio by decreasing the radial diffusion coefficient. The weak collisionality regime $\overline{\chi_0} < 0.1$ roughly corresponds to the range of the normalized collision frequency that appear in the measurements of the density peaking factor in H mode plasmas presented in Ref. [8]. The values of the poloidal velocity corresponding to the nonlinear regime are $\lesssim 1000 m/\sec$.

2.2 Density pinch

Due to the space dependence of the magnetic field, the $\mathbf{E}\times\mathbf{B}$ velocity has non-zero divergence and the density is compressible

$$\partial_t n + \mathbf{v}\cdot\nabla n = n\frac{\mathbf{v}\cdot\nabla B}{B}. \tag{4}$$

An average velocity $\mathbf{V}_c$ is obtained

$$\mathbf{V}_c = D\nabla\ln(B) = -\frac{V^2\tau_c}{L_B}\mathbf{e}_1 \tag{5}$$

where $D = V^2\tau_c$ is the diffusion coefficient and $\mathbf{e}_1$ is the unit vector along x_1 axis. This velocity is called curvature or turbulence equipartition pinch and appears due to the compressibility effect produced by the inhomogeneity of the confining magnetic field (the right hand side term in Eq. (4)).

We derive the equation for the average density from the stochastic advection equation (4) using the characteristics method and taking into account the effect of the inhomogeneous magnetic field on particle trajectories. The average density is obtained as

$$\overline{n(\mathbf{x},t)} = \int d^2x' \, n_0(\mathbf{x}') \exp\left(-\frac{x_1 - x_1'}{L_B}\right) P(\mathbf{x}-\mathbf{x}',t) \tag{6}$$

where the probability of the displacements P is defined by $P(\mathbf{x}-\mathbf{x}',t) = \langle \delta \left[\mathbf{x}' - \mathbf{x}(0;\mathbf{x},t)\right] \rangle$ and n_0 is the initial density. The two effects of the gradient of the magnetic field appear in Eq. (6): the compressibility determines the exponential factor while the modifications of the trajectories should be reflected in the probability P.

A general expression for the density pinch velocity that applies for quasilinear and nonlinear turbulence was derived in [5]

$$V_n = V_R - \frac{D}{L_B} = V_R + V_c. \tag{7}$$

Thus, the density pinch velocity is the sum of the ratchet and curvature pinches. Eq. (7) also shows that the curvature pinch in the nonlinear turbulence has the same structure as in the quasi-linear case but contains the effect of trajectory trapping in the diffusion coefficient $D = D(K)$. The density pinch velocity for the quasilinear turbulence is $V_n = -V^2\tau_c/2L_B$. This velocity is different from both curvature and ratchet pinches. It is half of the curvature pinch (5) and it has the amplitude of the ratchet pinch (**??**) but opposite direction (parallel to the gradient of the magnetic field).

The effect of the pinch velocity V_n on the average density profile appears in the dimensionless parameter $p = aV_n/D$ (where a is the minor radius) rather than in the absolute values. This parameter, the peaking factor, is an estimation of the average density gradient determined by the equilibration of the advective and diffusive transport when the boundary fluxes are negligible. One obtains in these conditions $a/L_n \cong p$, where L_n is the characteristic length of the average density. The peaking factor produced by the density pinch (7) is $p = |aV_R/D - a/L_B|$. This shows that the curvature pinch (second term) contributes to the peaking factor with a constant (small) value $p_c = a/L_B \cong a/R$. Density peaking can be driven by the ratchet pinch. It is important to note that this effect appears only in nonlinear turbulence $(K > 1)$ in the presence of poloidal rotation.

3 Self-consistent numerical simulations

Numerical simulations of the Hasegawa-Wakatani (HW) turbulence were performed to understand the fundamental process of impurity pinch. The HW system, despite its underlying simplifying assumptions, contains the basic elements to investigate transport, including a large

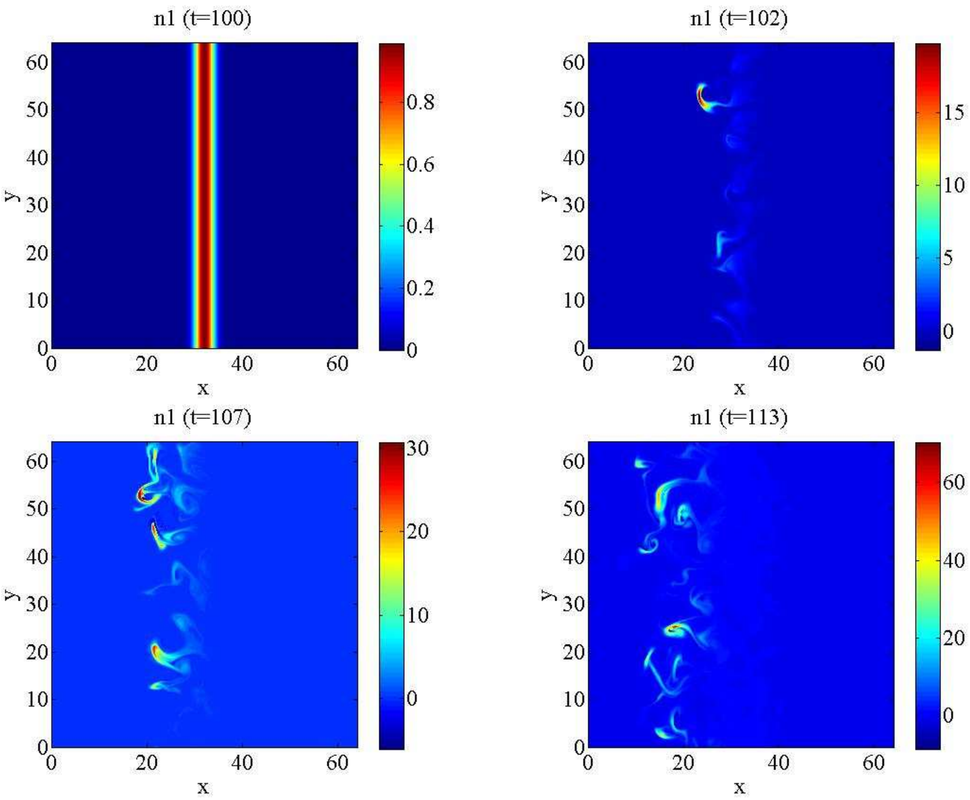

Figure 2: *Contour plots of the impurity density advected by the HW turbulence.*

spectrum of turbulent fluctuations and the spontaneous formation of coherent structures. It has been investigated by many authors in the last two decades [11, 12], In particular, HW system allows the study of cross-field transport by electrostatic drift waves. The phase difference between the potential and the density fluctuations is controlled by the parameter c. Depending on the value of this parameter two limits are distinguished. In the, $c \gg 1$ adiabatic limit, the electrons have a Boltzman distribution, there is no phase difference between n and ϕ, and the model reduces to the Hasegawa-Mima equation. On the other hand, in the $c \ll 1$ quasi-hydrodynamic limit, the system reduces to a two-dimensional Navier-Stokes equation describing the $\mathbf{E} \times \mathbf{B}$ flow, and a passive advection equation describing the density fluctuations.

Direct numerical simulations of HW system are performed for 512×512 grids with a square box size $L = 64$ with double periodic boundary conditions using a finite difference method for spatialy. The nonlinear terms are computed using a method developed by Arakawa [10]. The time stepping is performed using a predictor-corrector scheme. We have focused on the quasi-adiabatic regime obtained for $c = 0.7$, which is more relevant for the tokamak edge turbulence. Once the system reaches a well established saturated turbulent regime with stationary statistical properties, we inject Gaussian stripes as initial impurity puffs and then let them advected by the background turbulence according to Eq. (4).

We have determined the evolution of impurities that are initially localized in a narrow strip, and we have estimated from these results the time evolution of the average and the mean square displacement. A good agreement with the theoretical results was obtained.

4 Conclusions

Impurity accumulation (density peaking) can appear due to the gradient of the magnetic field only in the presence of trajectory trapping and of a slow poloidal rotation, with velocity of the order of $10^3 m/sec$ for JET plasmas. In these conditions, the presence of collisions determines a dependence of the peaking factor p that is similar to the JET H-mode database for the range of the effective collision frequency appearing there, and that p decays at weaker collisionality.

These studies strengthen the idea that the impurity transport in tokamak turbulent plasmas is a nonlinear process with characteristics far from the Gaussian ones, with intermittent behavior and memory effects.

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